NSW Department of Education

Numeracy guide

Years 3 to 8

A guide to support conversations about evidence-based practice for leadership teams

Literacy and numeracy

2024 update

Contents

This document is designed for online use.

Purpose of the resource

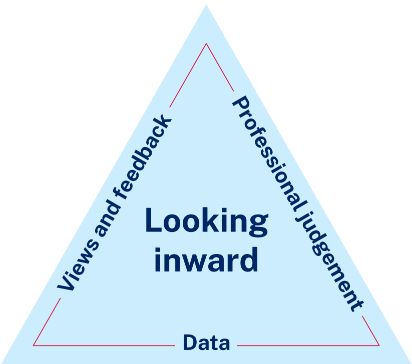
The purpose of this guide is to support directors, educational leadership, principals, school leadership teams and teachers to have informed conversations about evidence-based numeracy teaching across curriculum areas in primary and secondary school contexts.

This guide can:

- assist with an analysis of current practices
- help to inform planning for school improvement with numeracy
- suggest ways to build teacher capacity and understanding of numeracy with explicit classroom practices and professional learning resources.

Situational analysis

This guide can be used as part of the situational analysis in the following ways:



Looking inward includes analysis of data such as evidence of staff knowledge and perceptions around numeracy, and evidence of students' current skills in numeracy.

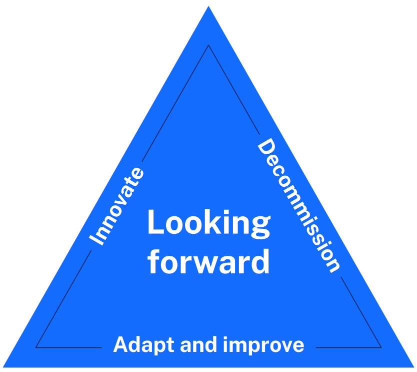
The guide should be used in conjunction with a thorough analysis of internal and external measures such as:

- School-based data
- Interview for Student Reasoning (IfSR)
- National Numeracy Learning Progression
- NAPLAN data
- Check-in assessment.

Looking outward includes comparing the school's approach on the teaching of numeracy to the research on effective teaching of numeracy.

This guide:

- explains the key aspects of numeracy
- describes evidence-based practices for effective teaching of numeracy
- describes the roles and responsibilities of all stakeholders.



Looking forward includes making decisions in response to information gained in the **Looking inward** and **Looking outward** phases.

This guide makes suggestions for ways to adapt and improve the teaching of numeracy across Years 3-8.

Introduction

This resource incorporates research on the connection between mathematics and numeracy, and the centrality of the idea of working mathematically drawn from the K-10 Mathematics syllabus. It also highlights evidence-based practices to inform teaching and learning in number and place value, patterns and algebra, additive thinking, multiplicative thinking and proportional thinking.

Numeracy and mathematics

Numeracy and mathematics are so intricately linked that it is difficult to define one without reference to the other. Whilst they are interdependent, the relationship between mathematics and numeracy changes as children progress through schooling.

Numeracy is considered as the confident application of mathematical skills, understanding and dispositions across areas of learning and within our daily lives. It involves recognising where mathematics can be used and being able to select the relevant mathematical tools and make sense of the solutions. As described by the NSW Mathematics K-10 syllabus (2022), 'By studying mathematics, students develop essential numeracy skills and fluency, while nurturing the ability to think logically, critically and creatively. They learn about patterns and reason about relationships, creating opportunities to generalise their solutions and to solve non-routine problems'. Mathematics provides the building blocks for the dispositions, understandings and skills needed for children to become numerate before extending out into developing deeper and richer understanding and more nuanced skills. The Every Student Podcast with Michelle Tregoning highlights the power and relevance of mathematics. Mathematics is necessary for numeracy, but numeracy is not all of mathematics. Both mathematics and numeracy are foundational for success in everyday life.

The distinction between numeracy and mathematics supports teachers to identify opportunities to assist students to become numerate as they develop the knowledge and skills to use mathematics confidently across learning areas at school and in their lives more broadly.

Working mathematically

Mathematics 3-6

Working mathematically through communicating, reasoning, understanding and fluency, and problem solving

Number and algebra	Stage 2	Representing numbers using place value	Additive relations	Multiplicative relations	Partitioned fractions
	Stage 3	Represents numbers	Additive relations	Multiplicative relations	Representing quantity fractions
Measurement and space	Stages 2 and 3	Geometric measure	2D spatial structure	3D spatial structure	Non-spatial measure
Statistics and probability	Stages 2 and 3			Data	Chance

Working mathematically in mathematics K-10

The working mathematically processes in the NSW Mathematics syllabus are:

- communicating
- understanding and fluency
- reasoning
- problem-solving.

Students learn to work mathematically by using these processes in an interconnected way. The coordinated development of these processes results in students becoming mathematically proficient.

When students are working mathematically it is important to help them to reflect on how they have used their thinking to solve problems. This assists students to develop mathematical habits of mind.[1]

Students need many experiences that require them to relate their knowledge to the vocabulary and conceptual frameworks of mathematics.

1 Cuoco A, Goldenberg EP and Mark J (2010) 'Contemporary Curriculum Issues: Organizing a Curriculum around Mathematical Habits of Mind', The Mathematics Teacher MT, 103(9):682-688, doi:10.5951/MT.103.9.0682.

Overarching working mathematically outcome

To highlight how these processes are interrelated, in Mathematics K-10 there is one overarching working mathematically outcome:

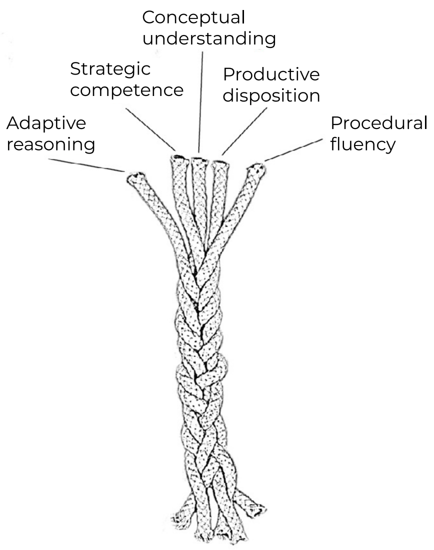
- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

The working mathematically outcome describes the thinking and doing of mathematics. In doing so, the outcome indicates the breadth of mathematical actions teachers need to emphasise.

In alignment with these Working Mathematically components, Kilpatrick, Swafford and Findell (2001) discuss the notion of 'mathematical proficiency' as a way to capture the necessary skills, knowledge and understandings a learner requires to learn mathematics successfully. They highlight 5 strands or components that are interwoven and interdependent in the development of proficiency in mathematics:

- conceptual understanding – comprehension of mathematical concepts, operations, and relations
- procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence – ability to formulate, represent, and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation, and justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Intertwined strands of proficiency



The Working Mathematically components and the five strands of mathematical proficiency are foregrounded in language use as students explore and develop their numeracy skills. Teachers need to be aware that students with differing English language proficiency may display different ways of working and thinking mathematically, and plan for explicit teaching for the language demands of numeracy.

For further information on the strands of mathematical proficiency see: Kilpatrick, Swafford and Findell (2001) Adding It Up: Helping Children learn mathematics, (Chapter 4: The strands of mathematical proficiency, p.116) National Research Council. 2001. Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press.

Learning across the curriculum (new K-10 Mathematics)

Numeracy

Numeracy involves understanding and applying mathematical knowledge and skills in a wide range of contexts. The application of mathematics across the curriculum enriches the study of other learning areas and helps to develop a broader and deeper understanding of numeracy.

A ‘numerate person’ (Hogan, 2000) uses a blend of:

- mathematical knowledge: concepts and skills within mathematics
- contextual knowledge: to recognise and link mathematics to broader situations
- strategic knowledge: to apply mathematics in situations and evaluate if the solution is reasonable.

Students are provided with opportunities to:

- develop knowledge and skills to use mathematics confidently at the school level and beyond
- develop the mathematical proficiencies of understanding, fluency, reasoning and problem-solving
- apply their knowledge of mathematics in a variety of contexts and circumstances, choosing the appropriate mathematical concepts, and critically evaluating its use.

Defining numeracy

Numeracy is complex and multifaceted. While the evidence base for understanding and teaching numeracy is largely shared with mathematics, some researchers have attempted to define numeracy and describe its components.

To support teacher reflection and planning to inform effective numeracy instruction, Goos, Geiger and Dole (2012) created a model for 21st century numeracy that outlines five main elements:

Element of model	Description of element
Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings.
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking.
Critical orientation	Use of mathematical information to make decisions and judgments; reason and support arguments.

The model supports teacher understanding of the complex interplay of components that contribute to student numeracy development, as well as the need to provide learning experiences that specifically target each element.

Building numeracy through effective mathematics teaching practices

It is generally accepted that becoming a confident, creative user and communicator of mathematics, particularly in the primary years of schooling, is foundational for numeracy. To support teachers to develop these foundations and nurture student development of 21st century numeracy, teachers and leaders need to enact effective teaching practices. The teacher plays a pivotal role, incorporating a mix of pedagogies 'including play based and structured activities' which provide students with high cognitive challenge and achievement' (Queensland Government, 2020, p13). An understanding of the continuity of learning as students begin school acknowledges prior mathematical skills and supports informed planning to build upon existing knowledge. In **Principle to Actions,** the National Council for the Teaching of Mathematics (NCTM, 2014), sets forth a set of research-informed teaching actions. These eight practices 'provide a framework for strengthening the teaching and learning of mathematics... [and] represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics' (NCTM, 2014, p.9).

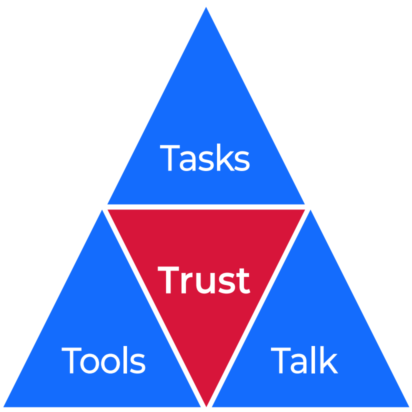
Mathematics teaching practices
Establish mathematics goals to focus learning Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analysing and comparing student approaches and arguments.
Pose purposeful questions Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Mathematics teaching practices
Build procedural fluency from conceptual understanding Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so students, over time, become skilful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

National Council for the Teaching of Mathematics (2014, p.10)

The teaching tripod

In his book **Transforming Primary Mathematics**, Mike Askew (2016) discusses that teachers should plan for communities of learners using the teaching tripod and its practical considerations. The teaching tripod focuses on three areas: tasks, tools and talk. He suggests that by attending to each of the three areas, lessons can be designed to be open enabling students to bring their mathematical knowledge to the forefront whilst still allowing teachers to guide the mathematical content that emerges.



Tasks are at the centre of teaching and learning. Tasks need to focus on developing students' active sense making, providing opportunities to construct knowledge inside of the learning whilst making connections to real life experiences. Mike Askew (2016) suggests this requires teachers to shift the structure of their lessons from demonstrations at the beginning of the lesson to allowing students to work on open-ended tasks with a certain amount of uncertainty so they 'engage mindfully and bring their sense making to the activity' (p.127).

The use of **tools** supports students' mathematical development. Tools can be physical manipulatives and can also include models that teachers introduce, such as open number lines or arrays, to scaffold the abstract nature of mathematics. Through the use of manipulatives and models, and in the presence of reasoning and discussion, the underlying mathematics can be made visible to students and they are able to 'see' and make sense of what is actually happening. Mike Askew (2016) suggests base ten blocks, interlocking cubes, 2D shapes and 3D objects, paper and the intentional use of colour as highly effective tools. Once students have a sound foundation in using tools for thinking about a particular concept, they can deal with the concept in more formal and abstract ways.

Talk is central to mathematics lessons. Students need to be encouraged to talk mathematics, not simply talk about mathematics which means that mathematical vocabulary becomes part of the classroom discourse. Mike Askew (2016) discusses three aspects of talk that aim to support mathematical thinking and these include:

- the importance of listening (as well as speaking) – teachers and students need to practice close and deep listening of each other's ideas. Students should be encouraged to listen attentively and build on each other's ideas as they contribute to discussions.
- recognising the difference between discussion and dialogue – where discussions are about establishing and defending an idea or view, dialogue is about the exchange of ideas and views. In mathematics, both are needed.
- focusing on mathematical reasoning – Mike Askew (2016) asserts that reasoning should not only happen after an answer has been obtained, and a student is explaining and justifying their solution, but happens as the solution is being found.

Teachers need to consider the needs of EAL/D students when planning for talk and the types of scaffolds that will support students to talk mathematically, which may include think, pair, share, word banks, sentence starters and anchor charts.

Central to Mike Askew's (2016) tripod of tasks, tools and talk is a fourth 'T' – **trust**. He explains:

> If we want children to engage with mindful, meaningful mathematical tasks then we, teachers, have to trust that they will come up with improvised solutions that can be collectively crafted into the canonical mathematics. They, the children, have to trust that we are genuinely interested in their thinking. And we have to trust ourselves to be able to make sense of what the children produce. (p.157)

By developing trusting classroom communities, teachers are working towards creating positive attitudes where students are able to take risks, think, reason, communicate, reflect and critique the mathematics they come across.

National Numeracy Learning Progression

National learning progressions describe the skills, understandings and capabilities that students typically acquire as their proficiency increases in a particular aspect of the curriculum over time.

They describe the learning pathway(s) along which students typically progress in particular aspects of the curriculum regardless of age or year level and are designed to help teachers ascertain the stage of learning reached, identify any gaps in skills and knowledge, and plan for the next step to progress learning. (National Numeracy Learning Progression, Version 3 2020, p.5).

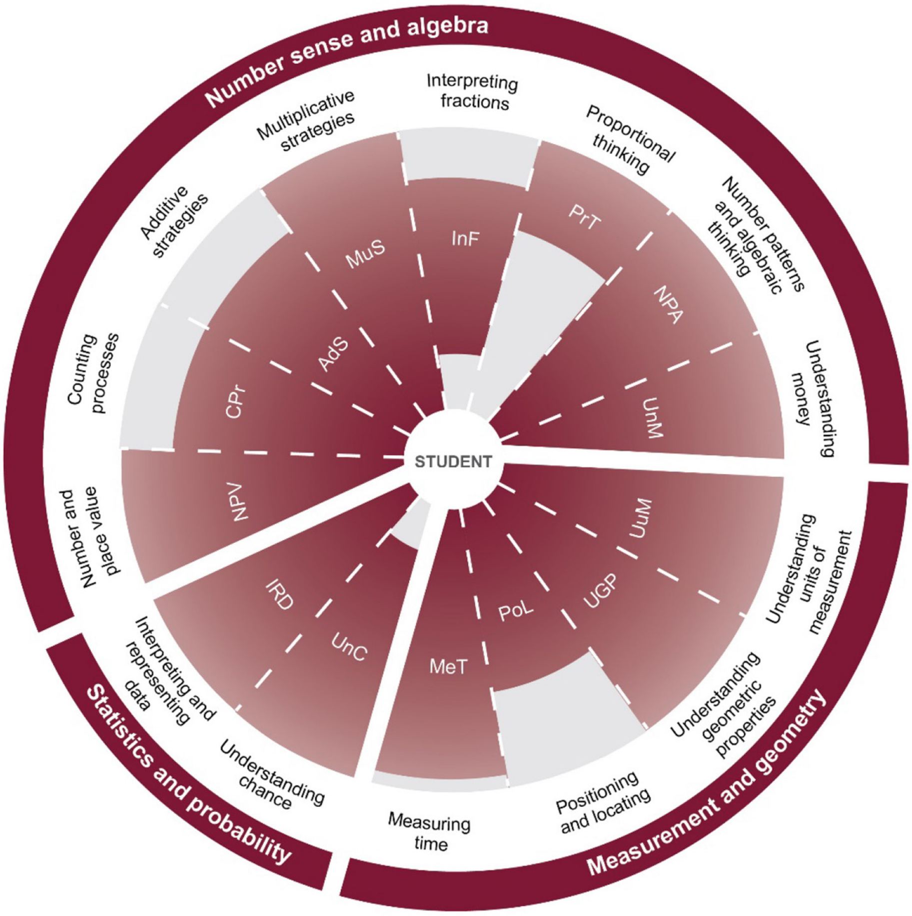


Figure 1. Elements and sub-elements of the National Numeracy Learning Progression

(ACARA, 2020, p.5)

Number sense and algebra

- Number and place value
- Counting processes
- Additive strategies
- Multiplicative strategies
- Interpreting fractions
- Proportional thinking
- Number patterns and algebraic thinking
- Understanding money

Measurement and geometry

- Understanding units of measurement
- Understanding geometric properties
- Positioning and locating
- Measuring time

Statistics and probability

- Understanding chance
- Interpreting and representing data.

Leading to improve numeracy

Principals and leadership teams have a pivotal role in driving numeracy improvement in schools. This guide is designed to support conversations about numeracy improvement in the context of school strategic planning.

Principal

To drive improvement in numeracy, a principal:

- ensures staff have a consistent understanding of evidence-based instruction and assessment of the numeracy demands within all key learning areas to embed effective teaching of numeracy
- builds numeracy leadership across the whole school, including the critical relationships across the curriculum
- promotes improvement as the shared responsibility of all staff in all key learning areas
- builds a structure where a shared culture of improvement is valued and supported across the school creating positive mindsets and dispositions for all stakeholders, including the school community
- leads a coherent and shared school understanding of numeracy development using the relevant NSW syllabus outcomes in conjunction with the National Numeracy Learning Progression
- uses the themes of the 'What works best: 2020 update' as a starting point to establish a consistent approach to teaching and learning
- facilitates collaboration and collegial approaches to whole school improvement by clearly identified structures and processes embedded in the school's strategic improvement plan that allow opportunities for leaders and teachers to:
    - collaborate with teams
    - engage in professional learning
    - explore and respond to research and evidence
    - co-plan, co-teach and co-evaluate including peer observations and lesson studies
    - engage in classroom 'learning walks and talks'
- ensures a powerful and collaborative whole school approach to data analysis and student progress monitoring in order to measure impact to target future teaching and learning
- budgets for professional learning, the purchase of resources and intervention programs for students who need further support
- creates positive, challenging learning environments for all learners
- recognises the cultural diversity of school communities and facilitates opportunities for staff to understand and respond to the numeracy learning needs of Aboriginal and/or Torres Strait Islander students and EAL/D learners

- ensures literacy and language learning is properly supported and emphasised, recognising these are fundamental to the development of student numeracy
- incorporates strategies within the school improvement plan that comply with the Aboriginal Education policy and supports staff to participate in learning pathways to build competencies in Aboriginal cultures
- ensures school policies and practices are consistent with the Multicultural Education policy and for including multicultural education strategies in their school improvement plans which provide teaching and learning programs that develop intercultural understanding, promote positive relationships and enable all students to participate as active Australian and global citizens
- drives a culture of inclusion and inclusive learning that benefits all students, including students with disability. Ensures reasonable adjustments are provided as required under the disability standards, so every student improves every year. For more information about adjustments visit the Disability, learning and support webpage
- leads the optimal talent development of high potential and gifted students across all domains of potential within supportive learning environments that develop the whole student. For more information visit the High potential and gifted education webpage.

Leadership team

To drive improvement in numeracy, a leadership team:

- engages with current research and evidence regarding the development of numeracy and mathematics to support teacher knowledge and understandings
- supports teachers to recognise the cultural diversity of their students and builds their capacity to understand and acknowledge the numeracy demands of students from Aboriginal and EAL/D backgrounds the numeracy learning needs of Aboriginal and/or Torres Strait Islander students and EAL/D learners , ensuring quality teaching and assessment practices and resources are culturally inclusive. For more information visit the Aboriginal education and communities webpage for Aboriginal education in NSW schools and the Multicultural education webpage for EAL/D education
- supports teachers to build a culture of inclusive education for all students to achieve learning outcomes, supported by reasonable adjustments and teaching strategies tailored to meet their individual needs. For more information about adjustments visit the Disability, learning and support webpage
- supports teachers in the assessment and identification of the specific numeracy learning needs of high potential and gifted students across all domains of potential and effective differentiation for those students. For more information visit the High potential and gifted education webpage
- ensures school teams and teachers have given consideration to transition to high school and the early identification of student numeracy needs

- reviews current teaching practices, teacher understanding and beliefs about numeracy and mathematics which can include:
  - systematic analysis of teaching using various models of observation and reflection
  - systematic analysis and refinement of teaching and learning programs and resources
  - clearly defined roles and responsibilities of staff to meet the needs of students
  - identifying, creating and implementing differentiated professional learning opportunities to build the capacity, knowledge, confidence and dispositions of teachers
  - ensuring current resources are supported by rigorous research evidence and enhance student learning and engagement
- identifies opportunities for numeracy to be enhanced in all key learning areas, ensuring dispositions, understanding and skills are transferred across a variety of contexts through:
  - establishing a shared understanding of evidence-based practices for the targeted teaching of numeracy (see Goos et al's model on p.8)
  - collaboratively identifying and embedding numeracy into teaching and learning programs
  - developing a shared whole-school responsibility and consistent use of language
- develops a deep understanding of current NSW K-10 Syllabuses and the links to the National Numeracy Learning Progression (currently mapped to the Mathematics K-10 Syllabus, 2022) to:
  - identify students' prior knowledge, current understanding, skills, and interests
  - plan for targeted teaching
  - monitor individual student progress
  - support students to successfully engage with the numeracy demands of the syllabuses
- establishes systems and structures to:
  - ensure assessment of numeracy across curriculum areas is balanced and ongoing, including mathematics
  - build leadership of colleagues by working shoulder to shoulder with teachers to investigate, plan programs, with embedded effective teaching strategies and the development of quality assessment
  - use data to make informed decisions about targeted teaching and differentiate learning
  - lead, plan and support high quality teaching and learning programs that engages students through inquiry-based, challenging tasks
  - engage in regular in-class support including classroom coaching, observation and feedback, learning walks and talks, co-teaching and the analysis of work samples to identify the impact of practice and programs on student outcomes and engagement
  - participate in professional conversations to evaluate and modify teaching strategies and programs.

Teacher

To drive improvement in numeracy, a teacher:

- engages students in purposeful tasks and learning experiences that require deep thinking about important concepts and relationships
- creates an environment that encourages collaboration, educative risk-taking, meaningful talk, and uses students' misconceptions and errors as building blocks for learning
- designs opportunities for students to regularly experience productive struggle, exploring ideas and concepts to develop and use an increasingly sophisticated range of skills
- designs opportunities for students to practise what they are learning whether it be to improve fluency, problem-solving skills, or enrich conceptual understanding
- intentionally chooses and uses tasks because they meet a specific mathematical purpose, offering appropriate levels of challenge and opportunities for feedback for all students
- facilitates and plans productive classroom dialogue that encourages and supports students to justify their thinking and actions, drawing on a range of pedagogical practices and representational competencies
- supports students in connecting different strategies, approaches, representations, and concepts
- uses everyday experiences to design teaching and learning activities
- explores and uses multiple, meaningful representations to develop communicating skills, and understanding
- models how to work flexibly with numbers, operations and other critical ideas
- provides opportunities to use an ever-increasing range of representations
- builds on students' existing thinking through questioning and modifying tasks to provide alternative pathways to understanding
- plans learning experiences that enable students to build on their existing proficiencies, interests, confidence and experiences
- selects concrete materials/manipulatives that engages students in mathematical thinking to support them to represent mathematical ideas explicitly and concretely
- provides students with opportunities to work as a whole class, in small groups, in focus groups and on their own to make sense of and use ideas
- exemplifies the importance and real-world application of numeracy and mathematical competencies
- draws clear and consistent links between abstract and concrete mathematical concepts
- supports and scaffolds the background knowledge, cultural assumptions and language demands of numeracy to meet the needs of students from Aboriginal and/or Torres Strait Islander backgrounds. For more information visit the Aboriginal education and communities webpage for Aboriginal education in NSW schools

- designs targeted teaching and learning numeracy tasks that address the specific learning and wellbeing needs of students from culturally diverse backgrounds, including newly arrived and refugee students. For more information visit the Multicultural education webpage for EAL/D education
- makes reasonable adjustments according to a student's personalised learning and support needs to access assessment, syllabus outcomes and content on the same basis as their peers to meet the numeracy demands of the curriculum. For more information about adjustments visit the Disability, learning and support webpage
- applies numeracy evidence-based approaches that extend and challenge high potential and gifted students beyond their current level of mastery across all domains of potential. Develops, designs and teaches differentiated learning programs and provides experiences that meet the advanced learning needs of students. For more information visit the High potential and gifted education webpage.

Number sense and place value

Number sense is considered a way of thinking about mathematical situations in order to make judgements, interpret data and communicate effectively (Booker, 2014). Number sense can be described as:

> a person's general understanding of numbers and operations along with the ability and inclination to use this knowledge in flexible ways. Number sense is crucial for making mathematical judgements and developing useful strategies for handling numbers and operations (McIntosh et al. 1997, p.3).

As such, number sense 'requires a deep knowledge of numbers and operations that can be used confidently and flexibly in multiple contexts, the capacity to explain and justify one's thinking and generalise, and an appreciation of pattern and mathematical structure.' (Siemon, Warren, Beswick, Faragher, Miller, Horne, Jazby, Breed, Clark and Brady, 2020, p.265).

Place value is foundational to developing a deep sense of number as students learn to appreciate the base 10 numeration system and that the value represented by a digit in a number is based on its position in the number. It is also about understanding the significance that '10 of these is one of those' and '1000 of these is 1 of those' (Siemon et al., 2019).

Number sense develops over a long period of time and requires meaningful, challenging experiences focussed on a broad range of critical ideas including:

- seeing mathematics as something we make sense of and use to share ideas
- noticing patterns and relationships
- making sense of numbers 0-9
- making sense of 10 and beyond (including place value)
- making sense of fractions (including decimals and percentages)
- using and making connections between different representations
- making sense of operations
- thinking multiplicatively.

Syllabus and progression links

Number sense underpins all aspects of the NSW Mathematics K-10 Syllabus (2022). Achievement of outcomes in number and algebra is dependent upon students having strong number sense.

NSW Syllabus	National Numeracy Learning Progression
All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including Mathematics, Science and technology K-6, Science 7-10, History, Geography and PDHPE. Numeracy is also embedded within Creative arts. Stage 2: MAO-WM-01, MA2-RN-01, MA2-RN-02, MA2-AR-01, MA2-MR-01, MA2-MR-02, MA2-PF-01, MA2-GM-01, MA2-NSM-01, MA2-NSM-02, MA2-DATA-01 Stage 3: MAO-WM-01, MA3-RN-01, MA3-RN-02, MA3-RN-03, MA3-AR-01, MA3-MR-01, MA3-RQF-01, MA3-GM-03, MA3-NSM-01, MA3-NSM-02, MA3-CHAN-01 Stage 4: MAO-WM-01, MA4-INT-C-01, MA4-FRC-C-01, MA4-RAT-C-01, MA4-IND-C-01, MA4-ARE-C-01, MA4-VOL-C-01 For more information on syllabus connections, see the Making connections in Mathematics document series on the NESA website.	Stage 2: NPV4 – NPV7, CPr6 – CPr7 Stage 3: NPV6 – NPV9 and CPr8 Stage 4: NPV9 – NPV10

Further support

Professional learning	Assessment tools and resources
• Becoming mathematicians: How numbers and fractions work (Professional learning video with accompanying resource) • Becoming mathematicians: Quantifying collections (Professional learning video with accompanying resource) • Improving reading and numeracy suite: ◦ Number and place value (primary) • Quality curriculum implementation K-6 (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6) • Mathematics 3-6 microlearning (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))	• IfSR-Number and place value (IfSR-NP) resources • IfSR-NP diagnostic online assessment – ALAN • Number knowledge resources • Numeracy resources on the Universal Resources Hub • National Numeracy Learning Progression • How to – technical guide to using PLAN2

What does number and place value look like in a Years 3-8 classroom?

These are some examples for the development of number and place value which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 Syllabus (2022) and key research. For further information see the NSW Mathematics K-10 Syllabus and the National Numeracy Learning Progression.

Number knowledge (including place value)

Students

- connect number words, symbols and quantities (those physically present and imagined) beyond 1000, smaller than 1 (fractions) and less than 0 (negative)
- acknowledge that numbers are a 'symbol' that represents meaning – numbers themselves could be replaced with other symbols that are recognisable and relate to cultural context and understanding
- identify, order, sequence, read and write numerals beyond 1000
- identify zero as both a number and a placeholder for reading and writing larger and smaller numerals. For example, 7000 or 0.007
- explain that the position of digits in a number determines its value

- identify whole quantities as the result of recognising smaller quantities with numerals beyond 1000 (for example, determines there is 954 in total as they know 954 is composed of 6 hundreds 5 tens 4 ones and 3 hundreds more)
- estimate large quantities
- explain that decimals are a way of representing fractional quantities
- identify, read and write decimal numbers applying knowledge of the place value with tenths, hundredths, thousandths and beyond
- read, represent and use negative numbers in problem solving (for example, explains that the temperature −10 °C is colder than the temperature −2.5 °C; recognises that negative numbers are less than zero; locates −12 on a number line)
- identify, read and interpret numbers of any size (for example, reads that the world population is estimated to be seven billion and interprets this to mean 7 000 000 000)
- identify and describe factors and multiples of whole numbers
- identify and describe properties of prime, composite, square and triangular numbers
  - explain if a whole is prime, composite or neither by finding the number of factors and provides a valid justification
  - model square and triangular numbers, explains how they are created and provides a valid justification
- flexibly regroup three-digit numbers according to their place value (for example, renames 247 as 2 hundreds, 4 tens and 7 ones; 2 hundreds and 47 ones; 24 tens and 7 ones; 1 hundred, 14 tens and 7 ones and so on)
- explain that numbers can be partitioned (broken apart) in different but equivalent ways. For example, 256 can be partitioned into 2 hundreds, 5 tens and 6 ones. It can also be partitioned into 0 hundreds, 25 tens and 6 ones
- flexibly partition any given number by their place value, such as thousands, hundreds, tens and ones
- represent and name one-tenth as its decimal equivalent 0.1, zero point one
- estimate and round whole numbers to the nearest ten thousand, thousand, hundreds
- compare, order and represent decimals including rounding decimals and investigating terminating and recurring decimals
- use a number line to locate negative numbers in relation to zero
- use knowledge of place value to compare and order numerals up to 5-digit numbers and beyond
- relate place value parts to indices
- understand place value relationships such as ten thousand is the same as 100 hundreds, which is the same as 100 tens, which is the same as 10 000 ones.

Counting with understanding

Students

- recall word number sequences, in any sequence (forwards and backwards) from any given number
- recall number word sequences of tens and hundreds, applying knowledge of place value understanding, on and off the decade
- count flexibly forwards and backwards from any rational number (for example, counts in thirds such as $\frac{1}{3}$, $\frac{2}{3}$, 1, 1 and $\frac{1}{3}$, 1 and $\frac{2}{3}$, 2 …; counts backwards by 0.3 starting from four 4, 3.7, 3.4, 3.1 …)
- count backwards from zero understanding that the count can be extended in the negative direction (for example, 0, -1, -2, -3, -4)
- apply counting processes to any collection beyond the tangible (for example, systematically counts the number of possible outcomes of an event; applies a frequency count)
- identify the number before or after a given number for numbers of any size
- estimate the number of items in a collection to assist with determining group sizes for efficient counting (for example, decides that counting in twos would not be the most efficient counting strategy based on a quick estimate of the quantity and decides instead to use groups of ten).

Patterns and algebra

Patterning is fundamental in all mathematics learning. Amongst other skills, patterning involves the ability to identify and describe attributes of objects and similarities and differences between them (Papic, 2007). Noticing similarities and differences is a powerful way to teach mathematics and it assists in unveiling its structure (Siemon et al, 2019). Developing an intuitive awareness of patterns and structure is fundamental for all students and supports with learning across all mathematical areas and other learning areas. Patterning is integral, for example, to the development of determining 'how many?' by subitising and by counting. Patterning also helps us make sense of arithmetic structure, base ten and multiplicative concepts, units of measure, proportional reasoning, data and statistical reasoning.

A mathematical pattern may be described as any replicable regularity, involving number, space or logical relationships. Observing natural patterns in the world around us and creating and describing patterns support the development of early number concepts inclusive of repeating patterns (for example, ABABAB …), spatial structural patterns (for example, geometrical shapes), growing patterns (increasing and decreasing), units of measure or transformations (Mulligan 2010, Mulligan and Mitchelmore, 2009).

In the early stages of school, students explore number and pre-algebra concepts by pattern making, and by discussing, generalising and recording their observations. As students become increasingly able to connect patterns with the structure of numbers, they create a foundation for algebraic thinking (that is, thinking about generalised quantities). 'Generalising patterns is seen as a key to developing mathematical thinking and algebraic understanding' (Siemon et al, 2015 p.270).

Algebraic thinking is inextricably linked to patterning, and as such, it too is foundational to mathematical thinking. Algebraic thinking provides the language and structure to represent ideas, solve problems, model situations, generalise and prove. Like patterns, algebra is linked with the other strands in the mathematics syllabus and is applicable across a broad range of other learning areas. Algebraic thinking begins in the earliest years of schooling when students describe a repeating pattern as an AB pattern. It can also be found when students extend and copy patterns and when they sort and classify objects. We also see algebra in use when we record a situation, for example, when I combined 3 donuts with 7 donuts as '3 + 7 = 10'.

Algebraic understanding supports students with mathematical computation and developing a strong foundational understanding of our numerical system.

Examples of embedded patterning in Mathematics 3-8 syllabus	
Completes number sentences involving addition and subtraction by finding missing values MA2-AR-02	Use the equals sign to mean 'the same as', rather than to perform an operation Find the missing number in an equivalent number sentence involving operations of addition or subtraction on both sides of the equals sign (Algebraic reasoning)
Represents and uses the structure of multiplicative relations to 10 × 10 to solve problems MA2-MR-0	Model, describe and record patterns of multiples Create and continue a variety of number patterns that increase or decrease by a constant amount
Constructs and completes number sentences involving multiplicative relations, applying the order of operations to calculations MA3-MR-02	Complete number sentences that involve more than one operation by calculating missing numbers Use a given geometric pattern involving multiples to create a table of values
Locates and describes points on a coordinate plane MA3-GM-01	Plot and label points, given coordinates, in all 4 quadrants of the number plane
Generalises number properties to operate with algebraic expressions including expansion and factorisation MA4-ALG-C-01	Generate a number pattern from an algebraic expression Generalise the associative property of addition and multiplication to algebraic expressions
Applies Pythagoras' theorem to solve problems in various contexts MA4-PYT-C-01	Establish the relationship between the lengths of the sides of a right-angled triangle Solve practical problems involving Pythagoras' theorem before exploring a variety of related problems

Syllabus and progression links

Patterning is central to mathematics teaching and learning. Awareness of patterns and knowledge of structure helps learning across all mathematical areas and has a positive influence on mathematical achievement overall to enable a stronger foundation for algebraic thinking.

NSW Syllabus	National Numeracy Learning Progression
All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including Mathematics, Science and technology K-6, Science 7-10, History, Geography and PDHPE. Numeracy is also embedded within Creative arts. Stage 2: MAO-WM-01, MA2-RN-01, MA2-RN-02, MA2-AR-02, MA2-MR-01, MA2-MR-02 Stage 3: MAO-WM-01, MA3-MR-01, MA3-MR-02, MA3-GM-01 Stage 4: MAO-WM-01, MA4-EQU-C-01, MA4-LIN-C-01, MA4-LEN-C-01, MA4-PYT-C-01, MA4-ARE-C-01, MA4-VOL-C-01, MA4-ANG-C-01, MA4-GEO-C-01 For more information on syllabus connections, see the Making connections in Mathematics document series on the NESA website.	Stage 2: AdS6 – AdS7, MuS4 – MuS7 and NPA3 – NPA4 Stage 3: AdS7 – AdS9, MuS6 – MuS7 and NPA4 – NPA7 Stage 4: MuS7 and NPA6 – NPA8

Further support

Professional learning	Assessment tools and resources
• Becoming mathematicians: Exploring patterns (Professional learning video with accompanying resource) • Quality curriculum implementation K-6 (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6) • Mathematics 3-6 microlearning (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))	• IfSR-Number and place value (IfSR-NP) resources • IfSR-NP diagnostic online assessment – ALAN • Exploring patterns • Talking about patterns and algebra • Numeracy resources on the Universal Resources Hub • National Numeracy Learning Progression • How to – technical guide to using PLAN2

What does patterns and algebra look like in a Years 3-8 classroom?

These are some examples for the development of patterns and algebra which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 Syllabus (2022) and key research. For further information see the NSW Mathematics K-10 (2022) Syllabus and the National Numeracy Learning Progression.

Students

- explain that a pattern is a mathematical regularity that has a ‘core’ (repeating unit) that repeats over and over
- represent the core of a pattern in a variety of ways
- use inverse operations to solve problems and justify solutions
- create, describe, represent and continue a variety of patterns involving shapes, objects, data, spatial patterns, whole numbers and fractions (including decimals), fractions and decimals; describe the unit of repeat used to create growing and shrinking patterns
- explain = represents a relationship of equivalence
- balance equations (number sentences) involving one or more operations following conventions of order of operations (5 × 2 + 4 = 4 × 2 + ?, 5 + 2 × 3 = 11)
- sequence numbers to identify a pattern core (unit of repeat), identifying a single operation rule in numerical patterns and records it as a numerical expression (2, 4, 6, 8, 10 … is n + 2, or 2, 6, 18, 54 … is 3n)
- investigate the traditional patterns of land, water and culture:
  - the water/rain cycle (each part of the water cycle can be attributed a letter to represent that part, and the cycle itself can be an algebra equation)
  - the life cycle of a plant/flower
  - food chains life cycle of animals for example, frogs
  - traditional dance (each step is given a letter to represent that step/move and can also be turned into an equation)
- apply knowledge of factors associated with the row and column structure of arrays to explain the commutative property of multiplication (3 × 4 = 4 × 3)
- create, complete and interpret a table of values for geometric shapes and number patterns involving one operation (including patterns that decrease) and describe the pattern in words, for example:

□ , □□ , □□□ , □□□□ , …➔

number of squares	1	2	3	4	…	100
number of matches	4	8	12	16	…	

position of squares	1	2	3	4	5	6
value of term	4	5	6	7		

- develop the concept that pronumerals (letters) can be used to represent numerical values
- create and identify algebraic expressions from word problems involving one or two operations and one unknown
- solve algebraic equations that involve addition, subtraction, multiplication and division
- simplify algebraic expressions involving mixed operations and apply the order of four operations to simplify algebraic equations
- expand algebraic expressions by removing grouping symbols, for example:
  - $3(a+2)=3a+6$
  - $-5(x+2)=-5x-10$
  - $a(a+b)=a^2+ab$
- connect algebra with the distributive property of arithmetic to determine that $a(b+c)=ab+ac$
- extend and apply the distributive law to the expansion of algebraic expressions
- factorise algebraic expressions by identifying numerical and algebraic factors
- recognise that the Cartesian number plane consists of a horizontal axis (x-axis) and a vertical axis (y-axis), creating four quadrants and has a point of intersection of the two axes as the origin, having coordinates (0, 0)
- identify, record and describe the coordinates of given points in all four quadrants of the Cartesian number plane
- interpret and use formulae and algebraic representations that describe relationships in various contexts.

Additive thinking

Additive thinking specifically relates to the construction and representation of number, in particular part-part-whole knowledge which is underpinned by two big ideas: 'trusting the count' and 'place value'.

Part-part-whole knowledge of numbers to 10 is essential to reconfigure numbers in different ways for additive purposes, and this knowledge should be extended upon when decomposing larger numbers. This knowledge allows students to understand numbers as compositions of other numbers, the commutative property of addition and the inverse relationship between addition and subtraction.

Place value knowledge supports the development of mental computation strategies and there should be a focus on the value of working with place value units rather than counts of one. Students should also gain the critical understanding of the place value notion that '10 of these is 1 of those' and extend this understanding and apply it to working with decimals (Siemon et al, 2015, p.270).

The transition from counting by one to more flexible strategies requires students to work on problems in different ways, decomposing, composing, partitioning and/or combining number patterns.

Choosing a strategy appropriate to the task requires a deep understanding of the operations and a well-developed sense of number. Students should develop a repertoire of meaningful, efficient strategies to support problem solving involving whole numbers, decimals and fractions so that they can use confidently to justify their strategy relative to context. Students should be encouraged to engage in discussions that focus on mathematical reasoning to explain and justify the merits of various strategies. When students talk about their solution paths and others' solution paths, they have opportunities to make sense of ideas and become increasingly able to choose and use additive strategies for different purposes.

Syllabus and progression links

NSW Syllabus	National Numeracy Learning Progression
All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including Mathematics, Science and technology K-6, Science 7-10, History, Geography and PDHPE. Numeracy is also embedded within Creative arts. Stage 2: MAO-WM-01, MA2-RN-01, MA2-RN-02, MA2-AR-01, MA2-AR-02, MA2-GM-01 Stage 3: MAO-WM-01, MA3-RN-01, MA3-RN-02, MA3-AR-01, MA3-RQF-01, MA3-RQF-02, MA3-GM-02, MA3-GM-03, MA3-DATA-02 Stage 4: MAO-WM-01, MA4-INT-C-0, MA4-FRAC-01, MA4-ALG-C-01, MA4-EQU-C-01, MA4-LEN-C-01, MA4-ANG-C-01, MA4-DAT-C-02 For more information on syllabus connections, see the Making connections in Mathematics document series on the NESA website.	Stage 2: NPV5 – NPV6, AdS6 – AdS8, UnM4 – UnM6 and NPA3 – NPA4 Stage 3: NPV5 – NPV6, AdS6 – AdS8, UnM6 and NPA4 Stage 4: NPV9, CPr8 and AdS9 – AdS10

Further support

Professional learning	Assessment tools and resources
• Additive strategies – blended professional learning focuses on deepening understanding of the development of additive thinking through integrating practical ideas for classroom application. • Becoming mathematicians: flexible additive thinking (Professional learning video with accompanying resource) • Improving reading and numeracy suite: ◦ Additive thinking (K-8) • Quality curriculum implementation K-6 (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6) • Mathematics 3-6 microlearning (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))	• IfSR-Additive thinking (IfSR-AT) resources • IfSR-AT diagnostic online assessment – ALAN • Flexible additive strategies – Combinations to 10 • Flexible additive strategies – 2-digit numbers • Flexible additive strategies – 3-digit numbers • Flexible additive strategies – Decimals • Numeracy resources on the Universal Resources Hub • National Numeracy Learning Progression • How to – technical guide to using PLAN2

What does additive thinking look like in a Years 3-8 classroom?

These are some examples for the development of additive thinking which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022) and key research. For further information see the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

Students

- apply knowledge of place value to partition, rearrange and regroup numbers to at least 10,000 to assist calculations and solve problems, including standard and non-standard partitioning
- model and apply the associative property of addition to aid mental computation
- apply known single-digit addition and subtraction facts to mental strategies for addition and subtraction of larger numbers including using patterns to extend number facts, bridging the decades and partitioning
- perform simple calculations with money including calculating equivalent amounts of money, calculating change and rounding

- explore additive number patterns that increase or decrease from any starting point
- select and apply efficient mental, written and calculator strategies with three-digit numbers and beyond to solve addition and subtraction word problems
- give reasons why calculators can be useful when solving a problem
- choose and use multiple, appropriate strategies for solving everyday problems involving addition and subtraction
- use a formal written algorithm to record addition and subtraction calculations
- communicate and reason to explain how an answer was obtained and compare their own method of solution with the methods of other students using appropriate terminology and symbols to describe mathematical ideas
- reflect on chosen strategies and solutions, considering whether it can be improved
- use estimation to check the reasonableness of solutions to addition and subtraction problems, including those involving money
- use equivalent number sentences involving addition and subtraction to find unknown quantities
- check solutions to problems, including by using the inverse operation
- round numbers appropriately when obtaining estimates to numerical calculations
- use knowledge of addition and subtraction facts to create a financial plan, such as a budget and solve problems involving profit and loss, with and without the use of digital technologies
- apply a practical understanding of commutativity and associativity to aid mental computation
- describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction
- compare, order, add and subtract integers, interpreting the different meanings (direction or operation) for the + and – signs, depending on the context
- solve problems involving addition and subtraction of fractions with the same or related denominators, and extending into those with unrelated denominators
- communicate and reason to explain how an answer was obtained and compare their own method of solution with the methods of other students.

Multiplicative thinking

According to Siemon et al. (2019), multiplicative thinking is characterised by:

- a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole numbers, decimals, common fractions, ratio, and percentages)
- an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion
- the means to communicate this effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms).

In short, multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts (Siemon and Breed, 2005).

In order to think multiplicatively and work with multiplication and division formally, students need to be able to count large collections efficiently and also recognise the numbers 2-10 as composite units. Teachers can assist students to notice different counting strategies and patterns through regular subitising activities and opportunities to count large collections of items. Furthermore, students need to develop efficient, flexible mental strategies and be able to work with; part-part-whole knowledge to 10 (that is, recognise numbers to 10 in terms of their parts); mental strategies for addition and subtraction with numbers to at least 20 (for example, doubles, near doubles); place value ideas flexibly (including renaming numbers) and sharing concrete materials equally.

The transition from additive to multiplicative thinking and strategies requires a significant shift in thinking, from counting and additive processes (counting equal groups and repeated addition) to utilising the 'for each' and 'times as many' ideas for multiplication. The ability to mentally coordinate two composite units supports the critical shift in student thinking from counting groups to recognising and operating with factors.

Array-based strategies support the shift from an additive groups of model to a factor-factor-product model (multiplicative) which is needed to support fraction representation, the multiplication and division of larger whole numbers, fractions and decimals, and algebra. Arrays can be used for building multiplication facts in a meaningful way. Before developing multiplicative automaticity, children must understand how these facts are derived. Siemon et al. (2019) explain that 'Array and region representations provide a basis for a more generalised understanding of multiplication and division that can be used to support the multiplication of multi-digits, fractions and, ultimately, algebraic expressions' (p.387).

Multiplicative structures and situations include equal groups ideas for both multiplication and division context, partition, 'times as many' (multiplicative comparison), arrays (extending into area and region ideas) and the 'for each' idea (or Cartesian product idea).

The capacity to think multiplicatively underpins proportional thinking and provides the basis for working with the relationships between quantities for concepts such as fractions, decimals, percentages, rates and ratios.

Syllabus and progression links

NSW Syllabus	National Numeracy Learning Progression
All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including Mathematics, Science and technology K-6, Science 7-10, History, Geography and PDHPE. Numeracy is also embedded within Creative arts. Stage 2: MAO-WM-01, MA2-RN-01, MA2-RN-02, MA2-MR-01, MA2-MR-02, MA2-GM-02, MA2-2DS-02, MA2-3DS-02 Stage 3: MAO-WM-01, MA3-RN-01, MA3-RN-03, MA3-MR-01, MA3-MR-02, MA3-MR-03, MA3-RQF-02, MA3-GM-02, MA3-2DS-02, MA3-2DS-03, MA3-3DS-02, MA3-CHAN-01 Stage 4: MAO-WM-01, MA4-INT-C-01, MA4-FRAC-C-01, MA4-RAT-C-01, MA4-ALG-C-01, MA4-IND-C-01, MA4-EQU-C-01, MA4-LIN-C-01, MA4-LEN-C-01, MA4-PYT-C-01, MA4-ARE-C-01, MA4-VOL-C-01, MA4-GEO-C-01, MA4-DAT-C-02 For more information on syllabus connections, see the Making connections in Mathematics document series on the NESA website.	Stage 2: CPr6 – CPr7, MuS2 – MuS7 and NPA5 Stage 3: NPV9, MuS6 – MuS8 and NPA5 Stage 4: NPV9 – NPV10, MuS6 – MuS10 and NPA6 – NPA7

Further support

Professional learning	Assessment tools and resources
• Multiplicative strategies: blended learning (a combination of self-paced modules and facilitator-led, live sessions via Microsoft Teams) • Becoming mathematicians: Flexible multiplicative thinking • Improving reading and numeracy suite: ◦ Multiplicative thinking (K-8) • Quality curriculum implementation K-6 (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6) • Mathematics 3-6 microlearning (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))	• IfSR-Multiplicative thinking (IfSR-MT) resources • IfSR-MT diagnostic online assessment – ALAN • Strategies for sharing and forming groups • Multiplicative thinking with single digit numbers • Multiplicative thinking with multi digit numbers • Flexible strategies with rational numbers • Multiplicative strategies webpage • Numeracy resources on the Universal Resources Hub • National Numeracy Learning Progression • How to – technical guide to using PLAN2

What does multiplicative thinking look like in a Years 3-8 classroom?

These are some examples for the development of multiplicative thinking which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022) and key research. For further information see the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

Students

- use flexible strategies for multiplication and division (of whole numbers)
  - apply doubling (and halving) and repeated doubling strategies for example, using repeated doubling as a strategy for multiplying by 4 and 8
  - rename the number of groups that is, deals with number of groups in terms of part-part-whole understanding
  - use factors of a number to carry out multiplication and division
  - use multiplication and division as inverse operations
  - apply knowledge of place-value to solve multiplication and division problems. For example. renames two-digit numbers and beyond in terms of their parts and factorises multiples of 10

    - use known facts and multiples to mentally calculate multiplication and division problems
    - use knowledge of distributive property of multiplication over addition (8 × 63 equals 8 × 60 plus 8 × 3)
    - model and apply commutative (and associative) properties of multiplication to assist with mental computation
- apply knowledge of factors associated with the row and column structure of arrays to explain the commutative property of multiplication (3 × 4 = 4 × 3)
- recall multiplication (and division) facts and generalise strategies (for example, 4 of anything as double double)
- relate the division idea of partition (sharing) and quotition (or how many groups in) to multiplication
- use estimation and approximation, including rounding to check the reasonableness of answers of products and quotients
- represent and solve problems involving multiplication and division using efficient mental and written strategies
- recognise that the equals symbol means 'is the same as' or ‘is equivalent in value to’, rather than an indication to perform an operation and can be used to record equivalent number sentences, for example, 4 × 3 = 6 × 2
- communicate and reason to explain how an answer was obtained and compare their own method of solution with the methods of other students
- record and explain remainders as incomplete rows or multiples and identify what is left over from the division
- represent multiplication using arrays and regions to move from the additive, ‘groups of’ model, to a ‘factor-factor-product’ model
- apply the area model with multiplication and division of multi-digit numbers using knowledge of place value
- model and represent combination problems using the Cartesian product extending from the ‘for each’ idea including tree diagrams and tables
- apply the order of operations to solve problems and complete calculations involving grouping symbols, including mixed operations
- use efficient mental strategies to multiply and divide integers and rational numbers
- solve financial problems involving discounts, including the percentage discount and evaluate special offers
- work flexibly with both the number of groups and the number in each group and treat both (the multiplier and multiplicand) as composite units.

Proportional thinking

Proportional thinking is a complex form of reasoning that builds upon a number of interconnected ideas over a long period of time (Siemon et al. 2019). Proportional thinking requires skills in thinking multiplicatively and involves measures, rates and/or ratios expressed in terms of natural numbers, rational numbers, and/or integers. For example, **$\frac{2}{3}$ × $24 as two-thirds of $24**, or **3.5 × 68 as 3 and a half times 68**, (Siemon et al., 2019). Students need to understand and work with percentages, fractions, rates, ratio and represent the proportional relationship between quantities.

Students apply their number skills to a variety of situations, including financial situations and practical problems, developing a range of life skills important for numeracy. Ratios and rates underpin proportional reasoning needed for problem solving and the development of concepts and skills in other aspects of mathematics, such as trigonometry, similarity and gradient.

Proportional reasoning also includes tasks involving a comparison of different rates, for example, if one dog grows from 5 kilograms to 8 kilograms and another dog grows from 3 kilograms to 6 kilograms, which dog grew more compared to its original weight? Learning to reason using proportion is a complex process that develops over an extended period (ACARA, 2020).

Syllabus and progression links

NSW Syllabus	National Numeracy Learning Progression
All teachers have a responsibility to support students to develop the general and discipline-specific numeracy requirements of students in their curriculum area. Numeracy is embedded throughout K-10 syllabus documents as a capability. The capabilities can be found in syllabus documents, including Mathematics, Science and technology K-6, Science 7-10, History, Geography and PDHPE. Numeracy is also embedded within Creative arts. Stage 2: MAO-WM-01, MA2-RN-01, MA2-RN-02, MA2-MR-01, MA2-MR-02, MA2-PF-01, MA2-CHAN-01 Stage 3: MAO-WM-01, MA3-RN-01, MA3-RN-02, MA3-RN-03, MA3-MR-01, MA3-MR-02, MA3-RQF-01, MA3-RQF-02, MA3-GM-02, MA3-2DS-02, MA3-2DS-03, MA3-3DS-02, MA3-CHAN-01 Stage 4: MAO-WM-01, MA4-INT-C-01, MA4-FRAC-C-01, MA4-RAT-C-01, MA4-ALG-C-01, MA4-IND-C-01, MA4-PRO-C-01 For more information on syllabus connections, see the Making connections in Mathematics document series on the NESA website.	Stage 2: MuS2 – MuS7 and InF2 – InF5 Stage 3: MuS6 – MuS6, InF3 – InF8 and PrT3 – PrT4 Stage 4: MuS8 – MuS10, InF6 – InF9 and PrT2–PrT6

Further support

Professional learning	Assessment tools and resources

• Multiplicative strategies: blended learning (a combination of self-paced modules and facilitator-led, live sessions via Microsoft Teams)
• Becoming mathematicians: Multiplicative thinking
• Improving reading and numeracy suite:
  ◦ Fractions and proportional thinking (primary)
  ◦ Fractions and proportional thinking (secondary)
  ◦ Multiplicative thinking (K-8)
• Quality curriculum implementation K-6 (microlearning modules focused on evidence-based practices that underpin the curriculum planning and programming, assessment and reporting process K-6)
• Mathematics 3-6 microlearning (microlearning modules designed to support you with implementation of the Mathematics K-10 Syllabus (2022))
	• IfSR-Proportional thinking (IfSR-PT) resources
	• IfSR-PT diagnostic online assessment – ALAN
	• Numeracy resources on the Universal Resources Hub
	• National Numeracy Learning Progression
	• How to – technical guide to using PLAN2

What does proportional thinking look like in a Years 3-8 classroom?

These are some examples for the development of proportional thinking which are non-hierarchical. These components are developed together but are listed like this for ease of reading. The examples below are adapted using information from the National Numeracy Learning Progression (2018, 2020), NSW Mathematics K-10 syllabus (2022) and key research. For further information see the NSW Mathematics K-10 syllabus and the National Numeracy Learning Progression.

- work with fractions having denominators of 2, 3, 4, 5, 8 building to 6, 10, 12, 100 to establish patterns and relationships between fractions, decimals and percentages
- understand and explain the concept of per cent being out of 100, for example, 25% means 25 out of 100
- apply knowledge of, and flexibly use benchmark fractions, percentages and decimals, such as 10%, $\frac{1}{4}$, 0.5, 75%, 100% $= \frac{1}{1} = 1.0$
- work with fractions, decimals and percentages to find the quantity where the result is a whole number

- calculate with any fraction (decimal or percentage) to express answers that are not whole numbers, for $4 \div 5 = \frac{4}{5}$ (or 0.8 or 80%)
- increase and decrease quantities by a percentage and apply to problems, such as interest on loans, discounts, GST
- choose and use a variety of models, such as the Bar model to support and understand when problem solving – see reSolve
- understand the relationship between a fraction, decimal and percentage as different representations of the same quantity
- identify ratios as a part-to-part comparison, using equivalent fractions or percentage
- use ratio and scale factors to enlarge or reduce the size of objects
- recognise and solve problems involving ratios with and without digital technology
- interpret a rate as a comparison between two different types of quantities
- use a ratio to increase or decrease a quantity to maintain a given proportion
- use rates to determine how quantities change in a real world context, including best buy by comparing price per unit, or quantity per monetary unit, for example, 500 grams for $4.50 compared with 300 grams for $2.75.

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