Dr Peter Gould, Leader Numeracy, Early Learning and Primary Education Office, explores the nature of problem solving, the ways mathematics is used for everyday living and creative ways this can translate to mathematics in the classroom.

The will, the skill and the understanding

In talking about what makes a champion, Muhammad Ali once stated … *that they have to have the skill and the will*. He then added … *that the will must be stronger than the skill* (Quotes.net, 2016). But are the will and the skill sufficient when it comes to student learning? If you have no interest in solving a problem, you are unlikely to engage with the problem. If you do not possess the necessary skills, you are also unlikely to be successful in solving a problem. The will and the skill are certainly necessary conditions for solving problems, but are they sufficient?

Do I really need to learn this?

Some types of questions that children ask change little over time. In particular, children frequently wonder, why do I need to learn this? This question can be applied to even the most fundamental components of schooling, such as reading, writing and arithmetic. As adults we know how important these things are to living in a post-Guttenberg era. Yet sometimes we make use of reading, writing and arithmetic so frequently we do it without conscious thought. This is true of many ways that we use mathematics.

Who uses mathematics?

Although we rarely think of using mathematics as we navigate our way each day, we rely on it. A study carried out in 1998 involving 200 adult Australians who recorded the mathematical calculations they did in a typical 24-hour period found that most calculations (85%) were done mentally (Northcote & McIntosh, 1999). The research also showed that the most common purpose for calculation was the calculation of time — 25% of all calculations.

We regularly engage in estimating and measuring time and distance, as well as likelihood. Most often (about 60% of the time) our mental calculations require only an estimate rather than an exact answer. Although people may not be consciously aware of it, quantitative reasoning is required to cross a road, just as it is required to construct a road.

The way we interact with the world requires us to reason with number, measurement, probability, data and spatial sense. However, these interactions often value a timely approximate answer over a delayed exact answer.

The study by Northcote and McIntosh also found the second most common purpose for calculating was shopping. In particular, those interviewed generally found it much easier to do calculations with money, adding up and taking away, than doing similar sums in written form. Can we build on these preferences for mental computation and day-to-day financial transactions in teaching arithmetic?

The mathematics of commerce

In many developing economies there exists a large section of the economy known as the informal economy. The informal economy is sometimes described as the economy of the poor and frequently includes children.

In Brazil, which has a large informal economy, three researchers went out into the street markets with a tape recorder, posing as ordinary market shoppers (Nunes, Schliemann, & Carraher, 1993). At each stall, they presented the young stallholder with a transaction designed to test a particular arithmetical skill. The purpose of the research was to determine how effective traditional mathematics instruction was, which all the young market traders had received in school since the age of six.

About a week after they had recorded the children explaining the costs of various quantities at their stalls, the researchers went back and asked each of them to take a pencil-and-paper test that comprised exactly the same arithmetic problems that had been presented to them in the context of purchases the week before.

Although the children's practical arithmetic was accurate when they were at their market stalls, the percentage of correct answers dropped to 74% when doing similar verbal problems and they averaged only 37% when virtually the same problems were presented to them in the form of an arithmetic test.

One of the questions asked of a young vendor when he was selling coconuts costing 35 cruzeiros each, was: *I'm going to take four coconuts. How much is that?* The boy replied: *There will be one hundred five, plus thirty, that's one thirty-five … one coconut is thirty-five … that is … one forty.*

Purchases of two or three coconuts are quite common and these are values that street traders often know. The boy began by breaking the problem up into simpler quantities — in this case, three coconuts plus one coconut. This enabled him to start out with the fact he knew, namely that 3 coconuts cost Cr$105. Then, to add on the cost of the fourth coconut, he first adds Cr$30 to give Cr$135 before restating the cost of one coconut and adding the final Cr$5.

On the formal arithmetic test, the boy was asked to calculate 35 x 4. He worked mentally, vocalising each step as requested, but the only thing he wrote down was the answer. His problem solving was: *Four times five is twenty, carry the two; two plus three is five, times four is twenty*. He then wrote down 200 as his answer. Despite the fact that, numerically, it was the same problem he had answered correctly at his market stall, he got it wrong.

In a later study by Paterson and Bana (2005) looking at students in Years 3, 5, 7 and 9 in Western Australia, it was found that neither experience with nor the use of the context of money had any effect at all, except in Year 3. In Year 3 the students' performance was worse when dealing with money problems.

In our classrooms

Is the type of problems we ask the main challenge we face? In 2015, the Year 3 Numeracy Assessment for NAPLAN contained the following question.

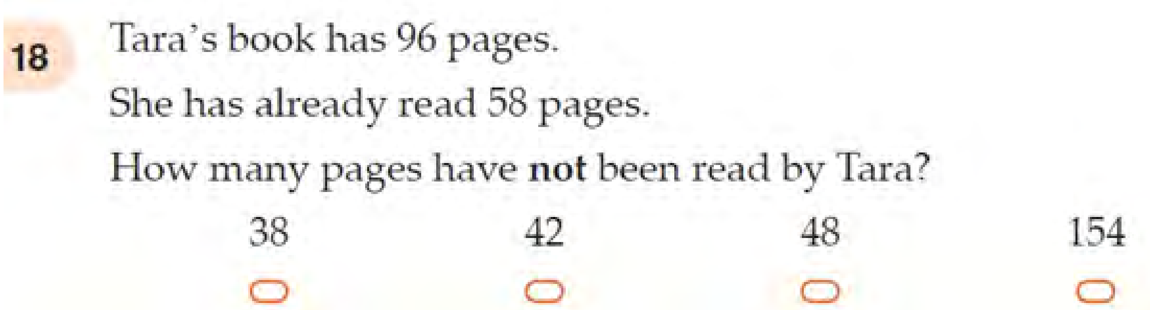
18 Tara's book has 96 pages.
She has already read 58 pages.
How many pages have **not** been read by Tara?
38 42 48 154

Figure 1. Q18 in the Year 3 Numeracy assessment in NAPLAN 2015

Although less than half of the Year 3 students in NSW correctly answered the question, even more troubling is that over one-quarter of Year 3 chose 42 for the answer. This error of subtracting the smaller number from the larger, that is subtracting 6 from 8 when attempting to subtract 58 from 96, appears to be widespread. This prompted a more in depth investigation of the problem, starting in our primary schools taking part in Early Action for Success.

When students were asked to show their working out in response to a number of questions designed to have students use place value when adding or subtracting, it soon became apparent how they were thinking. Figure 2 shows a Year 4 student's reasoning when attempting to subtract $27 from $53.

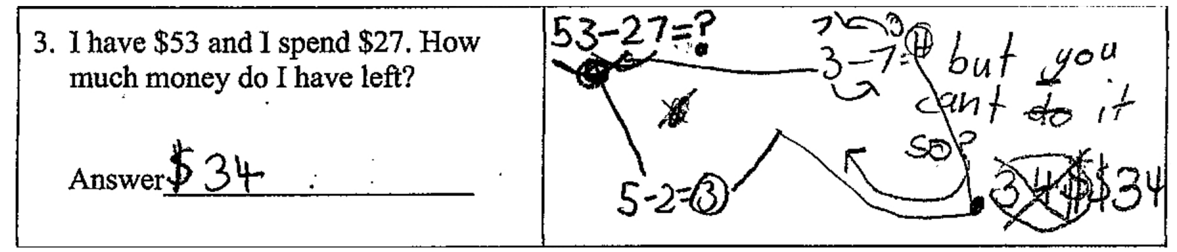
3. I have $53 and I spend $27. How much money do I have left?
Answer $34

Figure 2. A Year 4 student's explanation of the smaller from larger error

He considers subtracting the digits in the numbers separately. He first subtracts 2 from 5 to obtain 3, but when he attempts to subtract 7 from 3 he realises that he cannot, so he reverses the order of the subtraction to obtain 4. Educationally, this process of splitting the digits is a bad practise, as the digits no longer reflect their place value.

It is important to realise that this problem of splitting the digits in a subtraction and then losing their place value was not restricted to those students who are struggling with mathematics. The response shown in Figure 3 comes from a Year 3 student who achieved the top performance band for numeracy in the National Assessment Program – Literacy and Numeracy.

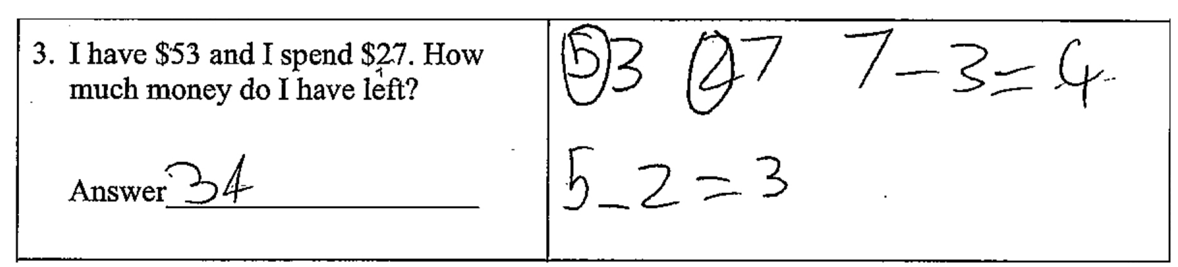
3. I have $53 and I spend $27. How much money do I have left?
Answer 34

Figure 3. A high-performing Year 3 student's response of splitting the digits

How common was the answer 34? In examining the responses of 711 Year 3 students to Question 3, out of 398 students who answered incorrectly, 117 answered 34. That is, 29% of the incorrect responses displayed the smaller-from-larger error. Moreover, the responses displaying this error were not restricted to students performing in the lower bands on NAPLAN. As in the Paterson and Bana study, the use of money in the question setting did not reduce the procedural errors the students made.

When the questions were not about money, the unusual modes of answering questions that required an appreciation of place value did not diminish. The response shown in Figure 4 was produced by a Year 3 student who achieved the second highest performance band for numeracy in NAPLAN.

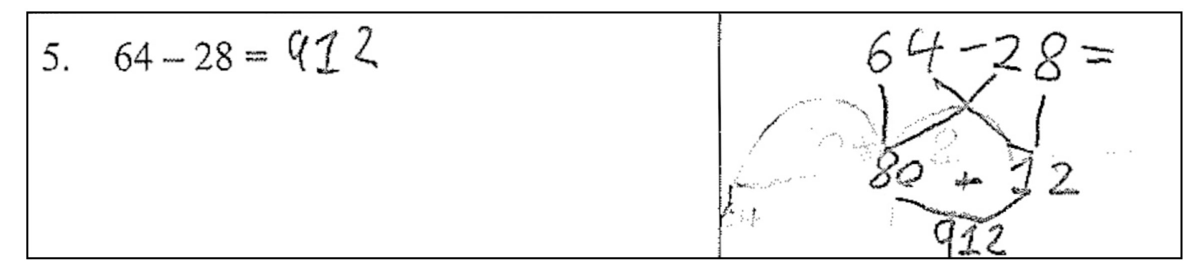
5. 64 – 28 = 912

Figure 4. A Year 3 student's response using an incorrect splitting procedure

The working in Figure 4 suggests that although the student has decomposed the numbers into the correct units of tens and ones, the student has added the component parts to obtain 80 and 12, before combining these results to produce an answer of 912. This answer of 912 to a subtraction involving two 2-digit numbers does not make sense.

The loss of sense-making can be a product of focusing solely on procedural knowledge — *what* without the *why*. Consider the answer provided to 64 – 28 in Figure 5.

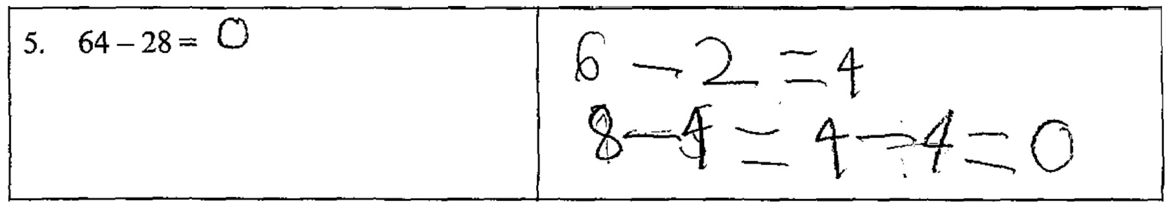

Figure 5. An answer displaying a lack of conceptual knowledge

If you look only at the answer of zero, it doesn't make sense. When you read over the steps that the student has followed to achieve this answer you can begin to understand how an answer that makes no sense was achieved. After separating the digits and subtracting the smaller from the larger, the student appears to have taken this one step further, and subtracted the resulting answers (4 – 4 = 0).

When students learn to do arithmetic it is possible to focus only on the procedures that need to be followed. However, without understanding why the procedure works we have no way of telling if an answer is reasonable. This has led me to frequently ask students if there is any way they can tell if their answer is correct. If the students respond no, I tend to think that they may not have a sense of the size of the numbers involved in the question.

We need to be very careful in our teaching that students *do not lose conceptual knowledge in the process of gaining procedural knowledge* (Narode, Board, & Davenport, 1993, p. 260). The student's response of 0 shown in Figure 5 appears to be a clear example of a student whose procedural knowledge has supplanted conceptual knowledge. Namely, that the difference between two numbers can only be zero if the numbers are the same.

Although the mathematics that we use every day is within very practical contexts, problems involving money are not always interpreted by students as anything associated with their lives. Moreover, many conscientious students appear to be learning and practicing calculation procedures that do not make sense.

Helping students to appreciate place value

If students provide responses with errors similar to those above, an additional question can help them to focus on the size of the answer before they start to respond to the question. Some teachers like to ask students to estimate the answer before they work it out. Lately, I have been trying a more specific question to prompt students' thinking about place value.

> *I have $53 and I spend $27.*
> *How much money do I have left?*
> *Is the answer more than $30 or less than $30?*

Before students begin to subtract $27 from $53 to indicate how much money is left, I want them to commit (usually by recording *more* or *less*) to the size of the answer, compared to the value of $30 provided. This recording becomes important in follow up discussions, particularly if a student answers 34. Think about the purpose for using the value of $30 in the prompt rather than $20 or $40. With 2-digit subtraction I always use a multiple of ten in my prompt question to focus on place value.

It is important to determine how students solve mathematics problems. Not just whether they have relevant skills but also if they understand what they are doing. When students practice procedures they do not understand there is a clear danger they will practice incorrect procedures. The longer students use incorrect methods to calculate the more difficult it is to learn correct ones.

References and further reading

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