# Mathematics Stage 5 – unit of learning – working with triangles



Contents

[Rationale 3](#_Toc130466419)

[Overview 4](#_Toc130466420)

[Outcomes 5](#_Toc130466421)

[Lesson sequence and details 7](#_Toc130466422)

[Learning episode 1 – 45-degree angles 7](#_Toc130466423)

[Learning episode 2 – larger and smaller angles 9](#_Toc130466424)

[Learning episode 3 – the tangent ratio 11](#_Toc130466425)

[Learning episode 4 – the sine and cosine ratios 13](#_Toc130466426)

[Learning episode 5 – I lost my calculator 15](#_Toc130466427)

[Learning episode 6 – finding missing sides 17](#_Toc130466428)

[Learning episode 7 – significant distances 19](#_Toc130466429)

[Learning episode 8 – mapping the classroom 21](#_Toc130466430)

[Learning episode 9 – surd is the word 23](#_Toc130466431)

[Learning episode 10 – distances 25](#_Toc130466432)

[Learning episode 11 – midpoints 27](#_Toc130466433)

[Learning episode 12 – trigonometry for navigation 29](#_Toc130466434)

[Learning episode 13 – how far around is one degree? 31](#_Toc130466435)

[Learning episode 14 – how steep is too steep? 33](#_Toc130466436)

[Learning episode 15 – slopes as gradients 35](#_Toc130466437)

[Learning episode 16 – gradients in the Cartesian plane 37](#_Toc130466438)

[References 39](#_Toc130466439)

## Rationale

The NSW Department of Education publishes a range of curriculum support materials, including samples of lesson sequences, scope and sequences, assessment tasks, examinations, student and teacher resource booklets, and curriculum planning and curriculum evaluation templates. The samples are not exhaustive and do not represent the only way to complete or engage in each of these processes. Curriculum design and implementation is a dynamic and contextually-specific process. While the mandatory components of syllabus implementation must be met by all schools, it is important that the approach taken by teachers is reflective of their needs and faculty/school processes.

NESA defines [programming](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming) as the process of ‘selecting and sequencing learning experiences which enable students to engage with syllabus outcomes and develop subject specific skills and knowledge’ ([NESA](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming) 2022). A program is developed collaboratively within a faculty. It differs from a unit in important ways, as outlined by NESA on their [advice on units](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming/advice-on-units) page. A unit is a contextually-specific plan for the intended teaching and learning for a particular class for a particular period. The organisation of the content in a unit is flexible and it may vary according to the school, the teacher, the class, and the learning space. They should be working documents that reflect the thoughtful planning and reflection that takes place during the teaching and learning cycle. There are mandatory components of programming and unit development, and this template provides one option for the delivery of these requirements. The NESA and department guidelines that have influenced this template are elaborated upon at the end of the document.

This resource has been developed to assist teachers in NSW Department of Education schools to create learning that is contextualised to their classroom. It can be used as a basis for the teacher’s own program, assessment, or scope and sequence, or be used as an example of how the new curriculum could be implemented. The resource has suggested timeframes that may need to be adjusted by the teacher to meet the needs of their students.

## Overview

**Description:** This program of learning addresses content from the focus areas Trigonometry A, Linear relationships A, Numbers of any magnitude, as well as Path focus areas Linear relationships C, and Indices C. The lessons and sequences in this program of learning are designed to allow students to use the properties of right-angled triangles to solve a range of geometrical, measurement and real world problems.

**Duration:** This program of learning is designed to be completed over a period of approximately 5 weeks, considering a range of factors and variables. This approach to timing can be adapted to suit the school context.

**Explicit teaching:** Suggested learning intentions and success criteria are available for some lessons provided. Learning intentions and success criteria are most effective when they are contextualised to meet the needs of students in the class. The examples provided in this document are generalised to demonstrate how learning intentions and success criteria could be created.

## Outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* applies trigonometric ratios to solve right-angled triangle problems **MA5-TRG-C-01**
* determines the midpoint, gradient, and length of an interval, and graphs linear relationships, with and without digital tools **MA5-LIN-C-01**
* solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures **MA5-MAG-C-01**

**Related Path outcomes:**

* describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems **MA5-LIN-P-01**
* describes and performs operations with surds and fractional indices **MA5-IND-P-02**

The identified Life Skills outcome that relates to this unit is **MALS-POS-01** – demonstrates knowledge of position and direction in everyday contexts.

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/syllabuses/mathematics-k-10-2022) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

**Prior to planning for teaching and learning, please consider the following:**

**Engagement**

* How will I provide authentic, relevant learning opportunities for students to personally connect with lesson content?
* How will I support every student to grow in independence, confidence, and self-regulation?
* How will I facilitate every student to have high expectations for themselves?
* How will I identify and provide the support each student needs to sustain their learning efforts?

**Representation**

* What are some different ways I can present content to enable every student to access and understand it?
* How will I identify and address language and/or cultural considerations that may limit access to content for students?
* How will I make lesson content and learning materials more accessible?
* How will I plan learning experiences that are relevant and challenging for the full range of students in the classroom?

**Expression**

* How will I provide multiple ways for students to respond and express what they know?
* What tools and resources can students use to demonstrate their understanding?
* How will I know every student has understood the concepts and language presented in each lesson?
* How will I monitor if every student has achieved the learning outcomes and learning growth?

## Lesson sequence and details

### Learning episode 1 – 45-degree angles

#### Teaching and learning activity

Students establish the constancy of the ratio between 2 sides of isosceles right-angled triangles and use this 1:1 ratio to measure heights of tall objects.

#### Syllabus content

* Verify the constancy of the sine, cosine and tangent ratios for a given angle by applying knowledge of similar right-angled triangles

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [**45-degree angles**](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-01-45-degree-angles.docx)**Duration**: 1–2 lessons**Learning intention*** To understand the constant properties of all right-angled isosceles triangles.

**Success criteria*** I can use a clinometer to measure angles to tall objects.
* I can use $45^{o}$, right-angled, isosceles triangles to measure the heights of tall objects.
* I can explain why the distance to a tree is the same as the height of a tree when the angle is $45^{o}$.
 | * Projector
* [*45-degree angles* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-1-45-degree-angles.pptx)
* Optional: Laptop per pair of students
* Class set of rulers
* Class set of protractors
* Grid paper
* Clinometers, one per group
* Large tape measures or trundle wheels
* Printed class set of [Appendix B](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-01-45-degree-angles.docx)
 |  |

### Learning episode 2 – larger and smaller angles

#### Teaching and learning activity

Students construct right-angled triangles with double, triple and other simple ratios between the opposite and adjacent sides, to find angles greater than and smaller than $45^{o}$ to use for measuring heights of tall objects.

#### Syllabus content

* Verify the constancy of the sine, cosine and tangent ratios for a given angle by applying knowledge of similar right-angled triangles

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| **[Larger and smaller angles](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-02-larger-and-smaller-angles.docx)****Duration**: 1–2 lessons**Learning intention*** To be able to explain that the ratio between sides of a right-angled triangle remains the same if the angle is the same, regardless of size.
* To be able to use ratios to find the heights of objects.

**Success criteria** * I can use a clinometer to find positions that give desired angles towards tall objects.
* I can use ratios in right-angled triangles to predict the heights of objects.
 | * Projector
* [*Larger and smaller angles* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-2-larger-and-smaller-angles.pptx)
* Optional: laptop per pair of students
* Class set of rulers
* Class set of protractors
* Grid paper
* Clinometers, one per group
* Large tape measures or trundle wheels
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-02-larger-and-smaller-angles.docx) and [Appendix B](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-02-larger-and-smaller-angles.docx)
 |  |

### Learning episode 3 – the tangent ratio

#### Teaching and learning activity

Students investigate the tangent function on their calculator and compare it with right-angled triangles they construct by hand or using digital tools, to establish the tangent ratio.

#### Syllabus content

* Identify and label the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation
* Define the sine, cosine and tangent ratios for angles in right-angled triangles and use trigonometric notation $\sin(θ)$, $\cos(θ)$, $\tan(θ)$
* Identify the sine, cosine and tangent ratios in a right-angled triangle

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [**The tangent ratio**](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-03-the-tangent-ratio.docx)**Duration**: 1 lesson**Learning intention*** To be able to use language associated with trigonometry to describe right-angled triangles.
* To know and be able to define the tangent ratio.

**Success criteria** * I can identify the hypotenuse, opposite and adjacent sides with respect to an angle in a right-angled triangle.
* I can explain the relationship between the value of $\tan(30^{o})$ found in my calculator and a right-angled triangle with a $30^{o}$ angle.
* I can write the tangent ratio for a given angle in a right-angled triangle.
 | * Projector
* [*The tangent ratio* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-3-the-tangent-ratio.pptx)
* Optional: laptop per pair of students
* Class set of rulers
* Class set of protractors
* Grid paper
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-03-the-tangent-ratio.docx)
 |  |

### Learning episode 4 – the sine and cosine ratios

#### Teaching and learning activity

Students investigate the sine and cosine functions on their calculator and compare them with right-angled triangles they construct by hand or using digital tools, to establish the sine and cosine ratios.

#### Syllabus content

* Identify and label the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation
* Define the sine, cosine and tangent ratios for angles in right-angled triangles and use trigonometric notation $\sin(θ)$, $\cos(θ)$, $\tan(θ)$
* Identify the sine, cosine and tangent ratios in a right-angled triangle

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [**The sine and cosine ratios**](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-04-the-sine-and-cosine-ratios.docx)**Duration**: 1 lesson**Learning intention*** To be able to use language associated with trigonometry to describe right-angled triangles.
* To know and be able to define the trigonometric ratios of sine, cosine and tangent.

**Success criteria** * I can identify the hypotenuse, opposite and adjacent sides with respect to an angle in a right-angled triangle.
* I can explain the relationship between the value of $sin30^{o}$ found in my calculator and a right-angled triangle with a $30^{o}$ angle.
* I can write 3 trigonometric ratios for a given angle in a right-angled triangle.
 | * Projector
* [*The sine and cosine ratios* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-4-the-sine-and-cosine-ratios.pptx)
* Optional: laptop per pair of students
* Class set of rulers
* Class set of protractors
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-04-the-sine-and-cosine-ratios.docx) and [Appendix B](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-04-the-sine-and-cosine-ratios.docx)
 |  |

### Learning episode 5 – I lost my calculator

#### Teaching and learning activity

Students develop and use a table of trigonometric ratios by constructing right-angled triangles with digital or measuring tools. Students develop an appreciation of scientific calculators as a database of trigonometric ratios that can be used to solve problems.

#### Syllabus content

* Identify and label the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation.
* Identify the sine, cosine and tangent ratios in a right-angled triangle.
* Find approximations of the trigonometric ratios for a given angle

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [**I lost my calculator**](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-05-i-lost-my-calculator.docx)**Duration**: 1 lesson**Learning intention*** To understand that the trigonometric ratios are calculated from the ratios of side lengths in right-angled triangles.
* To understand that values of trigonometric ratios are stored for use on scientific calculators.

**Success criteria** * I can construct a right-angled triangle from a given angle.
* I can measure 3 angles and 3 side lengths in a right-angled triangle.
* I can calculate the value of a trigonometric ratio with only a pencil, a ruler and a protractor.
 | * Optional: projector
* Optional: laptop per pair of students
* Class set of rulers
* Class set of protractors
* Grid paper
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-05-i-lost-my-calculator.docx)
 |  |

### Learning episode 6 – finding missing sides

#### Teaching and learning activity

Students explore the types of problems that trigonometry can help us solve, and the existing skills that support us to apply trigonometry to right-angled triangles. Students are explicitly taught how to use trigonometry to find a missing side of a right-angled triangle.

#### Syllabus content

* Apply trigonometry to find the lengths of unknown sides in right-angled triangles with a given angle including angles in degrees and minutes
* Solve a variety of practical problems involving trigonometric ratios in right-angled triangles

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [**Finding missing sides**](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-06-finding-missing-sides.docx)**Duration**: 2 lessons**Learning intention*** To be able to find a missing side in a right-angled triangle, given an angle and a side.

**Success criteria** * I can label the sides of a right-angled triangle as opposite, adjacent and hypotenuse.
* I can select an appropriate trigonometric ratio based on given information in a right-angled triangle.
* I can write an equation based on a trigonometric ratio.
* I can solve an equation to find a missing side in a right-angled triangle.
 | * Projector
* [*Finding missing sides* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-6-finding-missing-sides.pptx)
* Printed class set of [Appendices A, B, C and D](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-06-finding-missing-sides.docx)
 |  |

### Learning episode 7 – significant distances

#### Teaching and learning activity

Students learn about significant figures by calculating distances between the Earth and the stars using trigonometry, developing the need for a simpler way of expressing large distances.

#### Syllabus content

* Solve a variety of practical problems involving trigonometric ratios in right-angled triangles
* Apply the language of estimation appropriately, including the terms *rounding, approximate* and *level of accuracy*
* Round numbers to a specified number of significant figures

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Significant distances](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-07-significant-distances.docx)Duration: 2 lessonsLearning intention* To appreciate the usefulness of significant figures in expressing very large measurements.
* To be able to round very large numbers to a specified number of significant figures for comparison.

Success criteria * I can compare 2 very large numbers expressed to 2 significant figures.
* I can round a very large number to any number of significant figures.
 | * Projector
* [*Significant distances* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-7-significant-distances.pptx)
* Printed class set of [Appendices B, C and D](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-07-significant-distances.docx)
* A laptop or similar device with internet access

If devices are not available, provisions suggest the use of:* Class set of rulers
* Class set of protractors
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-07-significant-distances.docx)
 |  |

### Learning episode 8 – mapping the classroom

#### Teaching and learning activity

Students use a Cartesian plane over maps of their classroom and school and construct right-angled triangles to find distances and midpoints in the real world.

#### Syllabus content

* Plot and join 2 points to form an interval on the Cartesian plane and use the interval as the hypotenuse of a right-angled triangle
* Use the interval between 2 points as the hypotenuse of a right-angled triangle on the Cartesian plane and apply Pythagoras’ theorem to determine the length of the interval joining the 2 points
* Determine the midpoint of horizontal and vertical intervals on the Cartesian plane
* Apply the process for calculating the mean to find the midpoint, $M  $of the interval joining 2 points on the Cartesian plane

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Mapping the classroom](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-08-mapping-the-classroom.docx)Duration: 2 lessonsLearning intention* To be able to use Pythagoras' theorem to find the distance between 2 points in a Cartesian plane.
* To know that a midpoint is equidistant from 2 endpoints.

Success criteria * I can interpret a map of a known place.
* I can construct a right-angled triangle using horizontal and vertical lines and a desired diagonal line.
* I can apply Pythagoras' theorem to find an unknown distance in a Cartesian plane.
* I can locate a midpoint between 2 points on a Cartesian plane.
 | * Projector
* Laptops – one per pair
* Class set of rulers
* Tape measures, quantity optional dependent on how the activity is run
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-08-mapping-the-classroom.docx) and [Appendix D](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-08-mapping-the-classroom.docx)
* Optional: printed class set of [Appendix B](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-08-mapping-the-classroom.docx) and [Appendix E](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-08-mapping-the-classroom.docx)
 |  |

### Learning episode 9 – surd is the word

#### Teaching and learning activity

Students apply Pythagoras' theorem to measurement problems, investigating the effect of recording their solution as a surd, or rounded to a number of decimal places, as larger measurements are considered.

#### Syllabus content

* Describe the term surd as referring to irrational expressions of the form $\sqrt{x}$ where $x>0$ and is a rational number (Path)
* Recognise that a surd is an exact value that can be approximated by a rounded decimal (Path)

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Surd is the word](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-09-surd-is-the-word.docx)Duration: 1 lessonLearning intention* To recognise that a surd is an exact value that can be approximated by a rounded decimal.
* To understand the need for maintaining accuracy.

Success criteria * I can determine when a square root results in a surd.
* I can convert a surd into a decimal estimate.
* I know when to use surds and when it is appropriate to convert these to decimal estimates.
 | * Tape measure or trundle wheel, one per group
* Tent pegs, distance markers or chalk
* Printed copies of worksheets *Using Pythagoras’ theorem to find a shorter side* and *Using Pythagoras’ theorem to find a Hypotenuse* – one per group of 3
* Printed copies of problems *Pythagorean Shell* and *Open middle problem* or projector to display them
 |  |

### Learning episode 10 – distances

#### Teaching and learning activity

Students engage with Desmos graphs to develop definitions for the distance between 2 points on the Cartesian plane, applying practical methods and generalising to develop a formula.

#### Syllabus content

* Use the interval between 2 points as the hypotenuse of a right-angled triangle on the Cartesian plane and apply Pythagoras’ theorem to determine the length of the interval joining the 2 points
* Use graphing applications to find the distance between 2 points on the Cartesian plane
* Apply knowledge of Pythagoras’ theorem to establish the formula for the distance ($d$) between the 2 points $\left(x\_{1}, y\_{1}\right)$ and $\left(x\_{2}, y\_{2}\right)$ on the Cartesian plane: $d=\sqrt{\left(x\_{2}-x\_{1}\right)^{2}+\left(y\_{2}-y\_{1}\right)^{2}}$ (Path)
* Apply the distance formula to find the distance between 2 points on the Cartesian plane (Path)

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Distances](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-10-distances.docx)Duration: 1 lessonLearning intention* To be able to find the distance between 2 points on the Cartesian plane.

Success criteria * I can apply Pythagoras' theorem to find the distance between 2 points on the Cartesian plane.
* I can apply formulas to find the distance between 2 points on the Cartesian plane.
 | * Projector
* Class set of devices with internet access, at least one per pair (optional – the lesson includes provisions for using the online components via the teacher screen)
 |  |

### Learning episode 11 – midpoints

#### Teaching and learning activity

Students engage with Desmos graphs to develop definitions for the midpoint between 2 points on the Cartesian plane, applying practical methods and generalising to develop a formula.

#### Syllabus content

* Determine the midpoint of horizontal and vertical intervals on the Cartesian plane.
* Apply the process for calculating the mean to find the midpoint, $M$of the interval joining 2 points on the Cartesian plane.
* Use graphing applications to find the midpoint and gradient/slope of an interval.
* Apply the formula to find the midpoint of the interval joining 2 points on the Cartesian plane: $M\left(x,y\right)=\left(\frac{x\_{1}+x\_{2}}{2},\frac{y\_{1}+y\_{2}}{2}\right)$ (Path)

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Midpoints](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-11-midpoints.docx)Duration: 1 lessonLearning intention* To be able to find the midpoint between 2 points on the Cartesian plane.

Success criteria * I can apply averages to find the midpoint between 2 points on the Cartesian plane.
* I can apply a formula to find the midpoint between 2 points on the Cartesian plane.
 | * Class set of devices with internet access, at least one per pair (optional – the lesson includes provisions for using the online components via the teacher screen)
 |  |

### Learning episode 12 – trigonometry for navigation

#### Teaching and learning activity

Students form right-angled triangles on maps based on given angles for directions and distances travelled, to determine how far we have travelled in the 4 Cardinal directions.

#### Syllabus content

* Apply trigonometry to find the lengths of unknown sides in right-angled triangles with a given angle including angles in degrees and minutes
* Solve a variety of practical problems involving trigonometric ratios in right-angled triangles

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Trigonometry for navigation](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-12-trigonometry-for-navigation.docx)Duration: 1 lessonLearning intention* To be able to apply trigonometry to solve practical problems involving distance.

Success criteria * I can construct a right-angled triangle around a diagonal distance on a map.
* I can locate positions on a map in the direction of North, East, South and West.
* I can explain how an angle communicates direction from a given location.
* I can interpret information from a problem to construct a right-angled triangle.
 | * Projector
* Set of protractors, one per group
* Set of trundle wheels or tape measures, one per group
* Printed map of the school, – one per group or one per student
* Printed class set of [Appendix A](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-12-trigonometry-for-navigation.docx) and [Appendix B](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-12-trigonometry-for-navigation.docx)
 |  |

### Learning episode 13 – how far around is one degree?

#### Teaching and learning activity

Students explore the size of a single degree and learn about how technology can allow us to measure angles and direction with greater precision, introducing degrees and minutes.

#### Syllabus content

* Apply trigonometry to find the lengths of unknown sides in right-angled triangles with a given angle including angles in degrees and minutes

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [How far around is one degree?](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-13-how-far-around-is-one-degree.docx)Duration: 1 lessonLearning intention* To understand the concept of an angle as a measure of turning.
* To be able to use angles measured in degrees and minutes when solving problems.

Success criteria* I can describe what a particular angle looks like as a measure of turning.
* I can convert an angle represented as a decimal into degrees and minutes.
* I can solve problems involving finding missing sides in right-angled triangles, with angles represented in degrees and minutes.
 | * Projector
* [*How far around is one degree?* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-13-how-far-around-is-one-degree.pptx)
* Class set of rulers
* Class set of protractors
* Mini whiteboard or an A4 sheet of paper per student
* 1 copy of Appendix A, printed, cut up into individual cards.
* Printed class set of [Appendices B (optional), C and D](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-13-how-far-around-is-one-degree.docx)
* 1 large protractor
* Chalk
 |  |

### Learning episode 14 – how steep is too steep?

#### Teaching and learning activity

Students investigate the steepness of slopes around the school using a variety of methods, calculating angles using trigonometry and determining what angles describe safe slopes.

#### Syllabus content

* Find the size of an angle given one of the trigonometric ratios for the angle using digital tools
* Apply trigonometry to find the size of unknown angles in right-angled triangles including in degrees and minutes

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [How steep is too steep?](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-14-how-steep-is-too-steep.docx)Duration: 2 lessonsLearning intention* To understand how angles can be used to describe slopes.
* To be able to calculate the angle inside a right-angled triangle given 2 sides.

Success criteria * I can recognise and describe slopes physically and using angles.
* I can use trigonometry to find a missing angle inside a right-angled triangle.
 | * Projector
* [*How steep is too steep?.* PowerPoint](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-14-how-steep-is-too-steep.pptx)
* Trundle wheel or tape. measure, one per group of students
* Printed class set of [Appendices A, B and C](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-14-how-steep-is-too-steep.docx)
* Optional: stopwatch, marbles, round lollies, hard sloped surfaces
 |  |

### Learning episode 15 – slopes as gradients

#### Teaching and learning activity

Students investigate the steepness of slopes around the school represented as a gradient and compare safe slopes with angle measurements.

#### Syllabus content

* Apply the relationship gradient $m=\frac{rise}{run}$ to find the gradient/slope of the interval joining the 2 points

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Slopes as gradients](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-15-slopes-as-gradients.docx)Duration: 1 lessonLearning intention* To be able to calculate and represent a slope as a gradient.

Success criteria * I can calculate the gradient of a surface.
* I can explain the meaning of a calculated gradient in terms of rise and run.
 | * Projector
* Printed class set of [Appendices A, B and C](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-15-slopes-as-gradients.docx)
* Laptop per pair of students
* Tape measure, one per group
* Optional: stopwatch, marbles, Jaffas or Maltesers, hard sloped surfaces
 |  |

### Learning episode 16 – gradients in the Cartesian plane

#### Teaching and learning activity

Students engage with Desmos graphs to develop definitions for gradients between 2 points on the Cartesian plane, applying practical methods and generalising to develop a formula.

#### Syllabus content

* Apply the relationship gradient $m=\frac{rise}{run}$ to find the gradient/slope of the interval joining the 2 points
* Distinguish between intervals with positive and negative gradients from a diagram
* Explain why horizontal intervals have a gradient of 0 and vertical intervals have undefined gradients using the gradient relationship
* Use the relationship $m=\frac{rise}{run}$ to establish the formula for the gradient $(m)$ of the interval joining the 2 points $(x\_{1},y\_{1})$ and $(x\_{2}, y\_{2})$ on the Cartesian plane: $m=\frac{x\_{2}-x\_{1}}{y\_{2}-y\_{1}}$ (Path)
* Apply the gradient formula to find the gradient of the interval joining 2 points on the Cartesian plane (Path)

Table – lesson details

|  |  |  |
| --- | --- | --- |
| Visible learning | Required resources | Registration, adjustments and evaluation notes |
| [Gradients in the Cartesian plane](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s5-unit-2-lesson-16-gradients-in-the-cartesian-plane.docx)Duration: 1 lessonLearning intention* To be able to find gradients between 2 points on the Cartesian plane.

Success criteria * I can describe gradients of intervals as positive or negative.
* I can explain what a gradient of $0$ will look like.
* I can find a rise and run in the Cartesian plane and use this to find the gradient of an interval.
* I can apply a formula to find gradients.
 | * Class set of Appendix A (one between 2 would suffice), cut into cards.
* Class set of devices with internet access, at least one per pair (optional - the lesson includes provisions for using the online components via the teacher screen).
 |  |

## References

[NSW Mathematics K-10 Syllabus](https://curriculum.nsw.edu.au/syllabuses/mathematics-k-10-2022) © 2022 NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales.

NESA (NSW Education Standards Authority) (2022) ‘[Programming](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming)’, Understanding the curriculum, NESA website, accessed 14 March 2023.

**© State of New South Wales (Department of Education), 2023**

The copyright material published in this resource is subject to the *Copyright Act 1968* (Cth) and is owned by the NSW Department of Education or, where indicated, by a party other than the NSW Department of Education (third-party material).

Copyright material available in this resource and owned by the NSW Department of Education is licensed under a [Creative Commons Attribution 4.0 International (CC BY 4.0) licence](https://creativecommons.org/licenses/by/4.0/).



This licence allows you to share and adapt the material for any purpose, even commercially.

Attribution should be given to © State of New South Wales (Department of Education), 2023.

Material in this resource not available under a Creative Commons licence:

* the NSW Department of Education logo, other logos and trademark-protected material
* material owned by a third party that has been reproduced with permission. You will need to obtain permission from the third party to reuse its material.

**Links to third-party material and websites**

Please note that the provided (reading/viewing material/list/links/texts) are a suggestion only and implies no endorsement, by the New South Wales Department of Education, of any author, publisher, or book title. School principals and teachers are best placed to assess the suitability of resources that would complement the curriculum and reflect the needs and interests of their students.

If you use the links provided in this document to access a third-party's website, you acknowledge that the terms of use, including licence terms set out on the third-party's website apply to the use which may be made of the materials on that third-party website or where permitted by the *Copyright Act 1968* (Cth). The department accepts no responsibility for content on third-party websites.