# Finding missing sides

In this activity, students explore the types of problems that trigonometry can help us solve, and the existing skills that support us to apply trigonometry to right-angled triangles. Students are explicitly taught how to use trigonometry to find a missing side of a right-angled triangle.

This activity is designed on the premise that students are already familiar with solving equations that involve one step, labelling sides of right-angled triangles and writing trigonometric ratios.

## Visible learning

### Learning intention

* To be able to find a missing side in a right-angled triangle, given an angle and a side.

### Success criteria

* I can label the sides of a right-angled triangle as opposite, adjacent and hypotenuse.
* I can select an appropriate trigonometric ratio based on given information in a right-angled triangle.
* I can write an equation based on a trigonometric ratio.
* I can solve an equation to find a missing side in a right-angled triangle.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* applies trigonometric ratios to solve right-angled triangle problems **MA5-TRG-C-01**

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Please use the associated PowerPoint Finding missing sides to display images in this lesson.

## Activity structure

### Warm up

The purpose of this warmup is to activate prior knowledge, and to consider the versatility of what we already know how to do.

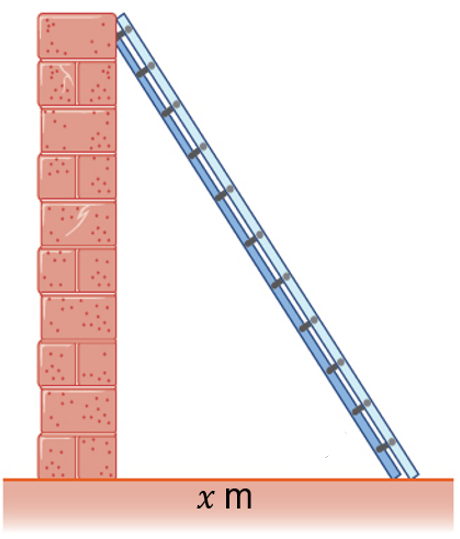
Students solve a range of one-step equations, resembling those needed to solve right-angled triangle problems using trigonometry.

1. Give each student a copy of [Appendix A](#_Appendix_A) or use the associated PowerPoint and ask them to read the examples individually. Students can place their hand on the desk as a thumbs up when they have finished reading.
2. Students are then to engage in a [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), discussing what is going on in each step of the worked solution. Students should attempt to answer the self-explanation questions.
3. Students should complete the ‘Your turn’ section in pairs.
4. Repeat steps 1–3 for the second page in [Appendix A](#_Appendix_A), focusing on equations that contain a trigonometric ratio.

### Launch

1. Display Figure 1 or use the associated PowerPoint.

Figure 1 – ladder against a wall

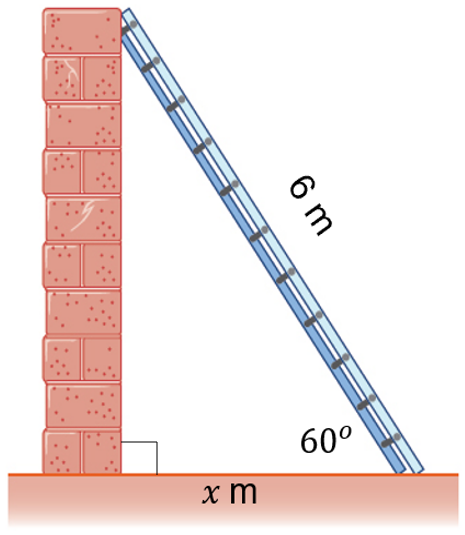


1. Explain to students the following situation: we have a 6 metre ladder and have been advised that it is safest if the angle from the ground is 60°. We want to know how far out from the wall the ladder should be to achieve this.
2. Using [mini whiteboards](https://bit.ly/miniwhiteboards) ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)), have students roughly sketch the triangle shown, and mark the information from the problem in the relevant places.
3. Use a strategy such as pause, pounce, bounce ([bit.ly/pausepouncebouncestrategy](https://bit.ly/pausepouncebouncestrategy)) to investigate student solutions.

It is likely that some students will overlook the fact that the wall and floor form a right angle. Teachers may choose to wait until after step 7 and pose the question ‘Will these trigonometric ratios still apply without the right angle?’ and emphasise the importance of this part of the triangle.

1. Display Figure 2 or use the associated PowerPoint.

Figure 2 – 6 m ladder against wall



1. Students engage in a [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) about how they might find the required distance from the wall, .
2. Display the worked solution of this situation below. Students are to repeat the Think-Pair-Share process discussing the self-explanation prompts at each step.

|  |  |  |
| --- | --- | --- |
| Step | Worked example | Self-explanation prompts |
| 1 | This is an image of a right-angled triangle, formed by a 6 metre ladder leaning up against a brick wall of unknown height. The angle the ladder makes with the grounds is 60 degrees, and the length of the floor between the foot of the wall and the foot of the ladder is the variable x which students will need to find. The ladder is also labelled Hypotenuse, the wall is labelled Opposite, and the floor is labelled Adjacent. | * Which strategies might have been used to label the sides as hypotenuse, opposite and adjacent? |
| 2 | This is an image of a right-angled triangle, formed by a 6 metre ladder leaning up against a brick wall of unknown height. The angle the ladder makes with the grounds is 60 degrees, and the length of the floor between the foot of the wall and the foot of the ladder is the variable x which students will need to find. The ladder is also labelled Hypotenuse, the wall is labelled Opposite, and the floor is labelled Adjacent. SOH CAH TOA is next to the image with CAH circled. | * Why was CAH circled in SOH CAH TOA? |
| 3 |  | * Why was cosine used? * Why was replaced with ? * Why was adjacent replaced with ? * Why was hypotenuse replaced with 6? * How might this equation have been solved differently? |

### Explore

1. Distribute copies of [Appendix B](#_Appendix_B) to all students and instruct that they are to use the method shown previously to find the missing distance in each of the 3 examples.
2. Have students work in groups to solve each problem.

In Building Thinking Classrooms in Mathematics, Peter Liljedahl outlines how groups who regularly experience working on vertical, non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) display greater productivity and likelihood of deep mathematical thinking.

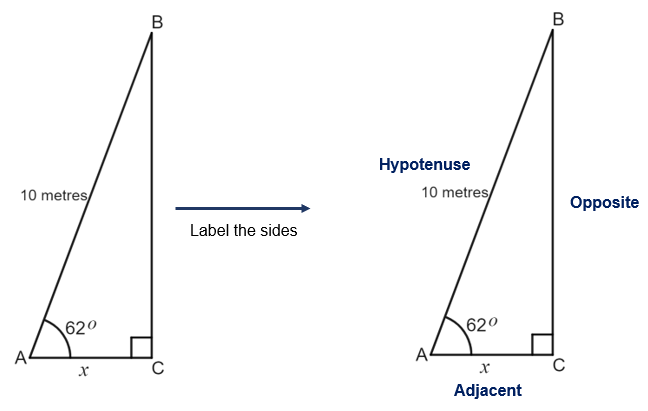
1. Reflection questions for discussion as a class:
2. How did you know which trigonometric ratio to choose?
3. How did you know which trigonometric ratio was definitely not helpful?
4. How do you know your solution is correct?

Take this opportunity to allow individuals to share what they noticed, for example, when the hypotenuse is not involved, we always choose tan.

### Summarise

1. Print and distribute [Appendix C](#_Appendix_C) to students.
2. Display [Example 1](#_Example_1), shown on the next page.
3. Have students construct a notice and wonder list ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) for Example 1.
4. Students should be challenged to articulate how in the example a trigonometric ratio has been chosen.
5. Model Example 1 for students, explaining the [faded example structure](https://bit.ly/fadedexamplesstrategy) ([bit.ly/fadedexamplesstrategy](https://bit.ly/fadedexamplesstrategy)).
6. Students should now attempt examples 2 through to 5.

#### Example 1

Choose a trigonometric ratio



##### Write the ratio

##### Solve the resulting equation

because

…

metres

I know this measurement is correct because the adjacent side AC should be less than the hypotenuse AB.

### Apply

1. Students complete the goal free problems in [Appendix D](#_Appendix_D). Students should be encouraged to find as much information as possible in each situation.
2. Reflection questions to discuss:
3. Did any students identify a mistake and correct it?
4. Did any students confirm their result by comparing to other values they had found, for example, knowing the hypotenuse should be the longest side?

## Assessment and Differentiation

### Suggested opportunities for differentiation

**Warm up**

* Algebraic tiles have been used in the worked examples in [Appendix A](#_Appendix_A) to aid students in solving one step equations.
* Students may need some assistance when is in the denominator in [Appendix A](#_Appendix_A).

**Launch/Explore**

* Images have been provided in the worked example to provide support for students.

**Summarise**

* Students may need some assistance in example 4 when appears in the denominator.

### Suggested opportunities for assessment

* Collect all handouts completed from students to review their understanding of how to find a missing side.
* Monitor student responses in the class discussions to check for understanding.
* [Appendix D](#_Appendix_D) could be used as an [exit ticket](https://bit.ly/exitticketstrategy) ([bit.ly/exitticketstrategy](https://bit.ly/exitticketstrategy)).

## Appendix A

### Solving equations involving one step

#### Worked example 1

|  |  |  |
| --- | --- | --- |
| Question | Method 1 | Method 2 |
| Solve | Two sets of balance scales with algebraic tiles. The first set of balanced scales has 4 algebraic tiles on the left, each labelled x and on the right is the number 12. Underneath this first is 4x=12. The second set of balanced scales has 4 algebraic tiles on the left, each labelled x and on the right is the four number 3's. Underneath this first is 4x=12, followed by x=3. | , because |
| Solve | Three sets of balance scales with algebraic tiles. The first set of balanced scales has an algebraic tile labelled x/2 on the left and on the right is the number 4. Underneath this first scale is x/2=4. The second set of balanced scales has 2 algebraic tiles on the left, each labelled x/2 and on the right is two number 4's. Underneath this second scale is 2x(x/2)=4x2. The third set of balanced scales has 1 algebraic tile on the left, labelled x and on the right is the number 8. Underneath this third scale is x=8. | , because |

##### Self-explanation prompts

* Which word could be used to replace the 'equals' (=) symbol?
* How might be said in words?
* How might be written differently using a symbol?
* Why do we divide when the original equation contains multiplication?
* Why do we multiply when the original equation contains division?
* Why is the final sentence important?

##### Your turn

Find the value of in each of the equations below.

#### Worked example 2

This image shows a table with worked examples of how to solve an equation involving one step, with non integer components, including trigonometry. In the first example, the problem to solve is x/4=0.25. The solution interprets the question as meaning something is divided by 4 to give the same result as 0.25. 0.25 is then multiplied by 4 to get an answer of 1, which is then reasoned to consider why the answer should be 1, which is that 1 divided by 4 gives 0.25. 
In the second example, the problem to solve is x/10=sin30. The solution interprets the question as meaning something is divided by 10 to give the same result as sin30. sin30 is evaluated to get 0.5, then 0.5 is multiplied by 10 to get an answer of 5, which is then reasoned to consider why the answer should be 5, which is that 5 divided by 10 gives 0.5, or sin30.

##### Self-explanation prompts

* Why do we multiply when the original equation contains division?
* Why does the example change the equation from into ?
* How might be written differently using a symbol?

##### Your turn

Find the value of in each of the trigonometric equations below.

## Appendix B

### Finding side lengths using trigonometry

Right-angled triangle scenario 1

This is an image of a problem constructed in Desmos. The image displays a situation, which read that "A helicopter can be seen from 100 metres away at an angle of 25 degrees. How high is the helicopter from the ground?"

The image also shows a helicopter in the sky and a right-angled triangle constructed around the position of the helicopter relative to the ground. The distance along the ground is marked as 100 metres, the angle from the ground is marked as 25 degrees and the height of the helicopter from the ground is marked as h. 

Right-angled triangle scenario 2

This is an image of a problem constructed in Desmos. The image displays a situation, which read that "A boat drops a 12 m anchor that makes an angle of 40 degrees with the waterline. How deep is the water?"

The image also shows a boat in the water and a right-angled triangle constructed around the position of the anchor and the sea level. The distance along the line of the anchor is marked as 12 metres, the angle from the waterline to the anchor is marked as 40 degrees and the depth of the water is marked as d. 

Right-angled triangle scenario 3

This is an image of a problem constructed in Desmos. The image displays a situation, which read that "A chairlift to the top of a 50 metre mountain is to be built at an angle of 15 degrees. How long will we need to make the cable (L) for the chairlift?"

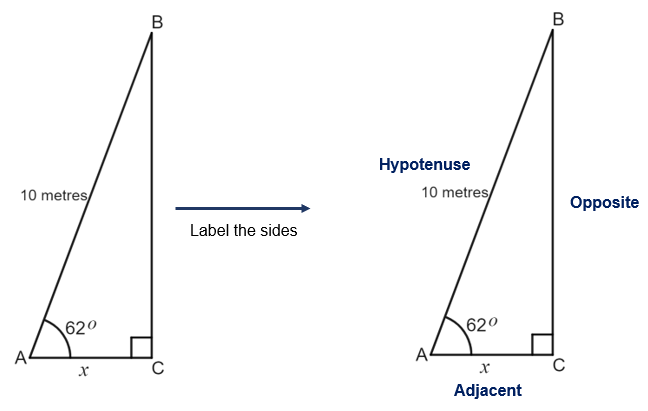
The image also shows a mountain and the chairlift approaching the top of the mountain and a right-angled triangle formed by the chairlift line, the ground and the height up the centre of the mountain. The distance along the chairlift is marked as L, the angle from the ground is marked as 15 degrees and the height of the mountain is marked as 50 metres. 

Images created using [Desmos](https://www.desmos.com/?lang=en) and are licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

## Appendix C

### Faded examples – Finding missing sides

#### Example 1



##### Choose a trigonometric ratio



##### Write the ratio

##### Solve the resulting equation

because

…

metres

I know this measurement is correct because the adjacent side AC should be less than the hypotenuse AB.

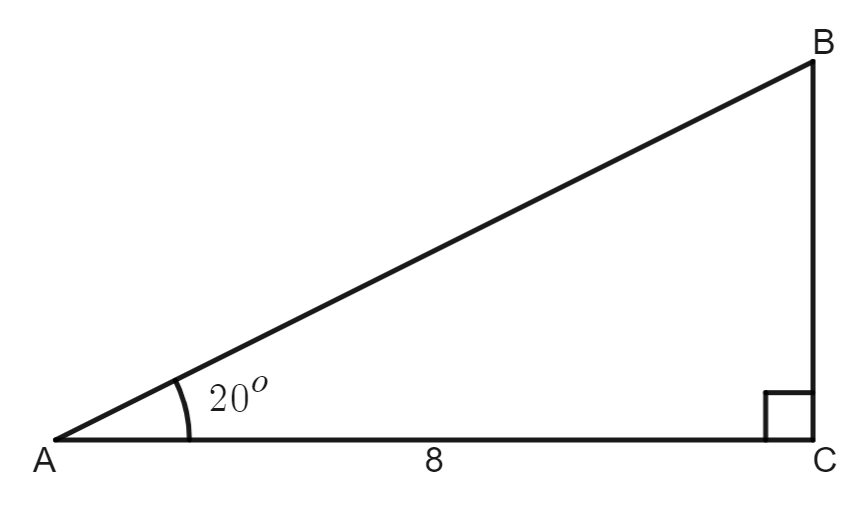
|  |  |  |  |
| --- | --- | --- | --- |
| Example 2 | Example 3 | Example 4 | Example 5 |
| This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side BC is labelled with 4.2 metres. The angle ABC is 78 degrees and the angle ACB is a right angle. AB is labelled Hypotenuse. | This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side AB is labelled with 12.6 metres. The angle ACB is 46 degrees and the angle ABC is a right angle. | This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side BC is labelled with 8 metres. The angle BAC is 35 degrees and the angle ABC is a right angle. | This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side BC is labelled with 3.6 metres. The angle BAC is 19 degrees and the angle ACB is a right angle. |
| Label the sides | Label the sides | Label the sides |  |
| SOH CAH TOA, the letter's H have a line through it. | SOH CAH TOA | SOH CAH TOA |  |
|  |  |  |  |
|  |  |  |  |
| because |  |  |  |
|  |  |  |  |
| metres |  | metres |  |
| I know this measurement is correct because the opposite side AC (19.76 metres) is opposite a much larger angle than the adjacent side BC | I know this measurement is correct because… |  |  |

## Appendix D

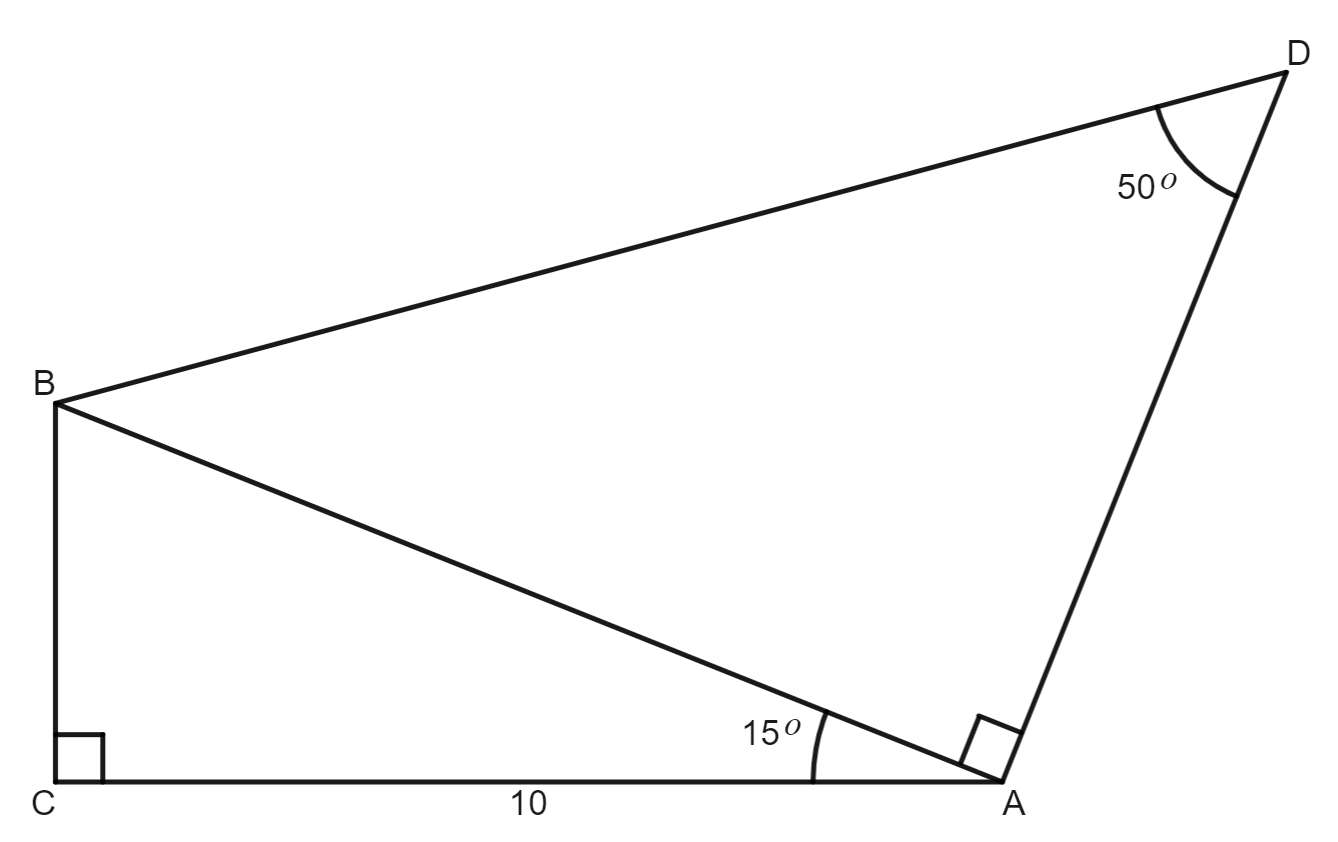
### Goal free problems

Find any information you can in the scenarios below. How do you know when you are finished?

#### Scenario 1



#### Scenario 2



## Sample solutions

### Appendix A

#### Worked example 1 – Your turn

#### Worked example 2 – Your turn



### Appendix B

Solution for right-angled triangle scenario 1

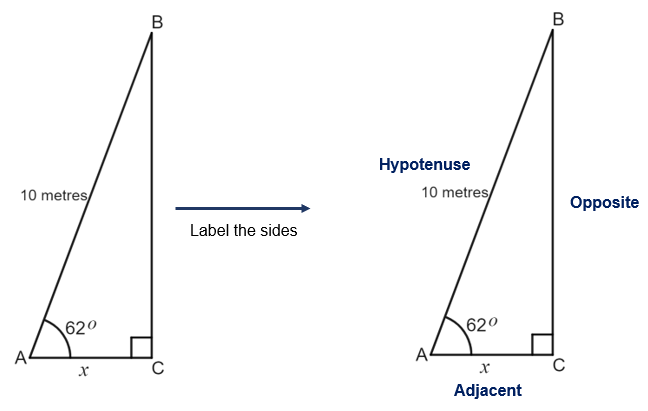
m

Solution for right-angled triangle scenario 2

Solution for right-angled triangle scenario 3

### Appendix C

#### Example 1



##### Choose a trigonometric ratio



##### Write the ratio

##### Solve the resulting equation

because

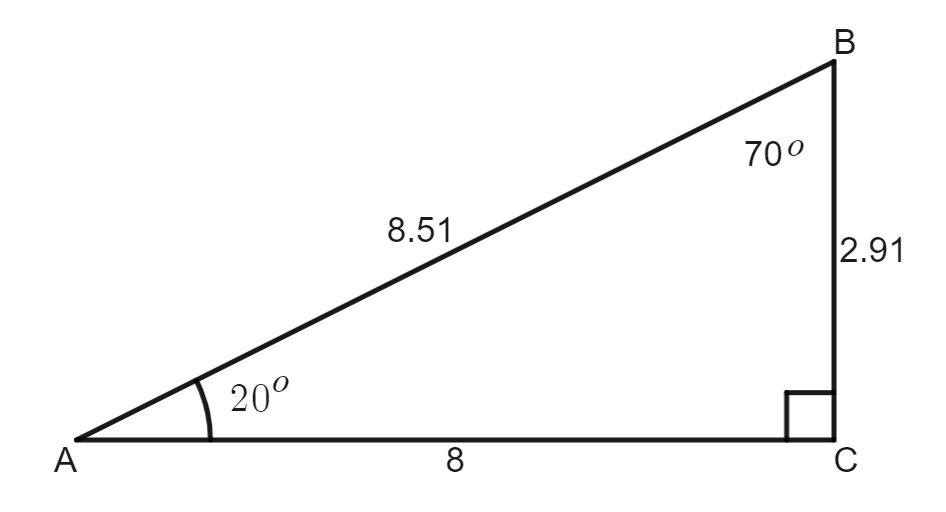
metres

I know this measurement is correct because the adjacent side AC should be less than the hypotenuse AB.

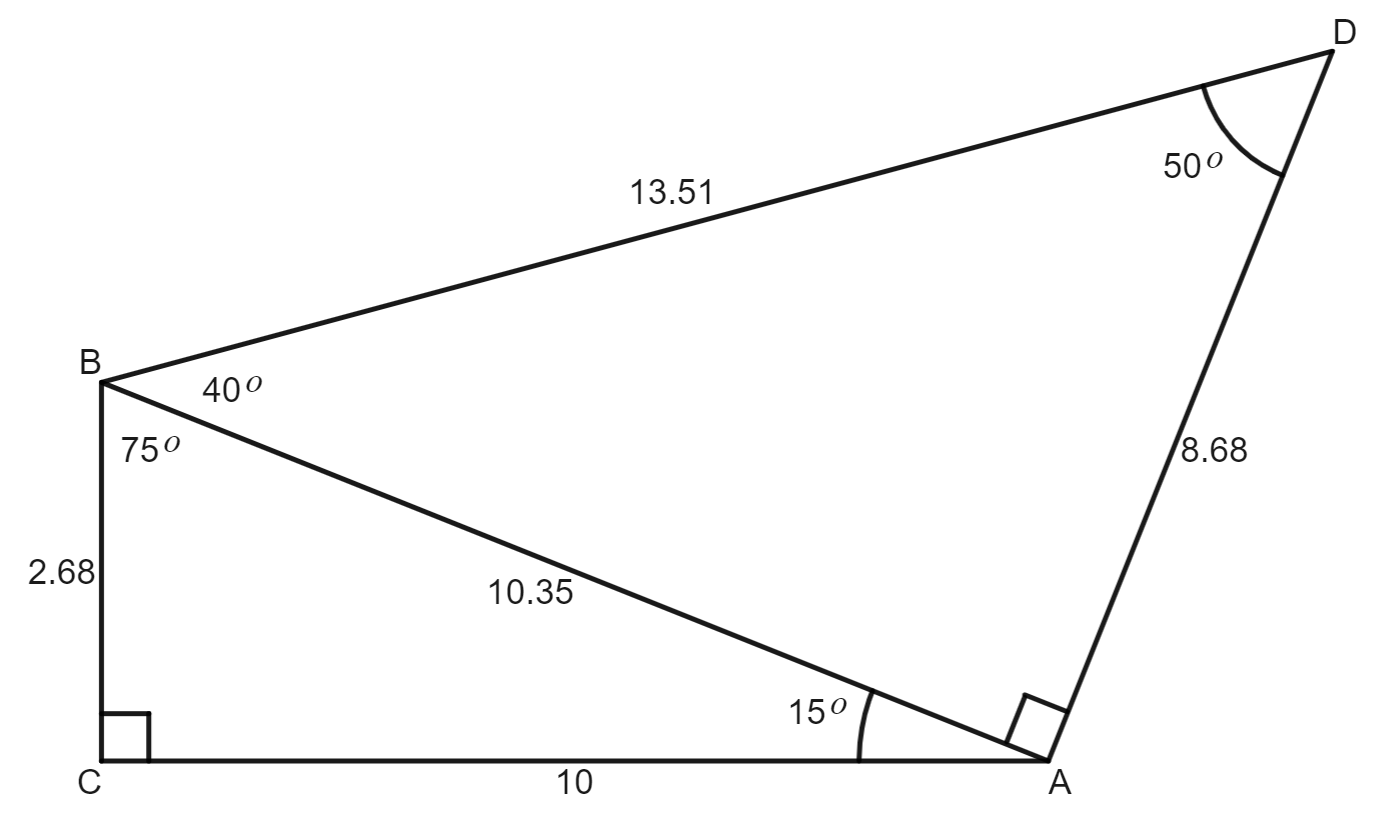
|  |  |  |  |
| --- | --- | --- | --- |
| Example 2 | Example 3 | Example 4 | Example 5 |
| This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side BC is labelled with 4.2 metres. The angle ABC is 78 degrees and the angle ACB is a right angle. AB is labelled Hypotenuse, BC is labelled 'A' and AC is labelled 'O'. | This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side AB is labelled with 12.6 metres. The angle ACB is 46 degrees and the angle ABC is a right angle. AC is labelled 'H', BC is labelled 'A' and AB is labelled 'O'. | This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side BC is labelled with 8 metres. The angle BAC is 35 degrees and the angle ABC is a right angle. AC is labelled 'H', AB is labelled 'A' and BC is labelled 'O'. | This is an image of a right-angled triangle, with vertices labelled A, B and C. The side AC is labelled x, the side BC is labelled with 3.6 metres. The angle BAC is 19 degrees and the angle ACB is a right angle. AB is labelled 'H', AC is labelled 'A' and BC is labelled 'O'. |
| Label the sides | Label the sides | Label the sides | Label the sides |
| SOH CAH TOA, H has a line through it and TOA is circled. | SOH CAH TOA, A has a line through it and SOH is circled. | SOH CAH TOA, A has a line through it and SOH is circled. | SOH CAH TOA, H has a line through it and TOA is circled. |
|  |  |  |  |
|  |  |  |  |
| because | because | because | because |
|  |  |  |  |
| metres | metres | metres | metres |
| I know this measurement is correct because the opposite side AC (19.76 metres) is opposite a much larger angle than the adjacent side BC | I know this measurement is correct because the opposite side should always be less than the hypotenuse. | I know this measurement is correct because the hypotenuse should be longer than the opposite side. | I know this measurement is correct because the opposite side is opposite a much smaller angle than the adjacent side BC. |

### Appendix D

#### Scenario 1



#### Scenario 2



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