# I lost my calculator

Students develop and use a table of trigonometric ratios by constructing right-angled triangles with digital or measuring tools. Students develop an appreciation of scientific calculators as a database of trigonometric ratios that can be used to solve problems.

This activity is designed to support students who already have some experience with trigonometric ratios and their definitions.

## Visible learning

### Learning intentions

* To understand that the trigonometric ratios are calculated from the ratios of side lengths in right-angled triangles.
* To understand that values of trigonometric ratios are stored for use on scientific calculators.

### Success criteria

* I can construct a right-angled triangle from a given angle.
* I can measure three angles and three side lengths in a right-angled triangle.
* I can calculate the value of a trigonometric ratio with only a pencil, a ruler and a protractor.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* applies trigonometric ratios to solve right-angled triangle problems **MA4-TRG-C-01**

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## Activity structure

### Launch

1. Students are to discuss with a partner, which of the following they know how to find the value of without a calculator. If you know any and your partner does not, show them how.
2. $124+59$
3. $431-54$
4. $8×46$
5. $189÷21$
6. $14^{2}$
7. $\sqrt{10} $
8. $\sin(42)$

The aim of this task is to present students with operations they do not immediately know the answer to and would usually do in a calculator, but that they can do by hand if challenged to do so. It is important to modify questions to suit the ability of students in the class such that this is achieved.

1. Lead a discussion around student's approaches and focus on the following questions:
2. Which values could be found by everyone?
3. Which values could be found by no one?
4. Which values were found using different approaches?
5. Is there anyone in the room who might use someone else's approach to solve the same problem next time?
6. How would we begin to find the value of $\sin(42^{o})$?

The focus from this point of the lesson will be on finding the value of trigonometric ratios, such as $\sin(42^{o})$ without a calculator. If any other expressions in the list above require revision, this would be a good opportunity to purposefully learn how to find squares or square roots (approximately) without a calculator.

### Explore

As an alternative to the exploration described below, teachers can choose to have students engage in this experience in Desmos via the activity, [I lost my calculator](https://bit.ly/desmosilostmycalculator) ([bit.ly/desmosilostmycalculator](https://bit.ly/desmosilostmycalculator)).

#### Equipment

* Pen or pencil.
* Ruler, 1 per student.
* Protractor, 1 per student.
* Grid paper.
* Copy of [Appendix A](#_Appendix_A).

#### Method

1. Ask students to complete a [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to answer the following question:
2. What do you expect to see when you are thinking about $\sin(42^{o})$?
3. Bring students back together and gather suggested answers.

While many responses would be accurate, the minimum that needs to be established as a group to move forward are right-angled triangles, an angle of 42°, opposite side length and hypotenuse.

1. Ask students to use the prompts on Appendix A to construct a triangle, measure and then calculate the value of $\sin(42^{o})$.
2. The teacher can use the animation in this Desmos graph, [Trigonometry without a calculator](https://bit.ly/DesmosTrignoncalc) ([bit.ly/DesmosTrignoncalc](https://bit.ly/DesmosTrignoncalc)) to demonstrate to students.

### Summarise

1. Verify the solutions to the table around the room. Conclude that students should all have different fractions, but still obtain the same value for the trigonometric ratio.
2. Discuss with the class the following reflection questions:
3. Were all of the class answers to the table exactly the same?
4. Can you find the quickest way to obtain values for angles of 80 and 10 degrees?

Students will hopefully identify that these 2 angles appear in the one triangle and can be led to generalise that any 2 angles that sum to 90 degrees will appear in the same right-angled triangle.

1. What do you think your calculator is doing when you enter $\sin(42^{o})$?

### Apply

1. Pose the question to students, ‘How did we find trigonometric values before calculators were invented?’
2. Discuss with students what they might do if they needed trigonometric ratio calculations regularly but did not have a calculator, leading them to the concept that you might find them once, and then record them so you no longer need to.
3. Hand students [Appendix A](https://schoolsnsw.sharepoint.com/sites/CurriculumReformResourceDevelopment/Shared%20Documents/1.%20Editorial/Accessibility%20and%20copyright/NESA%20Syllabus/Maths/Pending/Maths%20-%20Stage%205/Stage%205%20Unit%202%20Working%20with%20triangles/EDITORIAL%20%E2%80%93%20Maths%20%E2%80%93%20Stage%205%20%E2%80%93%20unit%202%20%E2%80%93%20lesson%2003%20%E2%80%93%20the%20tangent%20ratio.docx#_Appendix_A) and have them find the value of $\sin(42^{o})$.
4. Students then repeat this process in groups to complete the table in Appendix A.
5. Hand students a copy of [Appendix B](#_Appendix_B) and have them fill in the information they have found from Appendix A for angles 10°, 55° and 80°. Challenge students to go back to their triangles to find the remaining information for these angles.
6. In pairs, students can be challenged to draw one angle from the table in Appendix B, ensuring each group draws a different angle. Each pair finishes the triangle and completes this row of their table, and then shares with the class to complete Appendix B.

## Assessment and Differentiation

### Suggested opportunities for differentiation

**Launch**

* The expressions in the launch should be edited by teachers to support the abilities of their students, balancing access and challenge.

**Explore**

* The Desmos activity allows an exploration for students that does not depend on their skills with measuring tools.
* Using grid paper allows for easier and more accurate right-angled triangle construction. This in turn should lead to ratios that are more easily compared between students.

**Apply**

* When completing [Appendix B](#_Appendix_B), challenge students to seek out a method for obtaining the most trigonometric ratios from our table, with just one triangle. The maximum would be 6.

### Suggested opportunities for assessment

**Explore**

* The Desmos activity allows the teacher to record, review, and give feedback on student responses. There are multiple opportunities for students to express their thinking.
* Look for students who are finding calculator verification not to match their fraction value **and** review their work with constructing triangles.

## Appendix A

### Finding trigonometric ratios without a calculator

#### Finding sin42

1. Using a protractor, draw a $42^{o}$, with one line on grid paper.
2. Close off the angle to form a right-angled triangle.
3. Measure the lengths of the opposite side and the hypotenuse of your triangle and mark them on the diagram.
4. Write the fraction $\frac{opposite}{adjacent}$ using the numbers from your triangle.
5. Convert your fraction from step 4 to a decimal and compare with other students. Type $\sin(42)$ into your calculator to check your answer.

#### Finding further trigonometric ratios

1. Follow the steps above to construct triangles on grid paper to find the values of the trigonometric ratios in the table below.
2. Compare your values with a classmate. Your fractions will look different but use a calculator to check if they have approximately the same decimal value.

|  |  |  |
| --- | --- | --- |
| Trigonometric ratio | Fraction value from your triangle | Calculator check |
| sin55o |  |  |
| cos55o |  |  |
| tan55o |  |  |
| sin10o |  |  |
| cos10o |  |  |
| sin80o |  |  |
| cos80o |  |  |

## Appendix B

### Trigonometry table

|  |  |  |  |
| --- | --- | --- | --- |
|  | $$cosθ$$ | $$sinθ$$ | $$tanθ$$ |
| $$5^{o}$$ |  |  |  |
| $$10^{o}$$ |  |  |  |
| $$15^{o}$$ |  |  |  |
| $$20^{o}$$ |  |  |  |
| $$25^{o}$$ |  |  |  |
| $$30^{o}$$ |  |  |  |
| $$35^{o}$$ |  |  |  |
| $$40^{o}$$ |  |  |  |
| $$45^{o}$$ |  |  |  |
| $$50^{o}$$ |  |  |  |
| $$55^{o}$$ |  |  |  |
| $$60^{o}$$ |  |  |  |
| $$65^{o}$$ |  |  |  |
| $$70^{o}$$ |  |  |  |
| $$75^{o}$$ |  |  |  |
| $$80^{o}$$ |  |  |  |

## Sample solutions

### Appendix B

|  |  |  |  |
| --- | --- | --- | --- |
|  | $$cosθ$$ | $$sinθ$$ | $$tanθ$$ |
| $$5^{o}$$ | **0.9961** | **0.08714** | **0.08748** |
| $$10^{o}$$ | **0.98476** | **0.17365** | **0.17632** |
| $$15^{o}$$ | **0.96588** | **0.2588** | **0.26794** |
| $$20^{o}$$ | **0.93967** | **0.34201** | **0.36395** |
| $$25^{o}$$ | **0.90628** | **0.4226** | **0.4663** |
| $$30^{o}$$ | **0.86601** | **0.5** | **0.57733** |
| $$35^{o}$$ | **0.81915** | **0.57356** | **0.7002** |
| $$40^{o}$$ | **0.76604** | **0.64279** | **0.83911** |
| $$45^{o}$$ | **0.70709** | **0.70709** | **1** |
| $$50^{o}$$ | **0.64279** | **0.76604** | **1.19176** |
| $$55^{o}$$ | **0.57356** | **0.81915** | **1.42818** |
| $$60^{o}$$ | **0.5** | **0.86601** | **1.73201** |
| $$65^{o}$$ | **0.4226** | **0.90628** | **2.1445** |
| $$70^{o}$$ | **0.34201** | **0.93967** | **2.74738** |
| $$75^{o}$$ | **0.2588** | **0.96588** | **3.7319** |
| $$80^{o}$$ | **0.17365** | **0.98476** | **5.67067** |

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