# 45-degree angles

In this activity, students establish the constancy of the ratio between 2 sides of isosceles right-angled triangles and use this 1:1 ratio to measure heights of tall objects.

This activity is designed for students who have no experience with trigonometric ratios and aims to begin the process of introducing these concepts. Students will benefit from some knowledge of similar triangles.

## Visible learning

### Learning intentions

* To understand the constant properties of all right-angled isosceles triangles.

### Success criteria

* I can use a clinometer to measure angles to tall objects.
* I can use 45°, right-angled, isosceles triangles to measure the heights of tall objects.
* I can explain why the distance to a tree is the same as the height of a tree when the angle is 45°.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* applies trigonometric ratios to solve right-angled triangle problems **MA5-TRG-C-01**

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Please use the associated PowerPoint *45-degree angles* to display images in this lesson.

## Activity structure

### Launch

Teacher to initiate a discussion on the following questions, using strategies such as [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)):

* How tall is the tallest tree (or building) in our school?
* How do you know which is the tallest?
* How would you measure the tree?

Figure 1 – a tall tree



### Explore

Students investigate right-angled triangles with 45-degree angles to determine that 2 side lengths will always be equal, and how we can use this to find heights of objects.

This activity has been written with the intention of introducing the language of the sides of triangles in a future lesson. Teachers can choose to introduce this language now if they prefer.

The exploration below can be conducted online via Desmos in the activity [45 degree angles](https://bit.ly/Desmos45degreeangle) ([bit.ly/Desmos45degreeangle](https://bit.ly/Desmos45degreeangle)).

#### Equipment

* Class set of rulers
* Class set of protractors
* Grid paper (ideally 1 cm grid)

#### Method

1. Display the Desmos graph [The height of a tree](https://bit.ly/Desmos-treeheight) ([bit.ly/Desmos-treeheight](https://bit.ly/Desmos-treeheight)). Drag the character to show changes in the triangle but do not press the switch.
2. Have students engage in a [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) and discuss what we know and what we can measure with conventional tools.
3. Flick the switch in the Desmos graph to conclude that we can measure the distance to the tree from anywhere we stand with a trundle wheel, and the angle to the top of the tree with a clinometer.
4. It is vital to also draw attention to the assumed approximate right angle between the ground and the tree. Have students consider if this is always true.

This is a possible opportunity to acknowledge with students that to form an accurate triangle, we need to adjust for the height of our eyes above ground level.

Students can be shown how to make a clinometer, and what a clinometer is using the following videos:

* YouTube: [Clinometer (How to make and use) (2:10)](https://www.youtube.com/watch?v=FVqNEBWH4B0) ([bit.ly/Clinometerinstructions](https://www.youtube.com/watch?v=FVqNEBWH4B0)).
* Eddie Woo: [How to use a Clinometer (5:15)](https://bit.ly/wootubeclinometer) ([bit.ly/wootubeclinometer](https://bit.ly/wootubeclinometer)).
* [Appendix A: How to build and use a clinometer](#_How_to_build).

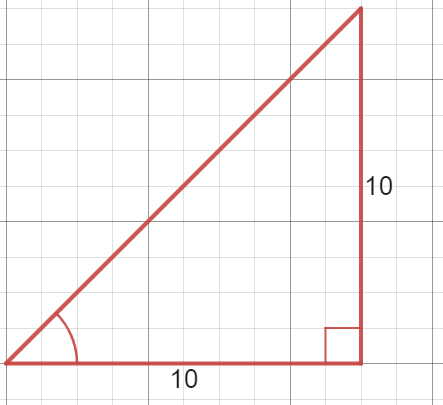
1. Given a ruler, protractor and grid paper, all students draw a right angle and measure to make the 2 sides the same (they can choose any length and count squares on the grid for lengths).

Figure 2 – 90-degree angle



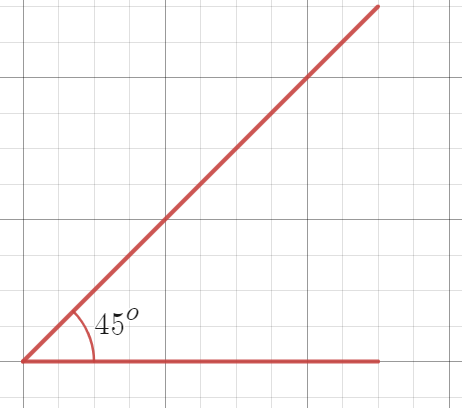
1. Join the sides to form a right-angled isosceles triangle and measure the base angle.

Figure 3 – right-angled triangle



1. All students should have different triangles and should be challenged to discuss and share what they find the same. (All triangles should have approximately 45-degree angles at the base).
2. Will 45-degree angles always make the sides equal in length? Challenge students to draw a 45-degree angle with their protractor on grid paper.

Figure 4 – 45-degree angle



1. Close the triangle at any size such that a right angle is formed and measure the 2 sides with a ruler or counting the square in the grid.
2. Again, students should discuss what they find in common.
3. Ask students to predict the height of a tree in the scenario below (this image is in the associated PowerPoint), where we know that we can see the height of the tree at a 45-degree angle, and we have measured the distance to the foot of the tree to be 15 m.

Figure 5 – person standing 15 m from a tree

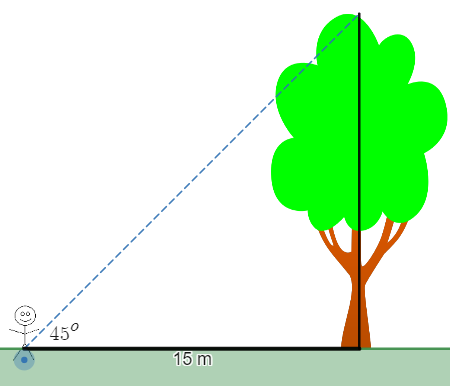


Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

### Summarise

#### Draw conclusions

1. Teacher to lead a discussion around significant conclusions from this investigation:
2. in every isosceles right-angled triangle, the base to height ratio is 1:1, meaning the 2 sides that form the right angle will be equal.
3. if we stand back from a tree until our clinometer reads 45°, the height of the tree will be the same as the distance along the ground to the tree, which we can easily measure with a tape measure or trundle wheel.
4. Have students write notes to their future self about these conclusions ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)).
5. Students complete exit ticket from [Appendix B](#_Appendix_B). This exit ticket is in the associated PowerPoint.

#### Consider why this always occurs

1. Students to engage in a [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), considering the question ‘how can you tell that these triangles are similar?’

Figure 6 – 2 right-angled triangles

This is an image of 2 right-angled triangles. The first, smaller triangle has vertices labelled A, B and C, with C on the right angle. Angle BAC is shown to be 45 degrees. Side lengths AC and BC are both shown to be 2 units long. 

The second, larger triangle has vertices labelled D, E and F, with F on the right angle. Angle EDF is shown to be 45 degrees. Side lengths DF and EF are both shown to be 4 units long. 

Alternatively, students could consider this same question, by looking at the triangles that they themselves have constructed during the ‘Explore’ section of this activity.

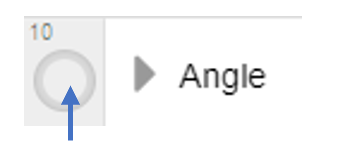
1. Lead a discussion connecting this conclusion of similarity to the previous topic regarding similar figures, and the minimum requirements for 2 triangles to be similar.
2. Evidence can include:
3. the common ratio .
4. the common pairs of angles and .

### Apply

Students use 45° in right-angled triangles to find the heights of tall trees and buildings around the school. The instructions for this are in the associated PowerPoint.

This process can be demonstrated to the class using the Desmos graphs, [Heights of trees](https://bit.ly/Desmos-heightsoftrees) ([bit.ly/Desmos-heightsoftrees](https://bit.ly/Desmos-heightsoftrees)). The instructions on the left of screen describe how to use the graph. Turn on folders by clicking on the circle beside the folder name.

Figure 7 – 'Angle' folder icon from Desmos



Additional examples include finding the heights of ceilings in classrooms and the school hall, or goalposts and basketball rings, which can then be checked against regulations.

#### Equipment

Clinometer, trundle wheel or tape measure

#### Method

1. Students stand back from a tall tree, looking at the top through a clinometer, until the angle becomes 45°.

This is a second opportunity to acknowledge measuring the angle from the ground and considering what can be done to compensate for measuring while standing, for example, adding the height of your eyes above the ground to the calculated height of the tree.

1. A partner measures the distance from this point to the foot of the tree.

Figure 8 – person standing distance from a tree

This is an image of a cartoon of a person standing an unknown distance away from the base of a tall tree. The angle from the person to the top of the tree is shown to be 45 degrees. 

The distance of the person from the tree and the height of the tree are both marked with the word 'distance', indicating they are the same. 

Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. This distance will be the same as the height of the tree.

#### Reflection questions

* Why are the distances the same? Try measuring the top angle in the triangles you drew.
* What would happen if you couldn't walk back to be at a 45° angle? How would you find the height in this circumstance?

## Assessment and Differentiation

### Suggested opportunities for differentiation

**Explore**

* The Desmos activity allows an exploration for students that does not depend on their skills with measuring tools.
* Using grid paper supports students to construct more accurate right-angled triangles. Advise students when drawing horizontal and vertical lines to choose lengths that are whole numbers of squares in length.

**Summarise**

* Challenge students to write a complete explanation as to how they know the 2 triangles are similar.

**Apply**

* Have groups of students collaborate by standing at the base of a tree and all moving back until they are at a 45° angle of inclination. Students can then measure via the process and compare their results.

### ****Suggested opportunities for assessment****

**Explore**

* Monitor the measurements found by students to assess their skills with using protractors and rulers.
* The Desmos activity allows the teacher to record, review, and give feedback on student responses. There are multiple opportunities for students to express their thinking.

**Summarise**

* Collect student exit tickets to assess their learning of the overall concept, that right-angled triangles with 45° angles have equal sides that can be used to solve problems.

**Apply**

* Monitor the measurements found by students to assess their skills with using larger practical measuring tools, clinometers, trundle wheels and tape measures.

## Appendix A

### How to build a clinometer

Equipment: straws; string; adhesive putty; protractor, sticky tape

1. Tie the string around the centre of the straw OR poke a hole in the straw and tie the string through the hole.

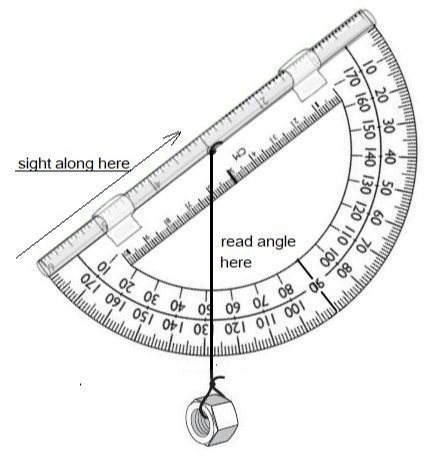


1. Use the sticky tape to attach the straw to the straight edge of the protractor.



1. Attach a ball of the adhesive putty to the other end of the string (or any other weight such as the nut shown below)





## Appendix B

### Exit ticket

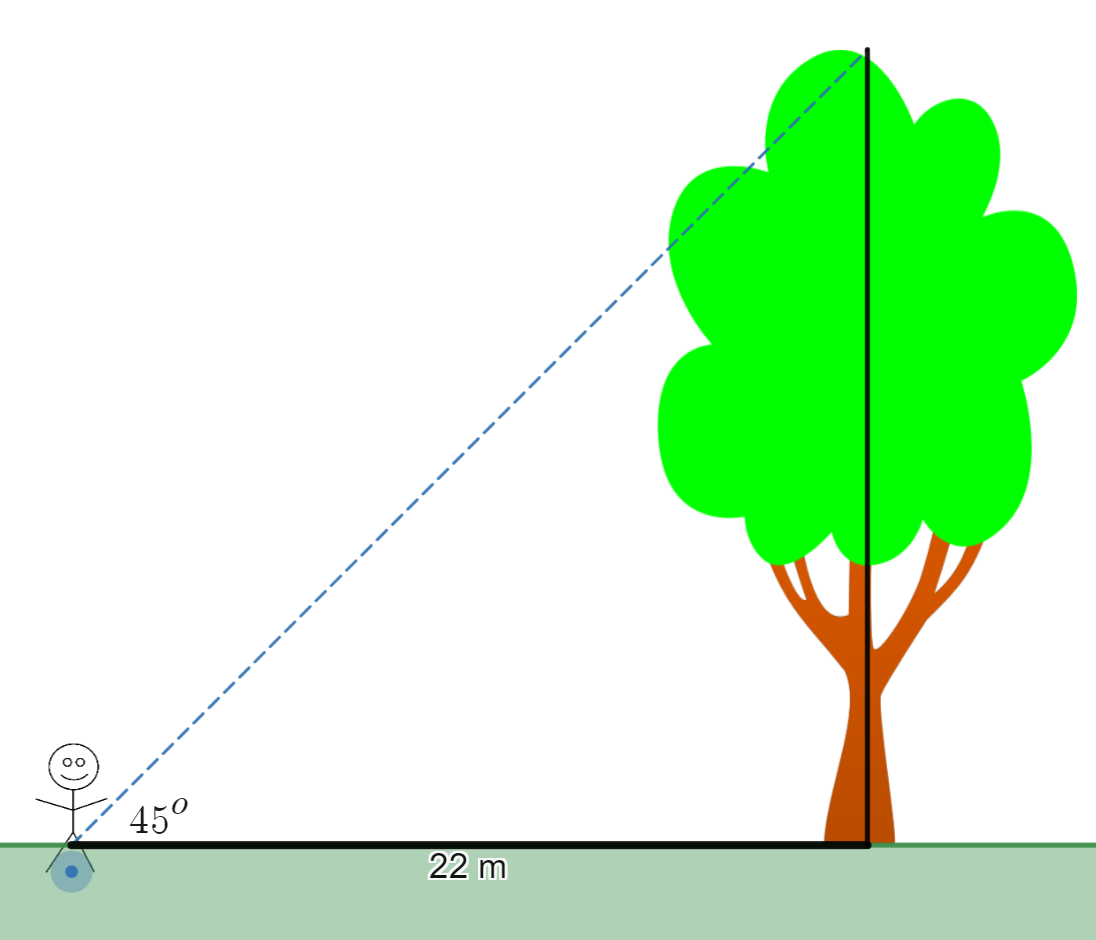


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1. How tall is the tree?
2. How do you know?

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