# Scale factor and area

This lesson provides students with opportunities to explore the relationship between scale factor and area. Students can take this further with an investigation into the relationship between scale factor and surface area and volume.

This lesson includes Path content.

## Visible learning

### Learning intention

* To solve problems involving similar figures and area.

### Success criteria

* I can solve problems involving similar figures and area.
* I can use scale factor to calculate the areas of similar figures.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* identifies and applies the properties of similar figures and scale drawings to solve problems **MA5-GEO-C-01**
* establishes conditions for congruent triangles and similar triangles and solves problems relating to properties of similar figures and plane shapes **MA5-GEO-P-01**

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## Activity structure

### Launch

1. Display the Which one doesn’t belong activity retrieved from [wodb.ca/images/shape19.jpg](https://wodb.ca/images/shape19.jpg). For more information about this strategy, refer to [bit.ly/wodbstrategy](https://aus01.safelinks.protection.outlook.com/?url=https%3A%2F%2Fbit.ly%2Fwodbstrategy&data=05%7C01%7CMaureen.OKeefe5%40det.nsw.edu.au%7Ca1af49665bad4ec2efa108db25b22bfb%7C05a0e69a418a47c19c259387261bf991%7C0%7C0%7C638145219414385317%7CUnknown%7CTWFpbGZsb3d8eyJWIjoiMC4wLjAwMDAiLCJQIjoiV2luMzIiLCJBTiI6Ik1haWwiLCJXVCI6Mn0%3D%7C3000%7C%7C%7C&sdata=Wr%2FebQ166Cu8EJORA9Fxs2Q0WES9qIQ4%2FwwcaMw1LKE%3D&reserved=0).
2. Have students justify which one doesn’t belong in a [Think-Pair-Share](https://bit.ly/thinkpairsharestrategy) ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)).
3. Lead students to discuss similar figures and scale factor. You could ask how many times bigger is the red or brown square than the white square.

### Explore

#### Pattern blocks

If you don’t have pattern blocks you can use [Polypad](https://mathigon.org/polypad#polygons) as an online alternative: [mathigon.org/polypad – polygons](https://mathigon.org/polypad#polygons)

1. Display [Appendix A](#_Appendix_A) and model making a square with scale factor 2. Each side will be 2 squares which creates a square with area 4.

Figure 1 – Square with scale factor 2



Image created using the free virtual manipulatives at Polypad.org

1. Students use pattern tiles to build each shape with the given scale factor (side length) and record the tiles used.

**Opportunities for extension:** Have students create a rule that could be used to relate scale factor and area of similar figures.

### Summarise

1. Display the [GeoGebra](https://www.geogebra.org/m/A8wW79ud) applet ([www.geogebra.org/m/A8wW79ud](http://www.geogebra.org/m/A8wW79ud)).
2. Ask students if they noticed any patterns between the scale factor and blocks used in the previous activity.
3. Lead students to develop the concept: The ratio of areas between 2 similar figures is equal to the square of the scale factor.
4. Demonstrate this on the GeoGebra applet by dragging the slider to increase the side lengths in rectangle Y.

### Apply

Print and distribute [Appendix B](#_Appendix_B).

Appendix B provides students with an opportunity to apply their learning in a routine context. If you would like your students to compare this with non-routine problems, there is an investigation below.

### Investigation

This investigation, [Appendix C](#_Appendix_C), explores scale factor in 3D objects and the ratios of their sides, areas, and volumes. This task also draws upon content about volume and surface area.

## Assessment and Differentiation

### Suggested opportunities and differentiation

**Explore**

* Students could spend the entire lesson exploring the relationship between side lengths, scale factor and area if that is the level they are working at. Provide manipulatives and more general prompts such as what happens to the area of a square if you keep doubling the side lengths?

**Investigation**

* As an extension the students could find their own products. As an assistance for students who are struggling, they could be given a product where the scale factor is a nice 2 or 3 like modelled above.

### Suggested opportunities for assessment

* Students could complete the investigation as an assessment task.
* Through their engagement with a new type of problem, students will likely present misconceptions about core skills in the topic.

## Appendix A

### Pattern blocks

Use pattern blocks to fill in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Scale factor | SquareShape, square  | TriangleShape - triangle | RhombusShape - rhombus |
| Scale factor | Square tiles | Triangle tiles | Rhombus tiles |
| 1 | 1 | 1 | 1 |
| 2 | 4 |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 10 |  |  |  |
| $$\frac{1}{2}$$ |  |  |  |

## Appendix B

### Thin slicing questions

For each question the original is on the left

**Question 1**

1. Find the scale factor
2. Find the area of each square



**Question 2**

1. Find the scale factor
2. Find the side length of each square



**Question 3**

Find as much information as you can for each set of rectangles below.









**Question 4**

The corresponding sides of 2 similar triangles are 7 cm and 21 $cm$. Find the area of the larger triangle, given the area of the smaller triangle is 14 $cm^{2}$.

**Question 5**

The diameter of a circle is doubled. What happens to its area?

## Appendix C

### Similar solids – what are similar solids?

Two three-dimensional (3D) objects are similar if you can multiply each of the side lengths in the smaller shape, by the same number (called the ‘scale factor’) to get the side lengths of the larger object.

For each of the following real-life objects:

* find out if they are similar solids, and if they are, find the scale factor
* if they aren’t similar, adjust the measurements of the larger shape to make them similar
* find the surface area and volume of each
* find out what the scale factor is for both the surface area and volume.

#### Your investigation

##### Task 1 – Gathering data

**Vegemite**

* 150 g: 4.5 cm radius and 7.5 cm tall
* 400 g: 7 cm radius and 9.5 cm tall

**SPC Fruit Salad**

* 410 g: 7.5 cm radius and 11 cm tall
* 825 g: 10 cm radius and 12 cm tall

**Baked Beans**

* 130 g: 5.3 cm radius and 6.8 cm tall
* 420 g: 7.5 cm radius and 11 cm tall

**Flour**

* 1 kg: 9.5 cm wide, 7 cm deep, 17 cm high
* 2 kg: 12 cm wide, 9.5 cm deep, 22 cm high

**Cornflakes**

* 460 g: 24 cm wide, 6.7 cm deep, 32 cm high
* 775 g: 25 cm wide, 9.5 cm deep, 35.8 cm high

**Toblerone**

* 50 g: 2.8 cm wide, 2.5 cm high, 16.5 cm long
* 100 g: 3.5 cm wide, 3 cm high, 21 cm long
* 200 g: 4.5 cm wide, 4 cm high, 27.5 cm long

**Duo Laundry Detergent**

* 500 g: 6 cm deep, 13 cm wide, 11.5 cm high
* 1.5 kg: 7.8 cm deep, 15 cm wide, 20 cm high

##### Task 2 – analysing data

What is the connection between the scale factor for side lengths and the scale factor for surface area and volume?

### Sample solutions

#### Pattern blocks

|  |  |  |  |
| --- | --- | --- | --- |
| Scale factor | SquareShape, square  | TriangleShape - triangle | RhombusShape - rhombus |
| Scale factor | Square tiles | Triangle tiles | Rhombus tiles |
| 1 | 1 | 1 | 1 |
| 2 | 4 | 4 | 4 |
| 3 | 9 | 9 | 9 |
| 4 | 16 | 16 | 16 |
| 10 | 100 | 100 | 100 |
|  |  |  |  |

#### Thin slicing questions

**Question 1**

1. Scale factor = 3
2. Area 1 = 4 Area 2 = 36

**Question 2**

1. Scale factor = 4
2. $x=1$ $y=4$

**Question 3**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| a. | Scale factor = 1/3 | Area 1 = 215 | Area 2 = 35 |  |
| b. | Scale factor = 1/2  | Side length 2 = 2.5 | Side length 1 = 5 | Area 1 = 60 |
| c. | Scale factor =1/4 | Side length 1 = 9 | Side length 2 = 2.25 | Area 2 = 11.25 |
| d. | Scale factor = 8/3 | Area 1 = 432 | Side length 2 = 6.75 | Area 2 = 60.75 |

**Question 4**

Scale factor = 3
Area scale factor = $3^{2}$ =9
14 × 9 = 126
Larger triangle has an area of $126 cm^{2}$

**Question 5**

The new area becomes 4 times larger.

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