# The 3 M’s

This activity is based on teaching resources by [NRICH](https://nrich.maths.org/mmandm/note).

This learning episode allows students the opportunity to consolidate their understanding of mean, mode, median and range. Students will be working mathematically to justify their responses.

## Visible learning

### Learning intentions

* To understand the effect that a data value has on the mean, mode and median of a data set.

### Success criteria

* I can find the mode and range of a dataset.
* I can find the median of a dataset.
* I can calculate the mean of a dataset.
* I can explain the effect of changing a data value in a data set, has on the mean, median, mode and range.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing, and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* analyses simple datasets using measures of centre, range, and shape of the data **MA4-DAT-C-02**

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## Activity structure

### Launch

1. Verbally present the following scenario to students, whilst writing Ben’s test scores on the board.

‘Ben scored 5, 3, 6 and 3 on his fortnightly maths tests (out of 10). His mum said that she would buy him a new PlayStation game if he could achieve an average of 4 over the course of the term. What does he have to achieve on his last test to get his PlayStation?’

1. Ask students to work in randomly chosen groups of 3 at vertical non-permanent surfaces, if possible, to discuss the problem and find a solution.

Students will probably naturally assume that *average* is referring to the mean. This is something that you can draw attention to in step 3 when you share solution strategies. What is society’s understanding of the word *average*. If they do not naturally make this assumption and find either median, mode or all 3, this is again something you can discuss in the next step.

1. As a whole class, with students still standing with their work, ask various groups to explain their solution strategy.
2. Present the following new scenario to students:

‘Ben’s mum does not understand that mean, median or mode, can be used to find the middle of a set of data. What would he need to achieve to get a median of 4 or a mode of 4?’

Students should come to the conclusion that it is impossible to get a mode of 4 by only adding one more test score, as there are already two 3’s in the dataset.

1. Students work in their same groups to solve the new problem.
2. Consolidate the activity by again sharing solution strategies. Was this an easier problem to solve than finding the mean? Why?

### Explore

1. Continuing in their random groups and at vertical non-permanent surfaces, ask students if they can find a set of 5 test scores with a mean = 4, mode = 3 and median = 3?
2. Challenge students to consider:
3. Is this the only set of test scores that fit these statistics?
4. Can they find others?
5. How many other sets of scores are there?

Students may find it useful to use visual representations such as towers made of counters or blocks to represent the scores.

Prompting questions to assist students could include:

* What if one of the scores was a 1?
* What if you had no 1’s?
* What if you only had one 3, two 3’s, three 3’s etc.?
* Which piece of information; mean, median or mode, do you think would be most useful to start with?
1. When students think they have found all the solutions ask:
2. Can you convince yourselves you have found all the solutions?
3. Do you think you could convince the class?

### Summarise

1. Use a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) so that students can examine the solutions of other groups.
2. Ask groups to share their solution strategies and to justify whether they have found all the solutions. See Appendix B for possible solutions.

### Apply

The mean = mode = median = range activity in Appendix A asks the students to think of 5 test scores where mean, median and mode fit specific criteria. Students should continue to work in random groups of three working on a vertical non-permanent surface, if possible.

## Assessment and Differentiation

### Suggested opportunities for differentiation

**Explore and Summarise**

* Students may find it useful to use visual representations such as towers made of counters or blocks to represent the scores.
* Students can be challenged to find more than one solution to any of the activities or to make the minimum number of changes possible to meet the given criteria.
* Students may need to revise median, mode, mean and range.

**Apply**

* Students may find it easiest to find a set of 5 scores where the mode = median. They could then be challenged to adapt this to include the range, before then including the mean as well.

### Suggested opportunities for assessment

* Monitor student responses during discussion to assess their understanding of mean, median, mode and range.
* Create an exit ticket asking students to calculate the mean, median and mode of some data.
* Create an exit ticket that asks students to create a set of data for a given mean, median and mode. The number of data values in the data set could be modified to meet student needs.

## Appendix A

### **Mean = Mode = Median = Range**

The weight of supermarket items such as chips and chocolate are never exactly what is advertised on their packaging. They are allowed to be slightly over or under the advertised weight.

**Question 1**

Here are 5 weights where the mean, mode, median and range are all equal, and would match the advertised weight on the packaging.



Can you find another set of 5 weights where mean = mode = median = range



**Question 2**

Can you find sets of 5 weights that satisfy the following properties?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mode < Median < Mean |  |  |  |  |  |
| Mode < Mean < Median |  |  |  |  |  |
| Mean < Mode < Median |  |  |  |  |  |
| Mean < Median < Mode |  |  |  |  |  |
| Median < Mode < Mean |  |  |  |  |  |
| Median < Mean < Mode |  |  |  |  |  |

### Challenge

Are there any of the above scenarios where you could not find 5 weights to meet the criteria? Can you explain why?

Can you explain why 4 weights, or 6 weights work for some activities?

## Appendix B: Possible solutions

**Question 1**

Groups of 5 scores with mean = 4, mode = 3 and median = 3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 3 | 3 | 3 | 10 |
| 1 | 3 | 3 | 4 | 9 |
| 1 | 3 | 3 | 5 | 8 |
| 1 | 3 | 3 | 6 | 7 |
| 1 | 3 | 3 | 2 | 11 |
| 2 | 3 | 3 | 3 | 9 |
| 2 | 3 | 3 | 4 | 8 |
| 2 | 3 | 3 | 5 | 7 |
| 3 | 3 | 3 | 3 | 8 |
| 3 | 3 | 3 | 4 | 7 |
| 3 | 3 | 3 | 5 | 6 |

Possible solutions where mean = mode = median = range

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Answer | #1 | #2 | #3 | #4 | #5 |
| 2 | 1 | 2 | 2 | 2 | 3 |
| 4 | 2 | 4 | 4 | 4 | 6 |
| 5 | 2 | 5 | 5 | 6 | 7 |
| 6 | 3 | 6 | 6 | 6 | 9 |
| 7 | 3 | 7 | 7 | 8 | 10 |
| 8 | 3 | 8 | 8 | 10 | 11 |
| 9 | 5 | 8 | 9 | 9 | 14 |
| 10 | 6 | 8 | 10 | 10 | 16 |
| 100 | 65 | 70 | 100 | 100 | 165 |

**Question 2**

Can you find sets of 5 weights that satisfy the following properties?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mode < Median < Mean | 2 | 2 | 5 | 10 | 11 |
| Mode < Mean < Median | 2 | 2 | 6 | 7 | 8 |
| Mean < Mode < MedianNo solutions with 5 numbers as the median must be the 3rd number. To make this bigger than the mode the first 2 numbers must be the same. The last 2 numbers can only be the 4th and 5th numbers making the mean > mode. |  |  |  |  |  |
| Mean < Median < Mode | 3 | 4 | 7 | 8 | 8 |
| Median < Mode < MeanNo solutions with 5 numbers. The median must be in the 3rd position. The mode must be in position 4 and 5. The mean will not be larger than the 2 largest scores. |  |  |  |  |  |
| Median < Mean < Mode | 4 | 5 | 6 | 10 | 10 |

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