Transcript:

Engineering studies

**(Duration 29 minutes)**

Hello and welcome to the HSC On Demand video for Engineering studies.

I would like to start by acknowledging our first Australians, the Aboriginal people who are the traditional custodians of the land. I would also like to pay my respects to elders both past and present and extend that respect to any other Aboriginal persons watching this presentation.

In this presentation, I'll be covering when and why it might be appropriate to use graphical solutions in the Engineering studies HSC exam. One of the reasons that it's important to know how to do a graphical solution is that it is a syllabus requirement and it is not unreasonable for the HSC examiners to ask you to provide a graphical solution. Sometimes it is easy to default to an analytical solution as you may find comfort in the numbers and calculations, however it is not always the most practical solution.

Graphical solutions can often easily be applied to three-force rule questions and we will have a look at some examples of this later in the presentation. Another thing to keep in mind is that if a diagram for a question is marked ‘to scale’, then any forces or vectors indicated in that question can have their lines of action extended and be used in your final graphical solution. Graphical solutions aren't just limited to force addition. They can also be applied to situations such as truss analysis and friction and it is also possible to use combinations of graphical and analytical solutions and we will have a look at an example of this also later in the presentation.

The Engineering studies exam is a three hour long exam and there are a total of 100 marks available to you and this equates to one minute and 48 seconds per mark in the exam. One of the benefits of using graphical solutions is that they're often faster and that will leave you with more time to access marks elsewhere in the exam.

Another benefit is that there is arguably less chance of error in the method as there are often fewer steps when compared to analytical solutions that will require more steps and involved calculations. Better students will be able to analyse a problem and decide whether a graphical solution or an analytical solution is the most appropriate method and then apply that method.

Part of using graphical solutions effectively is knowing when to apply them and some questions which you can ask yourself to help you identify if a graphical solution or an analytical solution is the most appropriate are listed below and we're going to go through each of these individually.

So the first question you should ask yourself is, is the problem a vector addition problem? Not all problems that have forces and vectors indicated in the problem will be vector addition problems. They could be a moment's problem or they could be a couple's problem. However, there are some problems that will present as a moment's problem that can actually be solved using the three-force rule and this means that a graphical solution can be applied.

The next question you should ask yourself is, is the diagram to scale? If the diagram is drawn to scale, this is a good indication that a graphical solution may be appropriate. Additionally, if the diagram is drawn to scale, any vector or force drawn in the diagram will be drawn with accurate sense and direction and as I mentioned earlier, you can extend the line of action of these vectors or forces and use them in your final graphical solution.

Next, check to see if the problem is a three-force rule problem. Three-force rule problems are often easily solved using graphical solutions and we have an example below of a three-force rule problem and a problem that is not a three-force rule problem. So the truss on the left has only three external forces acting on it. Therefore those three forces must be concurrent, which means that they must act through the same point and that means this can be solved with a graphical solution. The truss on the right has four external forces acting on it and therefore the three-force rule does not apply. So the forces do not have to act through the same point and therefore a graphical solution cannot be used. [A roof truss is shown on the left of the slide. The truss is supported on the left by a pin and on the right by a roller. There is a 15kN external force acting down and to the right on the truss. There is an identical truss shown on the right side of the screen. It has an additional force acting down on it. There is a statement below the trusses which is asking the examinee to calculate the reaction at the pin support.]

Finally, ask yourself, is the question asking me to use the method of joints? If the question is asking you the method of joints, it may be more appropriate to use a graphical solution opposed to an analytical solution. Graphical solutions for the method of joints are often quite straightforward when compared with analytical solutions that will often require you to solve multiple sums of X and Y components for a range of vectors.

Consider the problem shown. So there are two forces acting on an eye bolt and we need to find the resultant of these two forces. [An eye bolt is shown with two forces acting on it up and to the right. One of the force is a 100N force acting at 55 degrees to the horizontal. The other force is an 80N force acting at 30 degrees to the horizontal] This might be a good point to pause the video and try to solve the problem both graphically and analytically by yourself. I'll go through both solutions. Firstly, analytical and then the graphical and we'll compare both solutions afterward.

The standard method to solve this problem analytically is to first draw the free body diagram and label the vectors. Next it is often a good idea to draw a table that you can sum the X and Y components of the vectors in and the easiest way to draw one of these tables is to draw a naughts and crosses grid and add rows as you need for additional vectors and then one at the end for the sum. and then one at the end for the sum.

The next step in this method of solution is to break each of the vectors into their horizontal and vertical components and you will need to use trigonometry to do this. Once you have identified the value of the X and Y components of each of the vectors, write them down in the table and sum them together. [The X component of the 100N vector is shown to be 100 cosine 55, and the x component of the 80N vector is shown to be 80 cosine 30. The y component of the 100N vector is shown to be 100 sine 55 and the y component of the 80N vector is shown to be 80 sine 30] Note that it is important to use a sign convention when summing components of vectors and in this case, we have used to the right and up as positive and to the left and down as negative.

The final step is to arrange the summed vectors at a right angle, tip to tail as shown and the hypotenuse of this triangle will represent the resultant. [The summed x components are calculated to be 126.64N and the summed y components are calculated to be 121.92N. The two summed vectors are arranged at a right angle and joined to form a triangle. The hypotenuse of the triangle represents the resultant.] To calculate the magnitude of the resultant, you can use Pythagoras theorem and to calculate the direction of the resultant to the horizontal axis you need to use the inverse of tan.

As you can see, even with a simple force addition problem as shown, there are a considerable number of steps to actually calculating the resultant and the direction and if you are familiar with both analytical and graphical solutions, you can use your judgement to apply the method which is most appropriate.

To complete the graphical solution for this problem, we draw the 100 Newton force and let the length of the line represent magnitude of that vector and draw it with accurate direction using a protractor. Next, we translate the 80 Newton force so that it is arranged tip to tail with the 100 Newton force. Again, letting the length of the line represent the magnitude of the vector and drawn with accurate direction using a protractor. So in this case, we've used a scale of one millimetre is equal to one Newton. So the 100 Newton force would be drawn as a 100 millimetre long line and the 80 Newton force will be drawn as an 80 millimetre long line. [Vectors are shown arranged tip to tail and acting upward and to the right. The 100N vector acts at 55 degrees to the horizontal and the 80N vector acts at 30 degrees to the horizontal.] long line [Vectors are shown arranged tip to tail and acting upward and to the right. The 100N vector acts at 55 degrees to the horizontal and the 80N vector acts at 30 degrees to the horizontal.]

The next step is to draw a line from the original point of concurrency to the tip of the 80 Newton vector and this line will represent the resultant. To calculate the magnitude of the resultant, we simply measure the length of the line and multiply it by the scale and then to get the direction we measure the angle of the resultant from the horizontal axis using a protractor.

A couple of important things to note when doing a graphical solution is that you must use arrowheads to indicate the sense of each force in the force polygon. In the example shown, both forces are acting up and to the right and the arrowheads are used to indicate this. In a more complex problem with more vectors and forces that need to be added, if you get the sense of a vector incorrect, the final force polygon will be incorrect and any information that you are trying to discern will also not be correct. Additionally, it is important that you state the scale in your answer. So, as I mentioned in this case, the scale is one millimetre is equal to one Newton and therefore our 100 Newton force would be drawn 100 millimetres long and our 80 Newton force would be drawn 80millimetres long. It's also important that the scale is appropriate. So if your scale is too small the answer will be inaccurate and the answer that you deduce might fall outside the tolerance of what is deemed a correct answer that you deduce might fall outside the tolerance of what is deemed a correct answer.

So when we look at these two solutions side by side, it is clear that the graphical solution is simpler as there are fewer steps involved and there is arguably less chance of error in the method. [the force polygon for the graphical solution is shown next to the working for the analytical solution.] Again, I'll reinforce that for your graphical solution to be correct, you need to use the arrowheads to show the sense of all of the vectors in the polygon. You must state your scale and the scale must be appropriate.

Before looking at some more complex examples, let's quickly review what it means for a concurrent force system to be in equilibrium. First of all, there must be no resultant force in the system. The force polygon of a graphical solution will close and each of the equilibrium formulae will equal zero.

One rule that should always be considered as graphical solutions are often readily applied in this circumstance is the three-force rule and the three-force rule states that if three non-parallel forces act on a body in equilibrium, then they must be concurrent which means that they must act through the same point.

Consider the following problem. A simply supported truss is shown with a pin joint at one end and a roller at the other end and we are asked to calculate the magnitude and the direction of the reaction at pin joint A. At this point, you may want to pause the video and see if you can solve this problem both graphically and analytically. [An atypical roof truss with a pin joint at the left end (point A) and a roller joint at the right end (point E) is shown. There is an external 15kN force acting perpendicular to the raking plate (at point B) of the truss which is raked at 30 degrees. The force acts down and to the right]

(gentle upbeat music)

Again, I'm going to go through each of the solutions and we will compare them afterward. To do the analytical solution, we first need to calculate the reaction at E. We already know the direction of E is vertically up because it is a roller joint and the reaction at a roller joint will always act normal to the surface which it rolls along and in this case that surface is horizontal. To calculate the magnitude of the reaction at A, we need to take the sum of moments around point A so that we can ignore the reaction at point A in our calculations.

Summing the moments around point A reveals that the magnitude of the reaction at E is 4.34 kilo newtons. The next step is to sum either the horizontal or vertical forces acting on the truss and in this case, I've summed the vertical forces first. Doing the sum of the vertical forces reveals that vertical component of the reaction at A is 8.64 kilo newtons.

To find the horizontal component of the reaction at A, we simply need to sum the horizontal forces. Summing horizontal forces shows that the initial free body diagram showing the horizontal component of the reaction at A acting to the right is incorrect as a negative number was calculated and therefore the horizontal component of the reaction at A actually acts to the left. The magnitude of this force is calculated to be 7.5 kilo newtons.

The final step is to take the summed components of the reaction at A and arrange them tip to tail at a right angle as shown. The hypotenuse of this triangle represents the reaction. Again, Pythagoras' theorem can be applied to calculate the magnitude of the reaction and the inverse of tan can be used to calculate the direction. [The summed x components are calculated to be 7.5kN and the summed y components are calculated to be 8.65kN. The summed component are arranged tip to tail at a right angle and joined to create a triangle. The hypotenuse of the triangle represents the reaction.]

Now let's consider a graphical solution. Many students will not consider a graphical solution for this question as it is not perhaps immediately obvious that a graphical solution can be applied unless you have a sound understanding of the three-force rule. If you count the external forces, there are only three in this problem. There is the external reaction at E, the reaction at A and the external 15 kilo newton force acting at point B. This means that the external forces must be concurrent and act through the same point. Additionally, note that the diagram is drawn to scale. This means that any vectors shown in the diagram are drawn with accurate sense and direction and we can extend the line of action of these vectors and use them in our final solution. [A roof truss with a pin joint at the left end (point A) and a roller joint at the right end (point E) is shown. There is an external 15kN force acting perpendicular to the rafter (at point B) of the truss which is pitched at 30 degrees. The force acts down and to the right.]]

In this case, we know the direction of the 15 kilo newtons force and we can extend its line of action. We also know the direction of the reaction at E because it's a roller joint, so we can extend its line of action to see where it intersects with the 15 kilo newtons force and we've found the point of concurrency of the three-forces. of concurrency of the three-forces.

Now that we've found the point of concurrency. The next step is to extend the line from the point of concurrency to the joint A and this gives us the direction of the reaction at A. The next step is to apply an appropriate scale and the simplest way to do this is to use one of the vectors with known magnitude and direction and in this case, **it** is the 15 kilo newtons force acting at point B. Simply measure a distance from the point of concurrency along the line of action of that vector, a distance that will represent the magnitude of that vector. So in this case, I've measured from the point of concurrency along the line of action of the 15 kilo newtons vector, 150 millimetres. So that means that one millimetre on that line represents 100 Newtons. I've marked a cross where the line terminates. I've marked a cross where the line terminates.

The next step is to translate the reaction at E. We know the direction of the reaction is vertical. So we can simply draw a line from the cross that I have marked vertically down until it intersects the line which represents the reaction at A. The final step is to check that all of the forces are arranged from tip to tail with accurate sense and that the polygon closes.

Next, we can measure the length of the line, which represents the reaction at A from the point of concurrency to where the line which represents the reaction at E intersects it. We can multiply the length of this line by the scale to calculate the magnitude. In this case, the line is approximately 115 millimetres long. Multiplying this by 100 Newtons means that the reaction at A is approximately 11.5 kilo newtons. The last step is to measure the direction of the reaction from the horizontal using a protractor and in this case, it is approximately 49 degrees. and in this case, it is approximately 49 degrees.

If we compare the two solutions, it is evident that the analytical solution requires many more steps and involved calculations where errors may occur. [The force polygon for the graphical solution is shown next to the working for the analytical solution.] The graphical solution is fairly simple. However, it does require the examinee to have a sound understanding of the three-force rule.

Alright, so to solve this problem graphically, we'll first extend the lines of our vectors with known directions. [A roof truss with a pin joint at the left end (point A) and a roller joint at the right end (point E) is shown. There is an external 15kN force acting perpendicular to the rafter (at point B) of the truss which is pitched at 30 degrees. The force acts down and to the right]. So because the diagrams to scale, we can extend a line of action of this 15 kilo newtons vector. We also know the direction of the reaction at A so we can extend its line of action and we'll be able to find the point of concurrency. So we found the point of concurrency. Once we find the point of concurrency, we can draw a line from the point of concurrency to point A. we can draw a line from the point of concurrency to point A.

Next, we can measure along our vector that has known magnitude from the point of concurrency, a length that can represent the magnitude of that vector and in this case, I'm going to measure along 150 millimetres. So my scale will be one millimetre is equal to 100 Newtons. So this line is 15 kilo newtons

Okay, once we've done that, we can translate our vector that is the reaction at E. So we need to draw a line vertically down. Okay, so that's RE and this line will represent **reaction** A and we can draw our arrowheads to show that the polygon closes.

Next, we can measure the length of the reaction at A and multiply by the scale. So in this case, it's 115 millimetres multiplied by the scale gives us approximately 11.5 kilo newtons and the last step is to draw a line 90 degrees from the vertical axis of the reaction E. So in this case, it's approximately 49 degrees. So our final answer is that RA is approximately equal to 11.5 kilo newtons at 49 degrees upwards and to the left. Remember to use arrowheads to indicate the sense of all vectors in the polygon, state the scale and ensure that the scale is appropriate. state the scale and ensure that the scale is appropriate.

Let's consider another problem. A timber loading ramp between a truck and a step is shown. The ramp has a weight of 500 Newtons and a coefficient of static friction with concrete of 0.62. There is no friction at the roller support. [The ramp is 5 metres long and is inclined at an angle of 30 degrees to the horizontal.] Determine whether the ramp will slip or remain static for these conditions. This might be another good opportunity for you to pause the video and see if you can solve the problem both graphically and analytically.

(gentle upbeat music)

The analytical solution for this problem is quite complex and requires the examinee to have a sound understanding of friction, moments, components of vectors and reactions at roller supports and is simplified greatly by using a graphical solution. Again, I'm going to go through both solutions and we will compare them afterward. The first step to solving this problem analytically is to draw the free body diagram. The free body diagram gives us an insight into exactly what is happening in the problem before we apply a method of solution. [A free body diagram is shown which shows that there are three forces in the problem. The reaction at the ground (point G) the weight force of the ramp which acts through its centre of gravity, and the reaction at the roller (point R.)]]

In this case, we need to calculate the X component of the reaction at the ground required for equilibrium and then multiply the Y component of the reaction on the ground by the coefficient of friction to see if the maximum frictional force that can be generated is greater or less than the X component of the ground reaction. If the maximum frictional force is greater, the system will remain in static equilibrium and if the X component required for equilibrium is greater, the ramp will slide away from the truck. The analytical solution is fairly complex and if you have difficulty following this solution you may need to ask your teacher for assistance.

The first thing we have to solve in this problem is the magnitude of the reaction at the roller and we do this by taking the moments around point G which is where the ramp is in contact with the ground. Doing this allows us to ignore the ground reaction in our calculations. Summing the moments around point G reveals that the magnitude of the reaction at the roller is 216.5 Newtons.

The next step is to either sum forces vertically or horizontally and in this case we‘ve summed the forces vertically first. Summing forces vertically reveals that the Y component of the ground reaction is 312.5 Newtons. Summing forces horizontally reveals that the magnitude of the X component at the ground reaction is 108.25 Newtons.

Once we know the magnitude of the horizontal and vertical components at the ground reaction, we can solve the problem. If we multiply the coefficient of friction by the normal force which is equal to the Y component of the ground reaction, we find that the maximum frictional force which can be generated is 193.75 Newtons. We only require 108.25 Newtons at the X component of the ground reaction to remain in equilibrium. Therefore, the ramp will remain in static equilibrium.

Solving this problem graphically requires a small analytical calculation to compare the angle of static friction against the angle required for equilibrium. To do that we have to use a rearranged version of the friction formula which is not on the formulae sheet and that is that the angle of static friction is equal to the inverse of tan times the coefficient of friction.

Solving this problem graphically is made simple if you first draw the free body diagram, hopefully drawing the free body diagram will allow you to recognize that this too is a three-force rule problem and therefore the three forces in the problem must be concurrent. [a free body diagram is shown which shows that there are three forces in the problem. The reaction at the ground, the weight force of the ramp which acts through its centre of gravity, and the reaction at the roller.] We have one force which is the weight force of the ramp. We have the reaction at the roller and the reaction at the ground. Two of these forces have known direction and one has a known magnitude. We can use this information to find the point of concurrency by extending the lines of action on the diagram.

The weight force of the ramp has a known magnitude and direction so we can extend it vertically up. Next, we know the reaction at the roller support is going to act normal to the surface which it rolls along and in this case, it will act perpendicular to the ramp. Extending the force line of the reaction at the roller allows us to find the point of concurrency and use this to solve the problem.

Once we've established the point of concurrency of the 500 Newton weight force and the reaction at the roller we can draw a line from where the ramp meets the ground to the point of concurrency. This gives us the direction of the ground reaction and we can measure the angle of this vector from the vertical axis to see the angle required for equilibrium and compare it against the maximum angle of static friction.

Measuring the angle of the reaction at the ground gives us an approximate angle of 19 degrees required for equilibrium. If we use our rearranged friction formula which is the inverse of tan multiplied by the coefficient of friction, giving us the maximum angle of static friction, we can see that the maximum angle of static friction of 31.8 degrees.

Although we drew and measured our original angle, so it's not 100% accurate. There is a great enough difference between 19 degrees and 31.8 degrees that we can confidently say that the ramp will remain static.

Again, comparing the solutions, the graphical solution is arguably simpler. [The graphical solution showing the point of concurrency and angle of static friction calculation is shown next to the working for the analytical solution.] The analytical solution requires the examinee to have a sound understanding of friction, moments, components of vectors and reactions at rollers, while the graphical solution only requires the examinee to understand friction, the three-force rule and how to apply the graphical solution.

To summarize everything we've learned in this video, it is important to be familiar with the use of both graphical and analytical solutions and remember that it is not unreasonable for the examiners to ask you to provide a graphical solution. When you're faced with a problem, it is worthwhile to take a moment to analyse the problem and identify the most appropriate method of solution whether it be graphical or analytical. When adding vectors in a polygon, take care to arrange them tip to tail, otherwise your polygon will be incorrect and any information you are trying to deduce will also be incorrect. Practice answering questions both graphically and analytically so if you need to apply either solution, you can do so without hesitation. Ensure that you use arrowheads to indicate the sense of vectors and select an appropriate scale and state the scale in your answer.

Finally, remember to look for the three-force rule as these questions are often readily solved graphically. There's a short series of questions that align to this video to help develop your skills in answering questions both graphically and analytically, Try solving each question using both methods and it may also be a useful exercise to time each attempt.

Finally, thanks for watching this presentation and hopefully you'll feel more comfortable in applying graphical solutions and good luck with your HSC.

(gentle upbeat music)

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