Mathematics: Standard
Minimum spanning trees transcript

(Duration 12 minute 38 seconds)

This is the HSC Hub, Mathematics curriculum support for the New South Wales Department of Education and training. My name is Sarah Warry. Today we will look through question 28 from the 2019 Mathematics Standard One exam paper. This question also featured as question 30 in the Mathematics Standard Two exam paper. This question covers the common content found in topics N1 Networks and Paths from the Mathematics Standard one syllabus and N2 Network Concepts from the Mathematics Standard Two syllabus.

This question reads: "The network diagram shows the tracks connecting eight picnic sites in a nature park. The vertices A to H represent the picnic sites. The weights on the edges represent the distances along the tracks between the picnic sites, in kilometres. You will see that the network diagram on your screen contains eight vertices labelled A through to E and weighted edges connecting them. Please pause this video to allow yourself time to read through the questions again if you need to before we proceed.

The first thing to notice is that part A is worth 2 marks. This indicates that the marker will be looking for two pieces of information. When you read the question carefully, you will see that it is asking you to do two things. You will need to draw a minimum spanning tree and you will need to calculate the minimum length of water pipes.

When looking at the marking guidelines for this question, you will see exactly what the marker is looking for. Students will receive two marks for providing a correct minimum spanning tree and correct minimum length and only one mark if they calculate the minimum length from an incorrect network diagram or equivalent merit.

For the first part of this question, you need to draw a minimum spanning tree. In the Mathematics Standard syllabus, both for Standard One and Standard Two. It states that students "determine the minimum spanning tree by using Kruskal's or Prim's algorithm or by inspection". It is important to note that you can use any of these methods. For the purpose of this resource, I will outline both Kruskal's, and Prim's algorithm, but remember that you can use the method that you feel most comfortable with.

So let's get started. Let's look at Kruskal's algorithm first. Using Kruskal's algorithm, the first step is to choose the edge with the least weight. In this question we will choose the edge connecting G to F as it has a weight of one kilometre. Step 2 is to look at the remaining edges and choose the next smallest weighted edge. In this question we will now choose the edge connecting G to H with a weight of two kilometres. The last step is to continue this process until all the vertices are connected by one path. Remember, we do not want to form a cycle. So the next edge we will choose will be the edge that joins H to C with the weight of three kilometres.

We now look at the edges that have a weight of four kilometres. We will not choose the edge connecting C to G as it would form a cycle. We will however choose the edge connecting A to B. Looking now at the edges that have a weight of five, we have a few options. We will definitely choose the edge connecting B to C as that will connect vertices A and B to the rest of our path. To connect vertices E and D we have a few choices. We may choose the edge connecting H to E and then the edge connecting D to E. Once all the vertices are connected, we add all of the weights along the path together to determine the minimum length of pipe required. In this question, the minimum length of pipe required is twenty-five kilometres. Another option is shown on the right hand side of the screen.

Let's now take a look at Prim's algorithm on the same question. The first step in Prim's algorithm is to select a vertex to begin your path. You can select any vertex you like and I promise that no matter which one you choose, it will result in a minimum spanning tree. For this question I have chosen vertex A. Step 2 is to look at all the edges that connect to our chosen vertex. In this question, it's easy as there's only one edge connected to A. Therefore, let's choose the edge connecting A to B. Now we need to look at all of the remaining edges connected to vertices A and B and choose the edge with the least weight. In this question we don't have anymore edges coming from vertex A and we have two extra edges coming out of vertex B. Our options are we can choose the edge connecting B to C with a weight of five. Or the edge connecting B to G which has a weight of 6. Because we want the minimum spanning tree, we will select the edge connecting B to C as it has the smallest weight. Now we look at all the remaining edges coming from vertices A, B and C and choose the next smallest weight. You continue this process until you have a path that connects all of the vertices together.

So looking at our question, we still have an edge with weight six coming from vertex B that we haven't chosen yet and we have three remaining edges coming from vertex C with weights of four, three and seven. So the next edge we would choose is the edge connecting C to H as it has the smallest weight. Now, looking at the remaining edges coming from vertices A, B, C and now H, we will choose the edge connecting H to G with a weight of two kilometres, and then we will choose the edge connecting G to F with the weight of one kilometre.

It is at this point where we have a few possibilities, either one of the edges with weights of five kilometres that are leading from vertex H would be acceptable. I've chosen the edge connecting H to E in this sample solution. Once again, I now have a few options. I could choose the edge connecting E to D or the edge connecting H to D. It doesn't matter which one you choose. In this sample solution I've chosen the edge connecting H to D which also has a weight of five kilometres. Like Kruskal's algorithm, once we have drawn a minimum spanning tree, you add all the weights together to determine the minimum length of pipe. For this question, the minimum length of pipe is twenty-five kilometres.

 You will see that this is what NESA released as a sample solution in a Mathematics Standard One HSC marking guidelines in 2019. The solution is exactly the same for the equivalent question in the Mathematics Standard Two marking guideline. It supports that the minimum length of water pipes needed is twenty-five kilometres and provides a network diagram similar to the one shown in this video.

Let's now take a look at Part B of the question. Part B raids. One day the track between C&H is closed, state the vertices that identify the shortest path from C to E that avoids the closed track". Please pause this video so that you can carefully read through the question and analyse the network diagram.

You will notice that Part B has only been allocated one mark, which indicates that the marker is only looking for one piece of information. In Part B, the marker is looking for your ability to determine the shortest path from C to E. This is also reflected in the criteria from the marking guidelines. The syllabus does not mention a particular method that students need to use to determine the shortest path. Some students will be able to calculate the shortest path via inspection. Others may need a formal process to follow.

The topic guidance produced by NESA does outline Dijkstra's algorithm as an example. Before we get started on this question, there is one critical piece of information that we have to take into consideration. The track between C and H is closed. With this in mind, I put a cross through the edge connecting C to H as a reminder that the question told us that the track was closed. Next it is a good idea to mark the start and the end of the required path. After that, the first step is to look at all the edges that branch out from the starting vertex.

For this question you will see that vertex B is five kilometres away from vertex C. G is four kilometres away from C and D is seven kilometres away from C. From those vertices we then look at which vertices are now two edges away from our starting point. For the purpose of this question, we do not need to worry about the path from B to A as it is not going to be useful in getting us to vertex E. From vertex G, we could move to vertices H and F. To get to vertex H, we would need to add two kilometres to the four kilometres we have already travelled to G, which will result in six kilometres. To get to vertex F we would need to add one kilometre to the four kilometres we have already travelled to G, which will result in five kilometres. If we went to vertex E via D, we would need to add five to the seven kilometres that we have already travelled, which will result in twelve kilometres.

Although we have found a path to get to E, we need to investigate whether our other paths leading towards vertex E would in fact be shorter. We've already determined that we can get to vertex E via D in twelve kilometres. If we went to vertex E via D and H, it would take seventeen kilometres. If we went to vertex E via G and H it would take eleven kilometres. If we went to vertex E via G and F it would be twelve kilometres. Now that we have calculated the different ways you can get from C to E, it is clear that the shortest path goes from C to G to H, and then finally to E with a length of eleven kilometres.

Alternatively, you could determine all the possible paths and choose the shortest one from the list. This may be time consuming depending on the complexity of the network diagram. You'll see that once again the shortest path is C, G, H and E.

You will see that this is what NESA released as a sample solution in the Mathematics Standard One HSC marking guidelines in 2019, and the solution is exactly the same for the equivalent question in the Mathematics Standard Two marking guideline. It supports that the shortest path is from C to G to H to E, and providing this answer will award you with one mark.

This is the HSC Hub for the New South Wales Department of Education.

End of transcript