 NESA exemplar question solutions

S5 The normal distribution

Solutions for questions from the NESA topic guidance related to the normal distribution.

1. Packets of rice are each labelled as having a mass of $1$kg. The mass of these packets is normally distributed with a mean of $1.02$kg and a standard deviation of $0.01$kg. Complete the following table:

| Mass in kg | 1.00 | 1.01 | 1.02 | 1.03 | 1.04 |
| --- | --- | --- | --- | --- | --- |
| $z$-score | -2 | -1 | 0 | 1 | 2 |

1. What percentage of packets will have a mass less than $1.02$kg?

Solution*:* $50\%$

1. What percentage of packets will have a mass between $1.00$ and $1.04$kg?

Solution*:* $95\%$

1. What percentage of packets will have a mass between $1.00$ and $1.02$kg?

$Solution$*:* $47.5\%$

1. What percentage of packets will have a mass less than the labelled mass? $Solution: 2.5\%$
2. A machine is set for the production of cylinders of a mean diameter $5.00$cm, with standard deviation $0.020$cm. Assuming a normal distribution, between which values will $95\%$ of the diameters lie? If a cylinder, randomly selected from this production, has a diameter of $5.070$cm, what conclusions could be drawn?

Solution*:* When $\overbar{x}=5$, SD $=0.02$

| Diameter in cm | 4.96 | 4.98 | 5.00 | 5.02 | 5.04 |
| --- | --- | --- | --- | --- | --- |
| $z$-score | -2 | -1 | 0 | 1 | 2 |

$∴95\%$ of the cylinders have a diameter of between $4.96$ and $5.04$ cm

When $x=5.07, \overbar{x}=5$, SD $=0.02$

$$z=\frac{x-\overbar{x}}{SD}$$

$$z=\frac{5.07-5}{0.02}$$

$$z=3.5$$

$∴$ As $5.07$cm has a $z$-score of $3.5$ it is highly unlikely that it would be selected at random as only $0.15\%$ of values lie outside of $3$ standard deviations above the mean. It may also indicate a possible error in the measurement or analysis.

1. Find the probability that a person selected at random from a pool of people that took a test on which the mean was $100$ and the standard deviation was $15$ will have a score of:

Note: [Normal distribution tables](http://www.ztable.net) would be used to answer this question.

1. between $100$ and $120$

Solution*:* When $x=120, \overbar{x}=100$, SD $=15$

$$z=\frac{x-\overbar{x}}{SD}$$

$$z=\frac{120-100}{15}$$

$$z=1.3\dot{3}$$

Using the z-score table on page $6$, a $z$-score of $1.33$ indicates that the area to the left is $0.9082$ and the area to the left of the mean is $0.5$

$∴$The probability that the score is between 100 and 120 is:

$$= 0.9082-0.5$$

$$=0.4082$$

1. of at least $120$

Solution: From part a above. A score of $120$ results in a $z$-score of $1.3\dot{3}$. Using the $z$-table below it indicates that the probability to the left of $120$ is $0.9082$.

$∴$ The probability that the score is at least $120$ is $0.9082$.

1. of greater than $120$

Solution:

Probability of the score being greater than $120=1-0.9082$

Probability of the score being greater than $120=0.0918$

1. The lifetime of a particular lightbulb is normally distributed with mean of $1020$ hours and standard deviation $85$ hours. Find the probability that a lightbulb of the same make chosen at random has a lifetime between $1003$ and $1088$ hours.

Normal distribution tables would be used to answer this question.

Solution*:* When $x=1003, \overbar{x}=1020$, SD $=85$

$$z=\frac{x-\overbar{x}}{SD}$$

$$z=\frac{1003-1020}{85}$$

$$z=-0.2$$

When $x=1088, \overbar{x}=1020$, SD

$$z=\frac{x-\overbar{x}}{SD}$$

$$z=\frac{1088-1020}{85}$$

$$z=0.8$$

Using the $Z$-Table on page $6$, the area to the left of a $z$-score of $-0.2$ is $0.4207$ and the area to the left of a $z$-score of $0.8$ is $0.7881$.

$∴$ The probability that the lifetime of the lightbulb is between $1003$ and $1088$ $=0.7881-0.4207$

$∴$ The probability that the lifetime of the lightbulb is between $1003$ and $1088$ $=0.3674$