 NESA exemplar question solution

The circumcentre of a triangle is the centre of the circle that passes through each of the vertices. The centroid is the point of intersection of the angle bisectors of a triangle. Let $O$ be the circumcentre and $G$ the centroid of $∆ABC$. $H$ is the point of $OG$ such that $\vec{OH}=3 \vec{OG}$. Prove that $\vec{AH}⊥\vec{BC}$.



[www.geogebra.org/m/byvcmtvd](file:///C%3A/Users/dproctor2/AppData/Local/Microsoft/Windows/INetCache/Content.Outlook/RFXZX7ZT/www.geogebra.org/m/byvcmtvd)

Centroid $\overset{\to }{OG}=\frac{\overset{\to }{OA}+\overset{\to }{OB}+\overset{\to }{OC} }{3}$ and $\overset{\to }{OH}=3\overset{\to }{OG}$

$$∴\overset{\to }{OH}=\overset{\to }{OA}+\overset{\to }{OB}+\overset{\to }{OC}$$

$$\overset{\to }{OH}=\left(\begin{matrix}x\_{1}+x\_{2}+x\_{3}\\y\_{1}+y\_{2}+y\_{3}\end{matrix}\right)$$

$$\overset{\to }{AH}=\overset{\to }{OH}-\overset{\to }{OA}$$

$$=\left(\begin{matrix}x\_{1}+x\_{2}+x\_{3}\\y\_{1}+y\_{2}+y\_{3}\end{matrix}\right)-\left(\begin{matrix}x\_{1}\\y\_{1}\end{matrix}\right)=\left(\begin{matrix}x\_{2}+x\_{3}\\y\_{2}+y\_{3}\end{matrix}\right)$$

and $\overset{\to }{BC}=\overset{\to }{OC}-\overset{\to }{OB}$

$$=\left(\begin{matrix}x\_{3}\\y\_{3}\end{matrix}\right)-\left(\begin{matrix}x\_{2}\\y\_{2}\end{matrix}\right)$$

$$=\left(\begin{matrix}x\_{3}-x\_{2}\\y\_{3}-y\_{2}\end{matrix}\right)$$

Test $\overset{\to }{AH}.\overset{\to }{BC}=0$

LHS $=\overset{\to }{AH}.\overset{\to }{BC}$

$$=\left(x\_{2}+x\_{3}\right)\left(x\_{3}-x\_{2}\right)+\left(y\_{2}+y\_{3}\right)\left(y\_{3}-y\_{2}\right)$$

$$=(x\_{3})^{2}-\left(x\_{2}\right)^{2}+(y\_{3})^{2}-(y\_{2})^{2} $$

$$=\left(x\_{3}\right)^{2}+\left(y\_{3}\right)^{2}-(\left(x\_{2}\right)^{2}+\left(y\_{2}\right)^{2})$$

$$=\left|\vec{OC}\right|^{2}-\left|\vec{OB}\right|^{2}$$

$=OC^{2}-OB^{2}$ [$OC=OB$ (radii of circle)]

$$=0$$

$=$ RHS

Therefore $AH$ is perpendicular to $BC$