 Sample questions solutions

Integration by substitution

1. using the substitution .

Let

1. using the substitution .

Let

And

1. , where .

Let

**Note**: If you chose, a second substitution would have been required.

and

1. Find .

Let

Bounds:

When ,

When

Let

And

Note: This is one of the standard results on the student reference sheet.

1. using the substitution

Let

and

1. Use the substitution to show that .

Let

Bounds:

When

When

1. By completing the square, find:

Let

Note: that this solution references a standard integral that no longer appears on the reference sheet and has been redundant in the Extension courses for a few years.

The standard integral is

Integration of functions with a quadratic denominator



Let

Complete the polynomial division to obtain:

Let

Let

Let

Let

Integration using partial fractions

Let

When

When

Let

When

By matching the real and imaginary coefficients from the LHS and RHS gives

and

When

and

Let

=

When

and

and

When

Let

1. The expression can be written as where , and are real numbers.
   1. Find , and .

When

When

When

* 1. Hence find .

Let

1. Decompose into partial fractions and hence show that

When

When

Let

­

1. Decompose into partial fractions and hence show that .

When

When

and

When

Let

Integration by parts

Let

1. Use integration by parts to find *.*

Let

Let

*Let and ,*

Let

1. .

Let

Apply integration by parts a second time:

Adding to both sides:

Or

**Alternative solution:** Both functions can be easily integrate and differentiated so we could have chosen the opposite for and

Let

Apply integration by parts a second time:

Adding to both sides:

Or

1. Using two applications of integration by parts, evaluate

Let

Apply integration by parts a second time:

Adding to both sides:

Recurrence relationships



Adding to both sides:

Divide by n:

1. For let .

Show that for , .

Hence or otherwise calculate .