 ME-V1 Introduction to vectors

This document contains sample questions and solutions from the sample unit ME-V1 Introduction to vectors.

Identifying scalars and vectors

A scalar is a quantity that has magnitude only. A vector is a quantity that has both magnitude and direction.

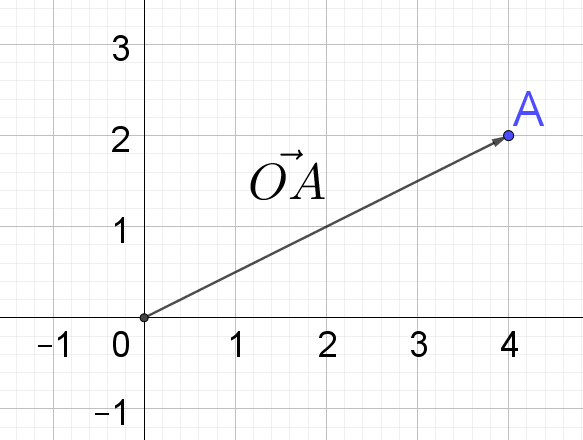
Identify each quantity as a scalar or vector: length, displacement, area, volume, velocity, acceleration, speed, time, mass, weight, density, force, temperature

| Scalar | Vector |
| --- | --- |
| Length | Displacement |
| Area | Velocity |
| Volume | Acceleration |
| Speed | Weight |
| Time | Force |
| Mass |  |
| Density |  |
| Temperature |  |

| Scalar | Vector |
| --- | --- |
| Length | Displacement |
| Area | Velocity |
| Volume | Acceleration |
| Speed | Weight |
| Time | Force |
| Mass |  |
| Density |  |
| Temperature |  |

Vector notation

1. Write vector as a column vector, an ordered pair and in component form.

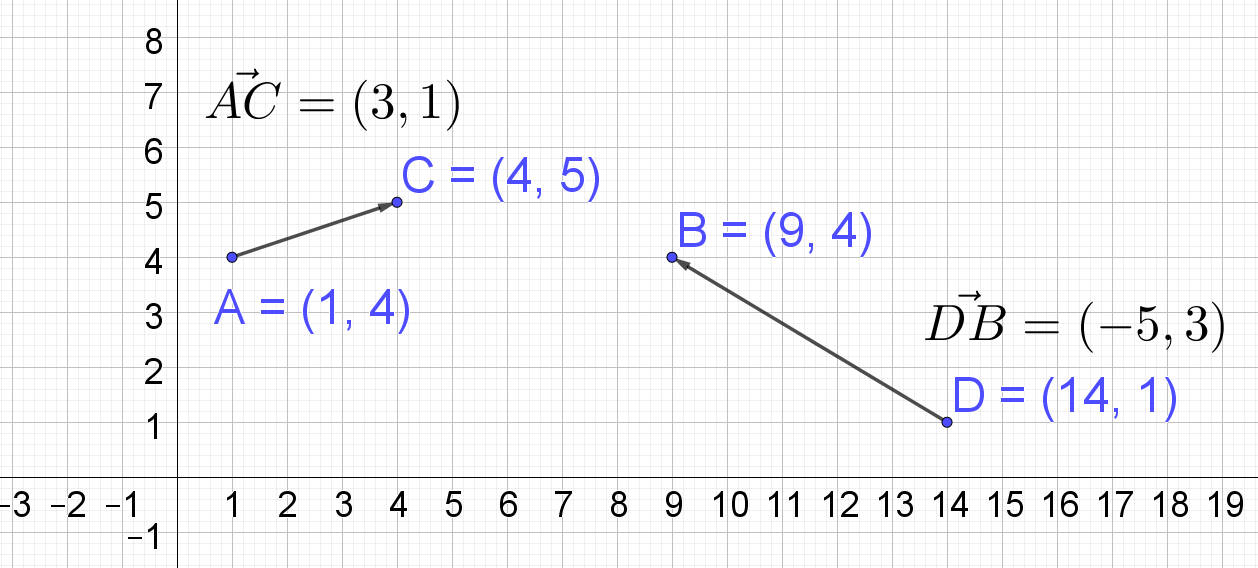


Column vector:

Ordered pair:

Component form:

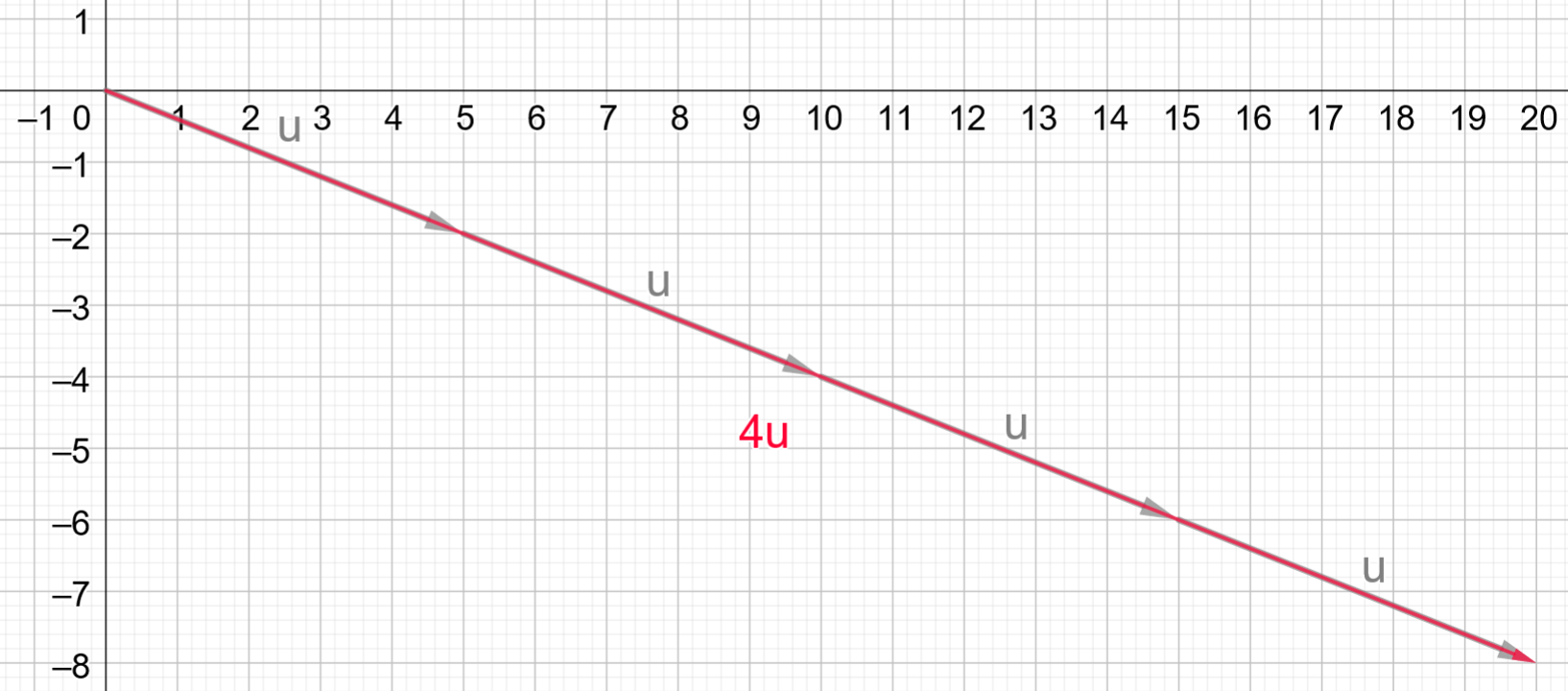
1. On a Cartesian plane:
   1. Plot two points and
   2. Plot two additional points and and draw vectors and such that and



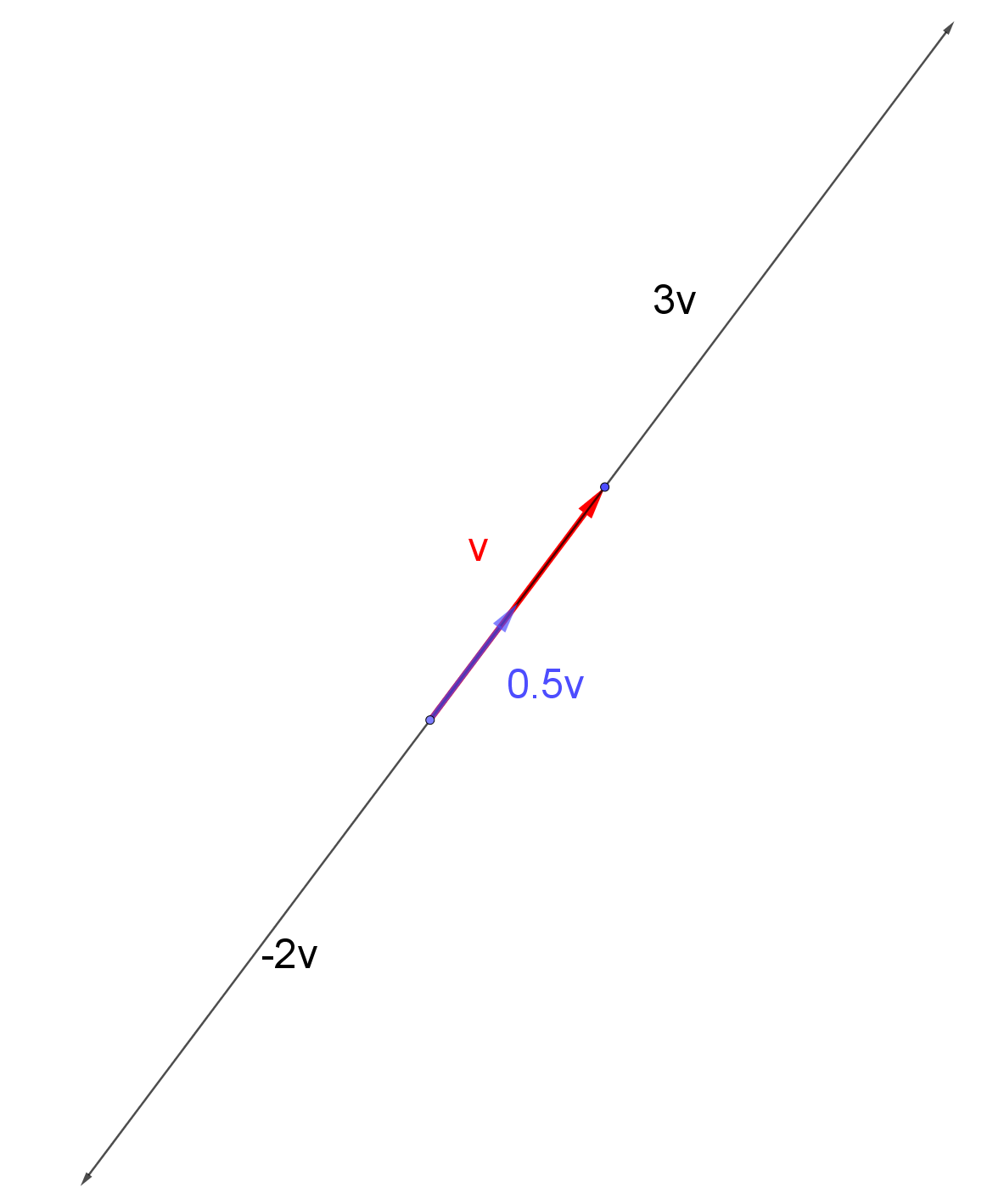
Multiplying a vector by a scalar

1. If calculate .

Geometrically, this can be shown as:



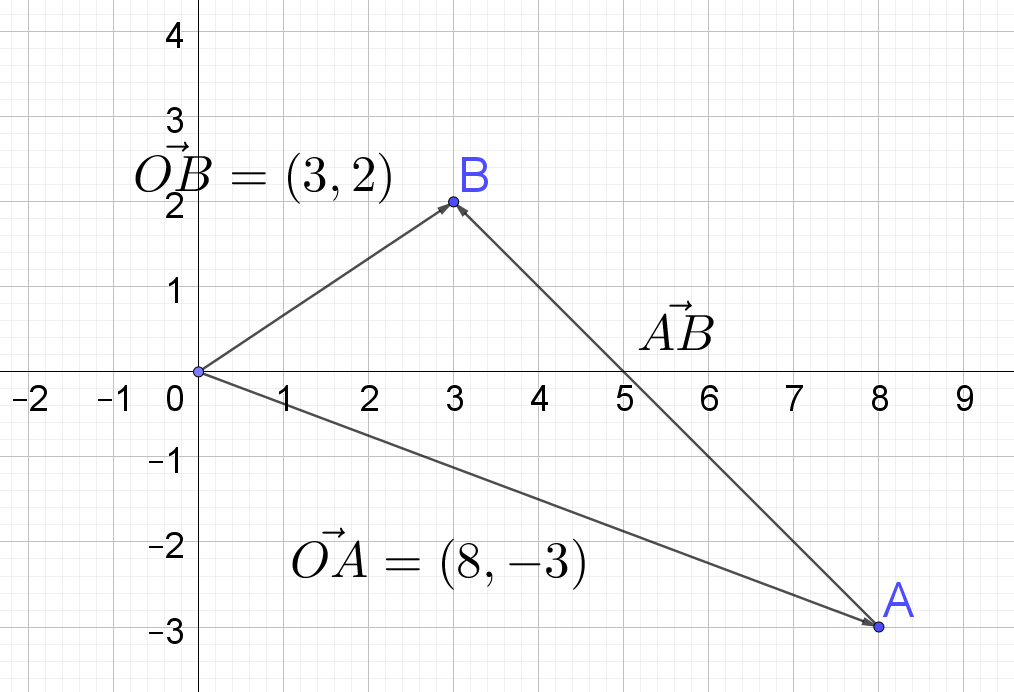
1. On a blank page draw a vector . Use this to draw vectors and .



1. If , find and .

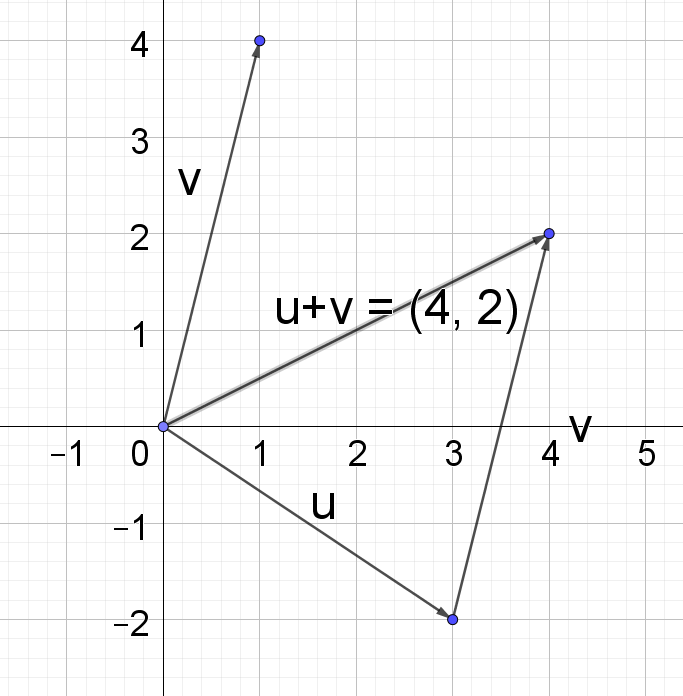
Adding and subtracting vectors

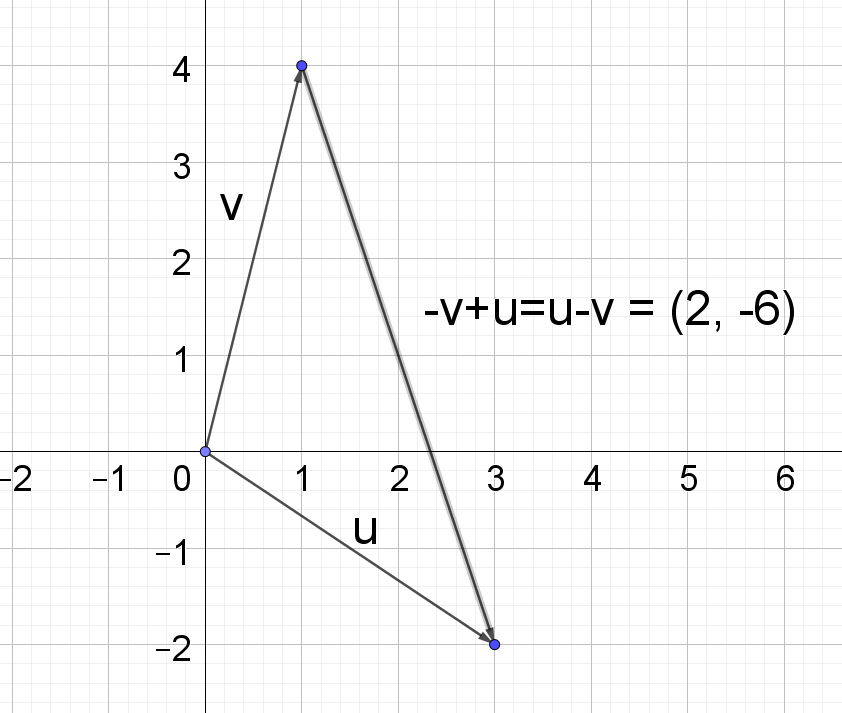
1. Given 8and :
   1. graph vectors and on a Cartesian plane

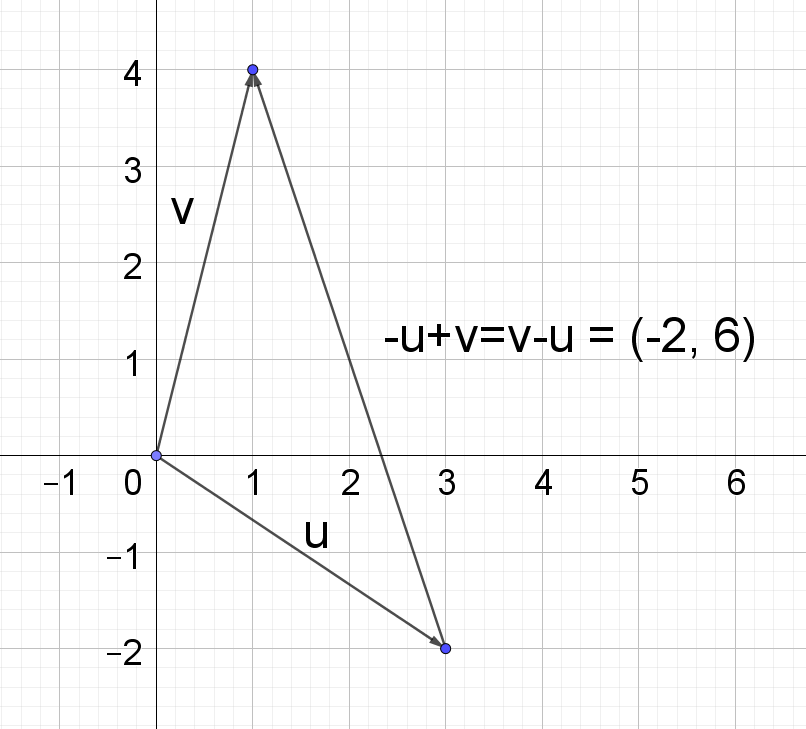


* 1. Find the vector
  2. Explain the relationship between , and
  3. Confirm the relationship numerically.

1. If and , calculate
2. Consider the vectors and . Find:
   1. Confirm the results for a. to c. by geometrically representing these on a Cartesian plane.







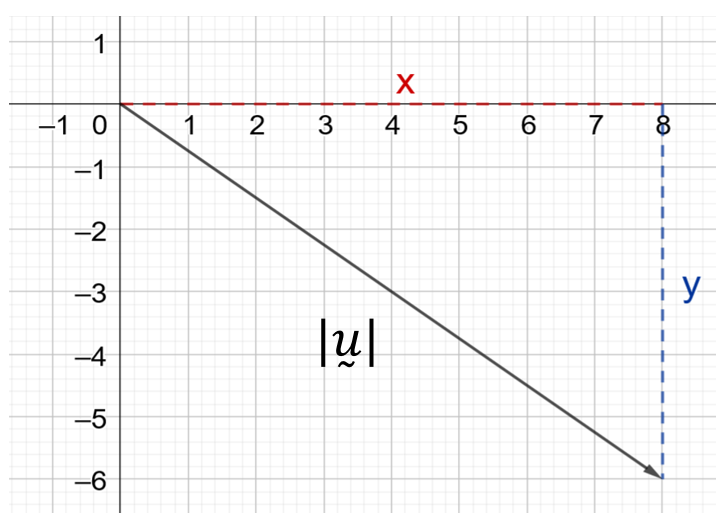
1. If , and match each statement to its component form.

| Statement | Component form expressions |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

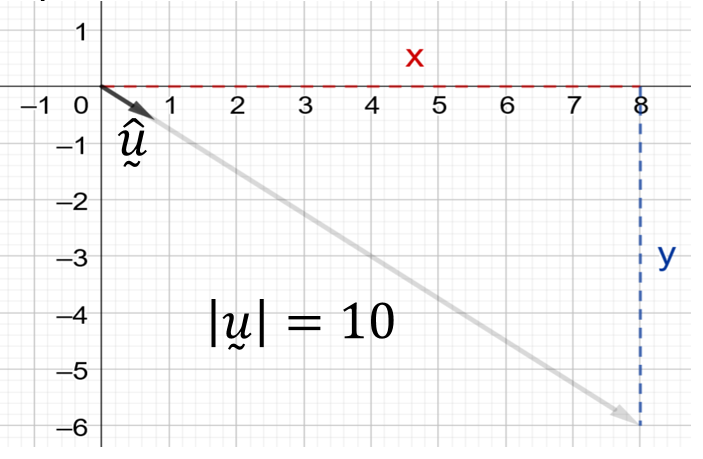
| Statement | Component form expressions – solutions |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Magnitude of a vector

1. If , find the:
   1. magnitude of the vector



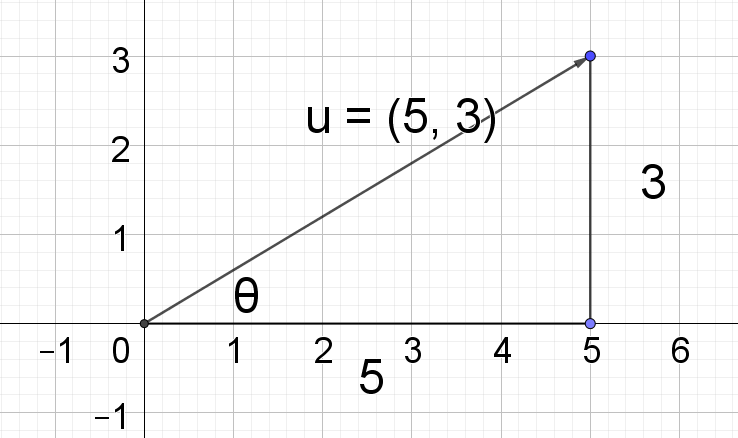
* 1. unit vector



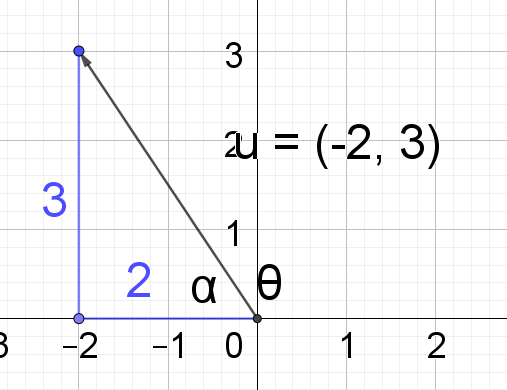
1. If , find the:
   1. magnitude vector .
   2. unit vector
2. Given 4and , find the magnitude of

Direction of a vector

1. A particle is projected from the origin with an initial velocity of . What is the angle of projection?



1. A particle is projected from the origin with an initial velocity of . What is the angle of projection?



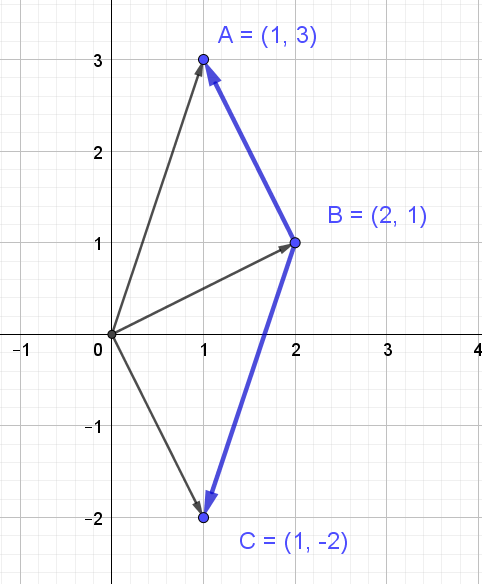
Scalar (dot) product

1. , If and
   1. Calculate
   2. Find the angle between and

.|||

* 1. Show the distributive property holds, i.e.

1. *A*, *B* and *C* are points defined by the position vectors , and respectively. Find the size of .



is the angle between vectors and

.|||

, or in our case, letting ,

1. If and , find and hence find the angle between and (to the nearest degree).

Parallel and perpendicular vectors

1. If , ,
   1. Show and are perpendicular

and are perpendicular

* 1. Show and are parallel

Show or if show

and are parallel.

Alternate method: Show

1. If is perpendicular to vector , find .

as they are perpendicular.

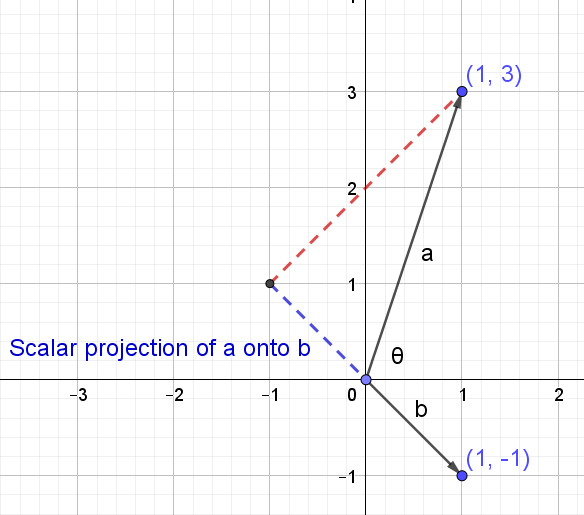
1. If is parallel to vector , find .

Two vectors are parallel if they are scalar multiples of one another.

Vector projection

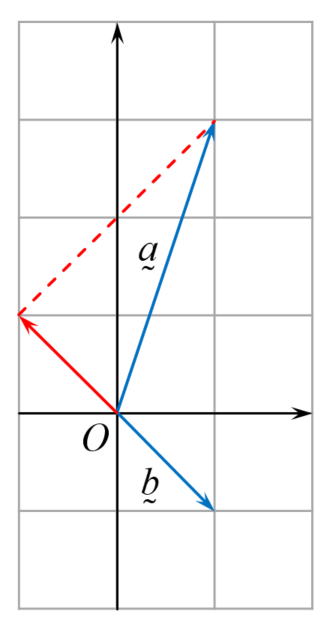
1. Let and ,
   1. find the scalar projection of onto .

Scalar projection of onto

  
**Note:** The scalar projection will be negative as opposes

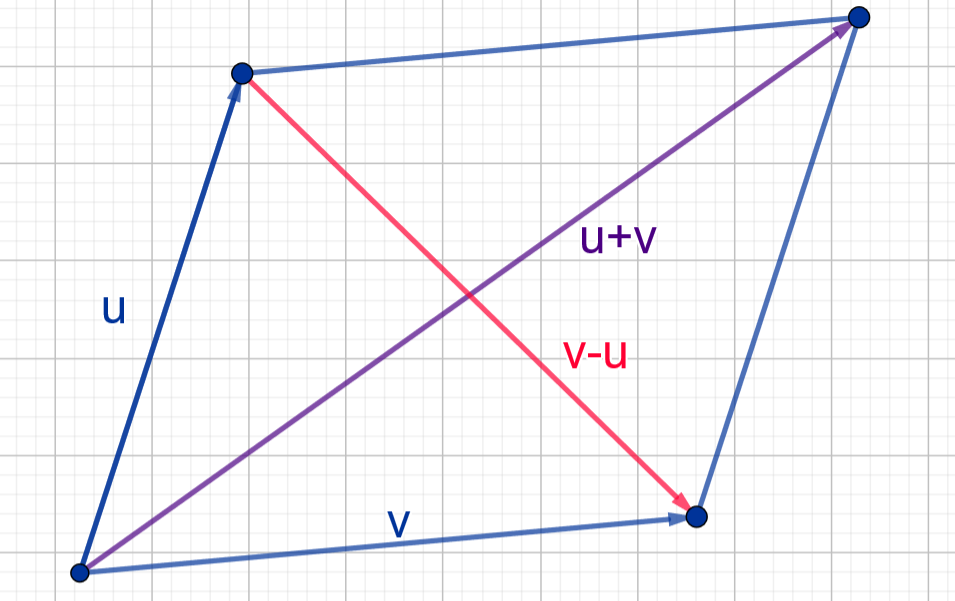
* 1. find the vector projection of in the direction of .

1. Let and , find the vector projection of in the direction .



Proving geometric results

1. The diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)

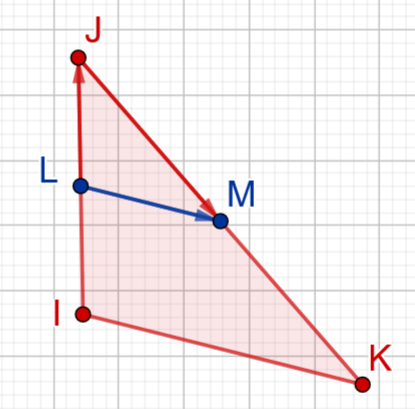


If the diagonals meet at a right angle then

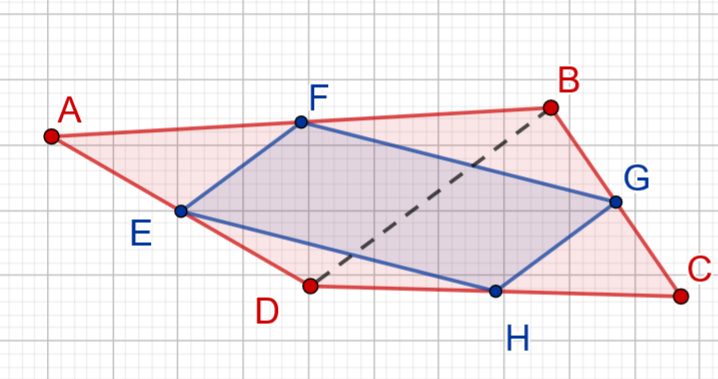
The parallelogram is a rhombus

1. The midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)

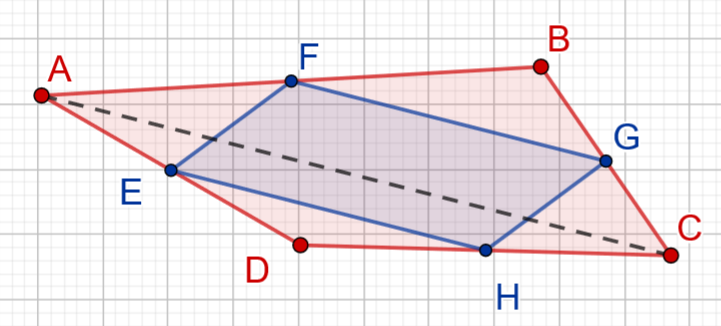
**Part 1:** First show the line joining the midpoints of two sides of a triangle is parallel to the third side and is half its length.



**Part 2:** Consider quadrilateral ABCD with midpoints F, G, E and H.



and

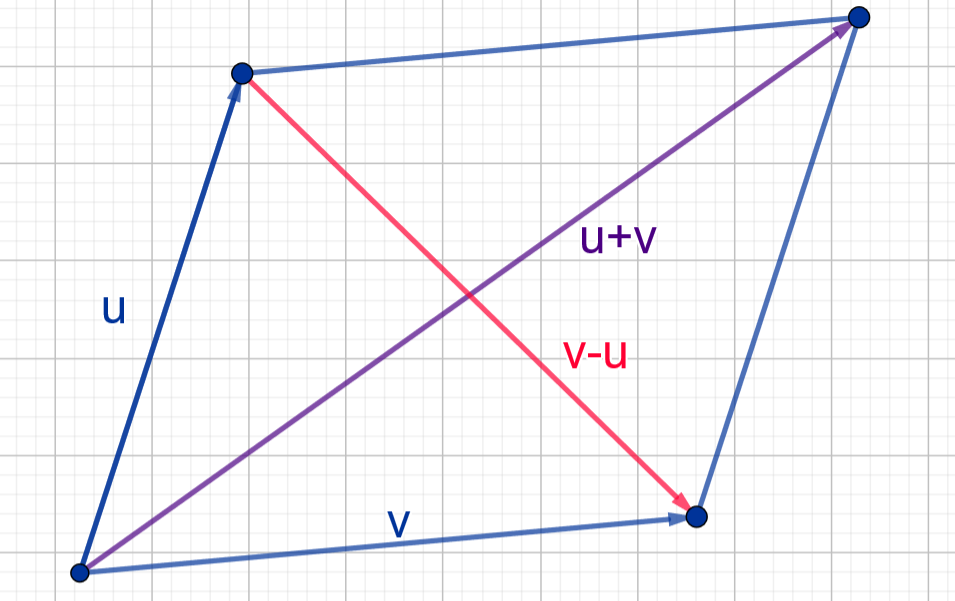


and

is a parallelogram as opposite sides are equal in length.

1. The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides (ACMSM041)

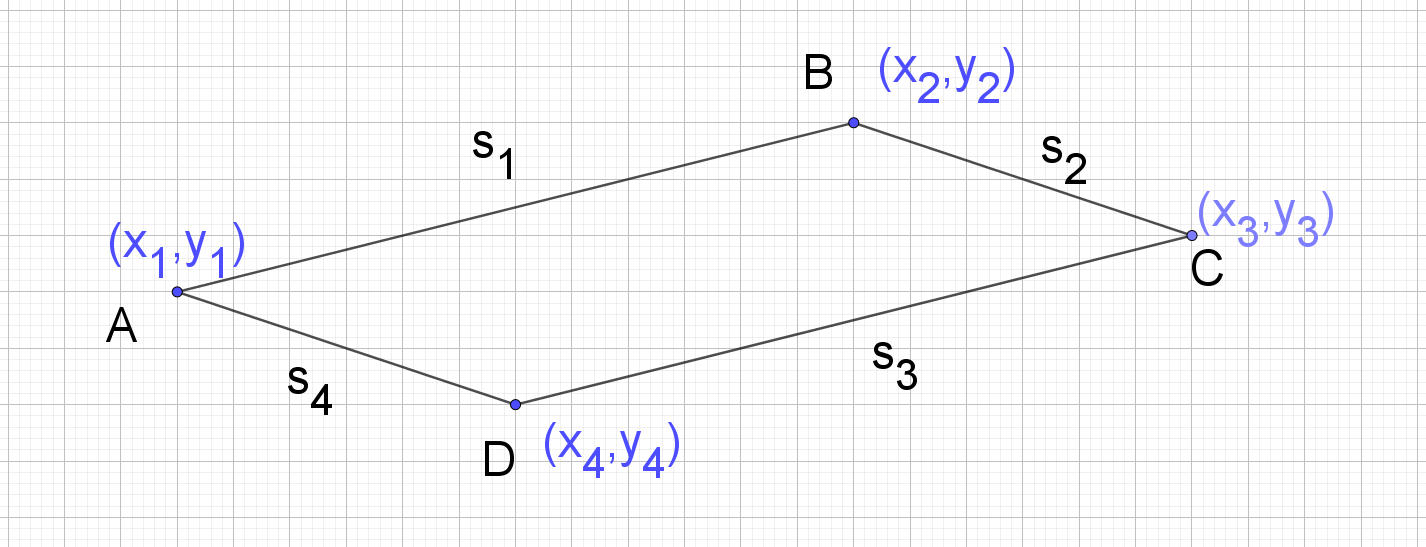
**Proof using vectors:**



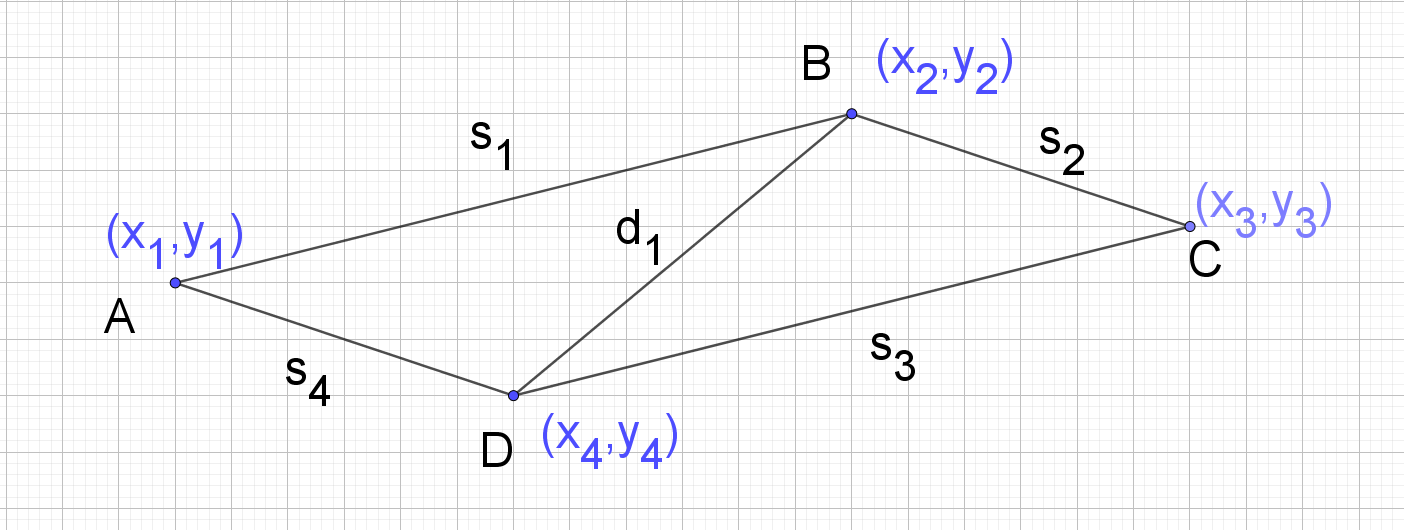
**i.e.** the sum of the squares of the length of the sides

The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides

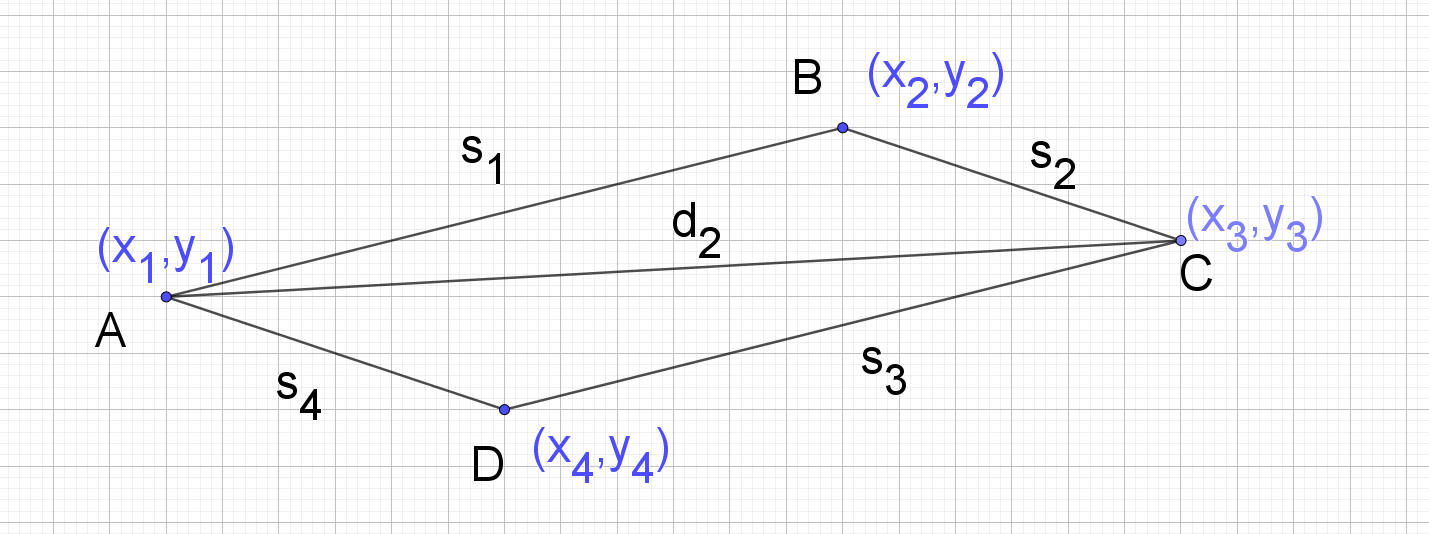
**Proof without vectors:**



Consider the cosine rule for angle C in triangle DBC:



Consider the cosine rule for angle D in triangle DAC:



(Cointerior angles are supplementary when BC||AB)

(Opposite sides of a parallelogram are equal)

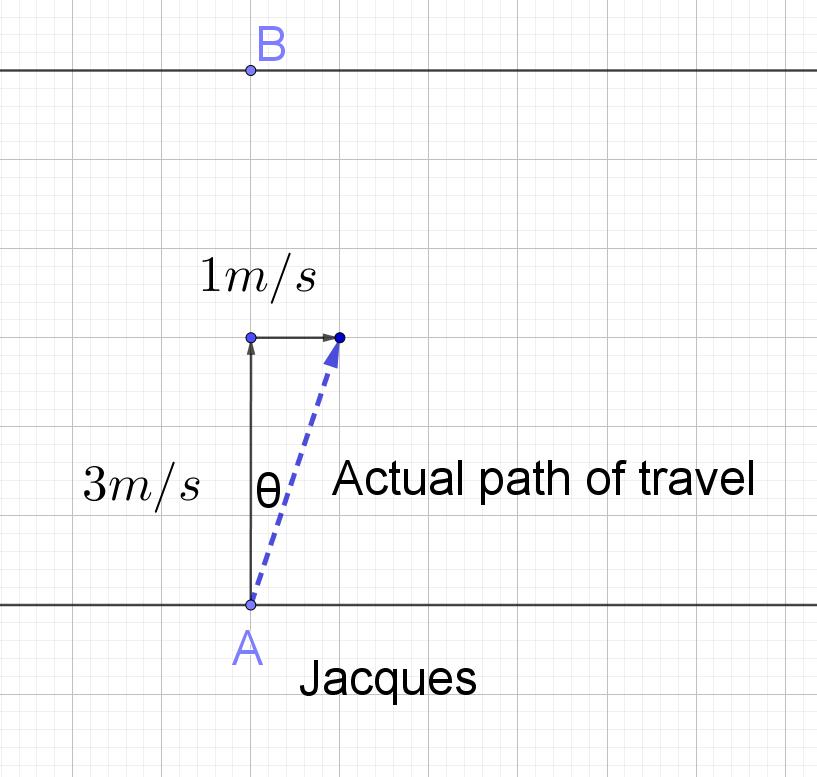
Multiply both sides by

(Opposite sides of a parallelogram are equal)

The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides.

Modelling motion

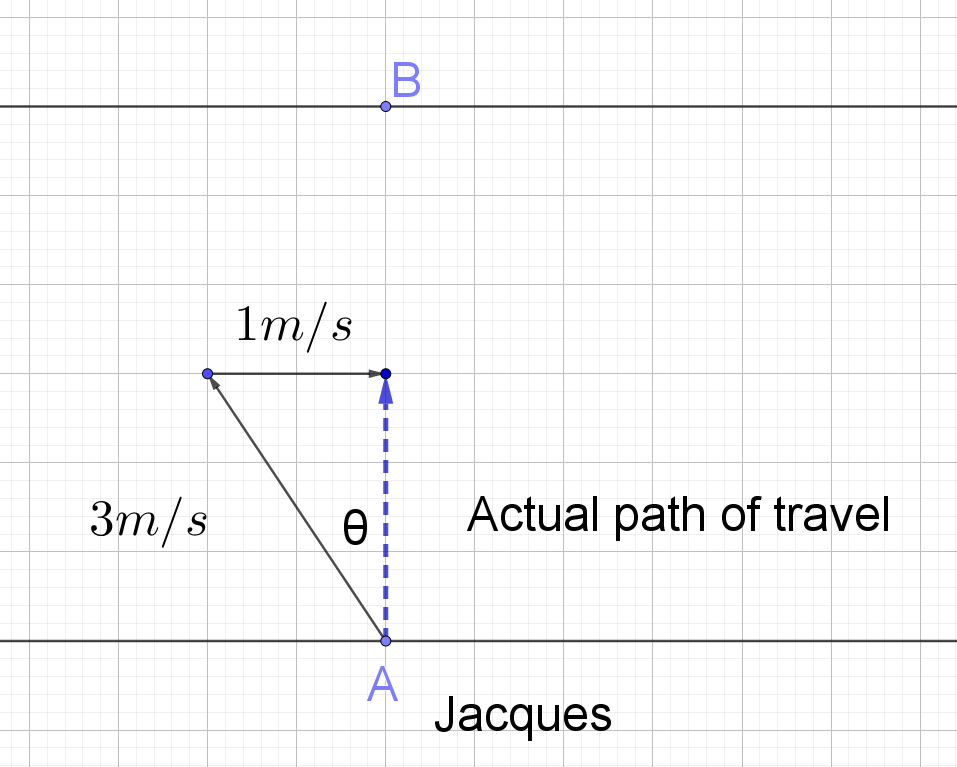
1. In still water, Jacques can swim at 3 m/s. Jacques is at point A on the edge of a canal, and considers point B directly opposite. A current is flowing from the left at a constant speed of 1 m/s. If Jacques dives in straight towards B, and swims without allowing for the  
   current, what will his actual speed and direction be? (answers correct to 1 decimal place)



Direction is the deviation of the desired path.

Let Using Pythagoras’ Theorem,

1. Jacques wants to swim directly across the canal to point B. At what angle should Jacques aim to swim in order that the current will correct his direction? And what will Jacques’ actual speed be? (answers correct to 1 decimal place)

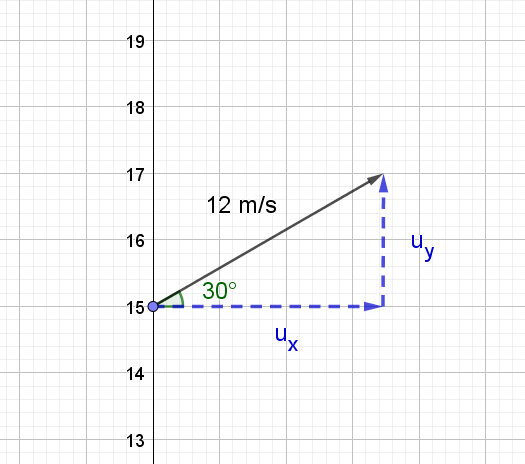


Direction Jacques should aim to swim:

Let Using Pythagoras’ Theorem,

Projectile motion

1. A projectile is launched off a cliff 15 above the ground at an angle of to the horizontal with an initial velocity of . Find:



* 1. Initial velocities
  2. Horizontal components

When

When

* 1. Vertical components

When

When

Note: The [projectile motion](https://www.geogebra.org/m/BXBMnZPS) Geogebra applet can be used to check solutions for a, b and c.

* 1. The time of the flight

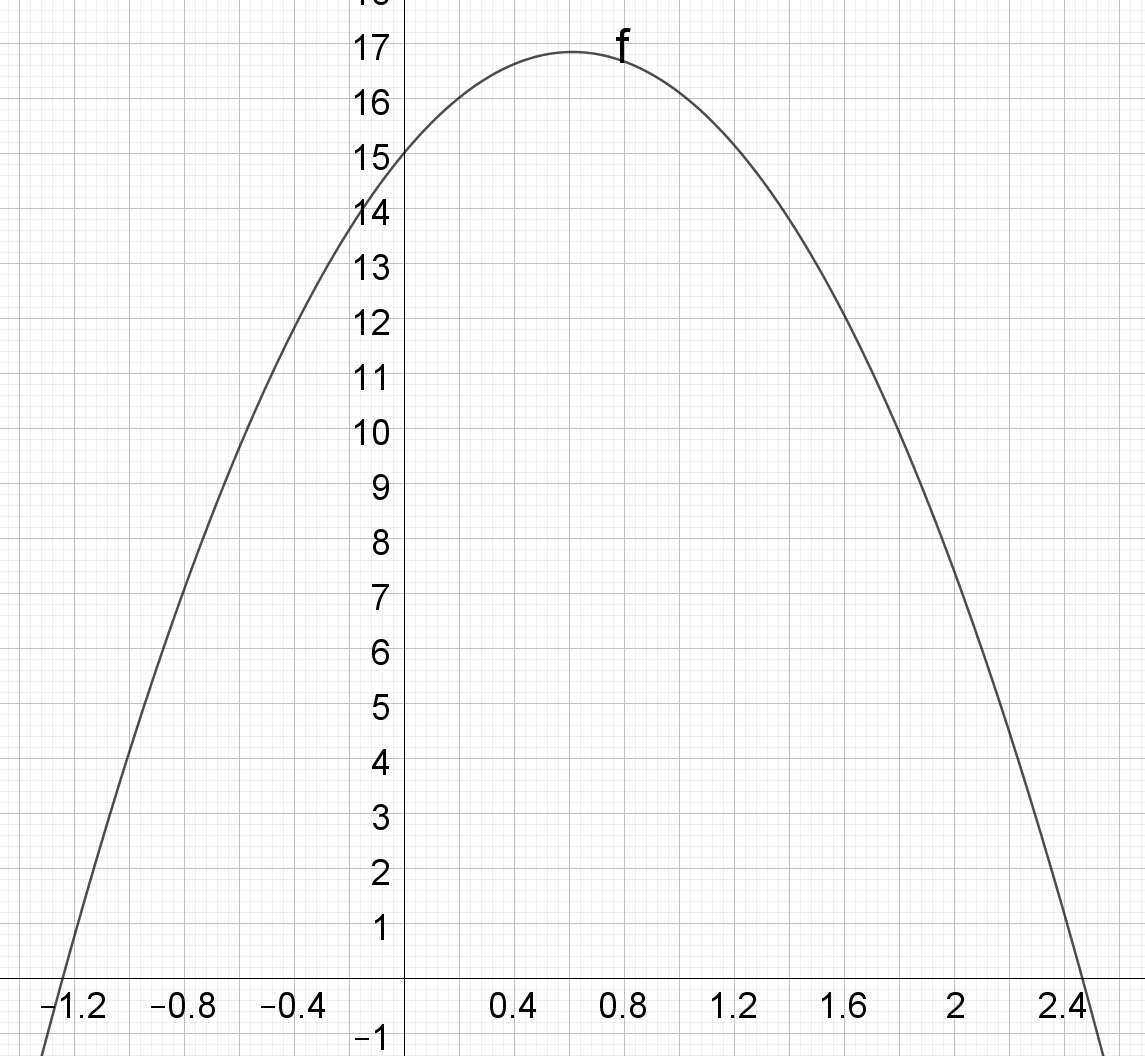
The time of the flight is the time taken for the projectile to hit the ground, **i.e.** the vertical displacement is zero.

Solve for when

Use the quadratic formula with

the time of the flight is (correct to 2 decimal places)

The solution can be checked by graphing using graphing software. (use in place of in the graphing software)



* 1. The range of the projectile

The range of the projectile is the horizontal displacement when the projectile strikes the ground.

Solve for when

the range of the projectile is (correct to 2 decimal places)

* 1. The final velocity of the projectile

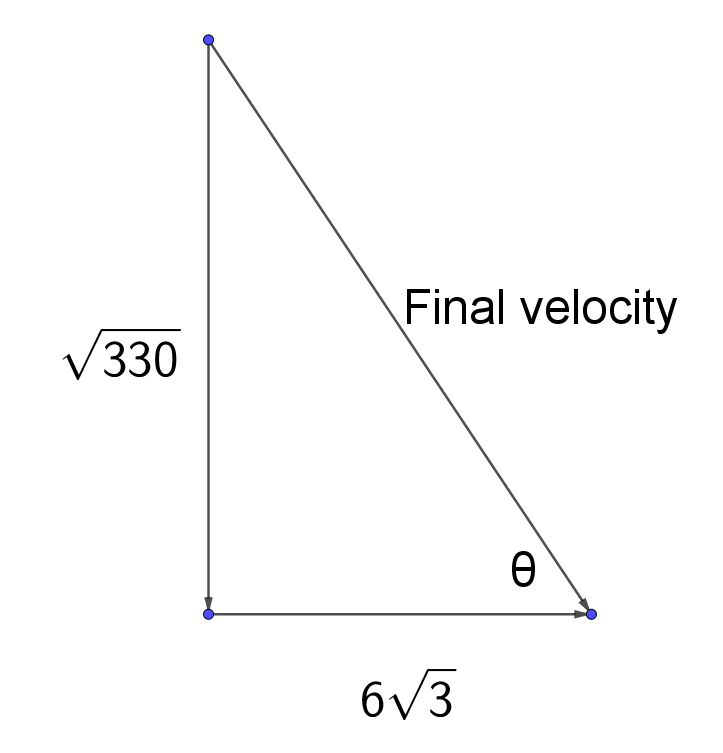
To find the final velocity we need to obtain the x and y velocity components when   
, the time when the projectile hits the ground.

The component of the velocity is a constant: .

To find the y component of the final velocity, solve for when . or

**Note:** The negative signifies the object is travelling downwards.

* 1. Final velocity



the final velocity is at an angle of to the horizontal.   
(correct to 2 decimal places)

* 1. The maximum height the projectile reaches

The maximum height of the projectile is the vertical displacement when

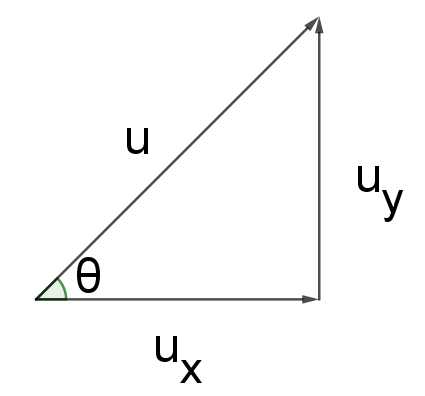
**Step 1:** Solve for when .

**Step 2:** Solve for when

Derive the Cartesian equation of the trajectory ( as a function of )

Substitute into

1. A projectile is launched from the ground at an angle of to the horizontal with an initial velocity of . Assuming no air resistance, show the maximum range is achieved when  
   .



* Initial velocities

* Horizontal components

When

When

* Vertical components

When

When

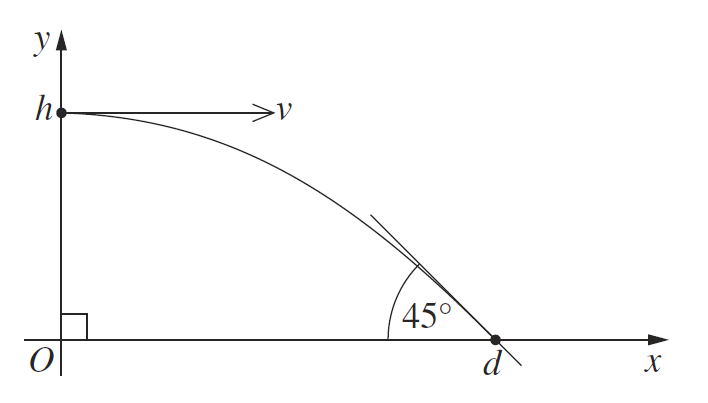
* Derive the Cartesian equation of the trajectory by substituting
* The range of the projectile is the horizontal displacement, , when the vertical displacement, , is .

Solve

* The maximum value of the functions is when 1 as

1. 2011 Mathematics Extension 1 HSC, question 6, part b

The diagram shows the trajectory of a ball thrown horizontally, at speed *v* ms–1, from the top of a tower *h* metres above ground level.



The ball strikes the ground at an angle of , *d* metres from the base of the tower, as shown in the diagram. The equations describing the trajectory of the ball are and, (Do NOT prove this.) where is the acceleration due to gravity, and is time in seconds.

* 1. Prove that the ball strikes the ground at time seconds.

The ball will strike the ground when

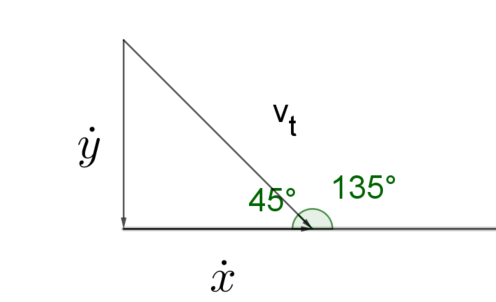
Solve for when

The ball strikes the ground at time, ( must be positive)

* 1. Hence, or otherwise, show that .

is the horizontal displacement when ,,

The ball strikes the ground at so the horizontal and vertical compoants of velocity are equal in magnitude.



Horizontal component of velocity

Vertical component of velocity

,

Substitute into

Substitute to find the horizontal displacement when the ball strikes the ground, .

,