 Year 12 Mathematics Extension 1

| ME-P1 Proof by mathematical induction | Unit duration |
| --- | --- |
| The topic Proof involves the communication and justification of an argument for a mathematical statement in a clear, concise and precise manner.A knowledge of proof enables a level of reasoning, justification and communication that is accurate, concise and precise.The study of proof is important in developing students’ ability to reason, justify, communicate and critique mathematical arguments and statements necessary for problem-solving and generalising patterns. | 6-8 lessons |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to explore and to develop the use of the technique of proof by mathematical induction to prove results. Students are introduced to mathematical induction for a limited range of applications so that they have time to develop confidence in its use.Students develop the use of formal mathematical language and argument to prove the validity of given situations using inductive reasoning. The logical sequence of steps in the proof technique needs to be understood and carefully justified, thus encouraging clear and concise communication which is useful both in further study of mathematics and in life. | A student:* applies techniques involving proof or calculus to model and solve problems ME12-1
* chooses and uses appropriate technology to solve problems in a range of contexts ME12- 6
* evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7
 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students would benefit by studying M1.2 Arithmetic sequences and series from MA-M1 Modelling financial situations, prior to this unit, however, students are not required to apply the formula for the sum of a series, as the formula are given in the initial statements. | Students could be issued a question bank of past HSC questions and examination-style questions for proof by mathematical induction and 10 questions could be given for a topic test. **Assignment:** how-does-mathematical-induction-help-us.DOCX |

All outcomes referred to in this unit come from the [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) syllabus
© NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017

Glossary of terms

| Term | Description |
| --- | --- |
| divisibility | Divisibility is a property when a number can be divided into a given number of groups without any remainders. |
| inductive step | An inductive step proves that, if the property holds for one natural number k, then it holds for the next natural number k + 1 |
| initial statement | An initial statement is a statement that holds true for the smallest integer value in a given range. |
| proof | Proof is a series of reasons given to prove a statement or a mathematical property. |
| mathematical induction ⚐ | Mathematical induction is a method of mathematical proof used to prove statements involving the natural numbers.Also known as proof by induction or inductive proof.The principle of induction is an axiom and so cannot itself be proven. |

| **Lesson sequence** | **Content** | **Suggested teaching strategies and resources**  | **Date and initial** | **Comments, feedback, additional resources used** |
| --- | --- | --- | --- | --- |
| Introduction to mathematical induction(1 or 2 lessons) | * Understand the nature of inductive proof, including the ‘initial statement’ and the inductive step (ACMSM064)
 | **Introduction to mathematical induction*** Introduce students to the meaning of induction by showing a video on Rube Goldberg machine, domino effect or paddle pop stick cobra explosion.

**Resources:** * + [Rube Goldberg machine](https://www.youtube.com/watch?v=beECeB0JVII) (duration 1:24)
	+ [domino effect](https://www.youtube.com/watch?v=4e2e9d6xDVQ) (duration 4:05)
	+ [paddle pop stick cobra bomb](https://www.youtube.com/watch?annotation_id=annotation_4013654337&feature=iv&src_vid=e7DEJ4V4hLc&v=GtnZc1dujgg) (duration 2:14)
* Brainstorm the idea that to make sure that all dominos fall, there are only two conditions that need to be met i.e. To prove anything to be true, if we can prove that being true in one instance indicates being true for the following instance, the remaining step is to simply prove one specific moment of it holding true.
* Students are shown a basic proof and record the example and their notes using the scaffold. **Resources:** proof-by-induction-scaffold.DOCX, prompt-for-proof-by-induction.DOCX
* Students model the domino effect using dominoes
* Students use paddle pop sticks to make a cobra stick bomb. They identify the cause as one stick shifting and the ‘effect’ being the next stick shifting. These two together are identified as the ‘inductive step’. **Resource**: [How to build a paddle pop stick cobra bomb](https://www.youtube.com/watch?v=T5vYrxC5kmg) (duration 4:07)
* Students annotate a proof by induction with the labels ‘initial statement’, ‘first case’, ‘assumption’ and ‘inductive step’ and complete the proof with guidance from teacher. **Resource:** identifying-steps-of-induction.DOCX
 |  |  |
| Proving results for sums(1 or 2 lessons) | * prove results using mathematical induction Critical and creative thinking icon
	+ prove results for sums, for example

$1+4+9+…+n^{2}=\frac{n(n+1)(2n+1)}{6}$ for any positive integer $n$ (ACMSM065) | **Proving results for sums*** To first develop the skills of effectively communicating the steps of mathematical induction, teachers could use scaffolded proofs for students to complete.
* Introduce problems including tower of Hanoi, diagonals in polygons, number of handshakes and intersecting lines, and guide students to the relationship between the series and the given or developed formula.

**Resources:** [Tower of Hanoi video](https://www.youtube.com/watch?v=5_6nsViVM00) (duration 7:18) tower-of-hanoi.DOCX, number-of-handshakes.DOCX* Demonstrate the use of Geogebra to construct intersecting lines and diagonals in polygons to investigate these problems.

**Resources**: diagonals-of-shapes-worksheet.DOCX, intersecting-lines-worksheet.DOCX * Demonstrate the use of playing cards to investigate the tower of Hanoi problem. Students investigate problems including tower of Hanoi, diagonals in polygons, number of handshakes and intersecting lines. They develop both an understanding of the series representation of these problems and the usefulness of the given or derived formula. Students use mathematical induction to prove these results.

**Resource:** question-bank-for-proof-by-induction.DOCX |  |  |
| Prove divisibility(1 lesson) | * prove results using mathematical induction Critical and creative thinking icon
	+ prove divisibility results, for example $3^{2n}-1$ is divisible by 8 for any positive integer $n$ (ACMSM066)
 | **Prove divisibility*** Introduce the triomino division problem and guide students to the relationship between the physical investigation and the proof of the division problem

**Resource**: triomino-division.DOCX, [triomino division problem](https://undergroundmathematics.org/divisibility-and-induction/triominoes)**Syllabus example**Prove by mathematical induction that $3^{2n}-1$ is divisible by 8 for any positive integer $n$**Solution*** 1. Prove true for $n=1$

$3^{2(1)}-1=8$, which is divisible by 8* 1. Assume true for $n=k$

$3^{2k}-1=8M$, where M is an integer* 1. Prove true for $n=k+1$

$3^{2(k+1)}-1=8P$, where P is an integer$$LHS=3^{2k+2}-1$$$$LHS=3^{2k}×3^{2}-1$$Now, $3^{2k}=8M+1$, from part b$$LHS=(8M+1)3^{2}-1$$$$LHS=72M+9-1$$$$LHS=72M+8$$$LHS=8\left(9M+1\right)$, which is divisible by 8If true for $n=k$, proven true for $n=k+1$Since true for $n=1$, true for $n=1+1=2$, $n=2+1=3$, … Therefore, true for any positive integer $n$.**Resource:** question-bank-for-proof-by-induction.DOCX |  |  |
| Identifying errors(1 lesson) | * identify errors in false ‘proofs by induction’, such as cases where only one of the required two steps of a proof by induction is true, and understand that this means that the statement has not been proved
 | **Identifying errors*** Introduce the proposed formula for finding prime numbers and lead students through the false proof of its validity. Students review proofs with mistakes and communicate what the error is and what is required to fix the proof.

**Resource**: finding-the-mistake.DOCX* Students review the proof of a formula for prime numbers on the worksheet and explain the errors in this proof, using spreadsheets to investigate counterexamples.

**Resource**: a-formula-for-prime-numbers.DOCX, prime-number-formula.XLSX* Students engage in peer critique of one another’s proofs, annotating the work of their peers with one piece of constructive feedback for improvement. The rules around this task are important. Examples include: each student may make exactly one positive comment on another student’s work; students use prepared notes that begin with the words, “In my proofs I…” for students to suggest their own methods.
 |  |  |
| Identifying inappropriate use of induction(1 lesson) | * recognise situations where proof by mathematical induction is not appropriate Critical and creative thinking icon
 | **Identifying inappropriate use of induction*** Lead students through the proof that all rabbits are the same colour and facilitate a discussion around the errors in this claim and its proof.

**Resource**: the-colour-of-all-rabbits.DOCX |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.