 Year 11 mathematics extension 1

| ME-F2 Polynomials | Unit duration |
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| The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities. This topic provides the means to more fully understand the behaviour of functions, extending to include inequalities, absolute values and inverse functions. A knowledge of functions enables students to discover connections between algebraic and graphical representations, to determine solutions of equations and to model theoretical or real-life situations involving algebra. The study of functions is important in developing students’ ability to find, recognise and use connections, to communicate concisely and precisely, to use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes in areas such as finance, economics and weather. | 1.5 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to explore the behaviour of polynomials algebraically, including the remainder and factor theorems, and sums and products of roots. Students develop knowledge, skills and understanding to manipulate, analyse and solve polynomial equations. Polynomials are of fundamental importance in algebra and have many applications in higher mathematics. They are also significant in many other fields of study, including the sciences, engineering, finance and economics. | A student:* uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
* manipulates algebraic expressions and graphical functions to solve problems ME11-2
* uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
* communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7
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| Prerequisite knowledge | Assessment strategies |
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| Students should have studied the concepts explored in MA-F1 and ME-F1.1-1.3. | * Students could work in pairs or small groups determine a variety of parametric pairs for a series of linear and quadratic Cartesian equations.
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All outcomes referred to in this unit come from [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| polynomial | A polynomial is defined by or . It is a type of function in the form . A polynomial is a series of powers of .The degree of a polynomial, , is defined by the highest power of , ie) in the definition above.Generally, polynomials are written in descending powers of . The first term written in this form is called the leading term and it is the term with the highest power of .The leading coefficient is the coefficient of the leading term, ie) in the definition above. |
| Multiplicity of a root **** | Given a polynomial , if and is a positive integer, then the root has multiplicity . |

Lesson sequence

| Lesson sequence | ContentStudents learn to: | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Introduction to polynomials (1 lesson) | * define a general polynomial in one variable, , of degree with real coefficients to be the expression: , where
	+ understand and use terminology relating to polynomials including degree, leading term, leading coefficient and constant term
 | **Introduction to polynomials*** A polynomial is a function defined for all real involving positive powers of in the form

where is a positive integer or zero* has degree where – degree means the highest power
* are the coefficients
* is known as the leading term, is the leading coefficient and is the constant term
* if then is called a monic polynomial
* if the coefficients all equal to 0. For example

 then is the zero polynomial |       |       |
| Division of polynomials (1 lesson) | * use division of polynomials to express in the form  where and is a linear or quadratic divisor, the quotient and the remainder
	+ review the process of division with remainders for integers
	+ describe the process of division using the terms: dividend, divisor, quotient, remainder
 | **Division of polynomials*** Review the process of long division and terms such as dividend, divisor, quotient and remainder
* A polynomial can be written in the form where is the dividend, is the divisor, is the quotient and is the remainder
* The degree of the remainder is always less than the degree of the divisor . If the degree of the remainder is higher than the degree of the divisor, then can still be divided by .
* The Khan Academy clip [Dividing Polynomials 1](https://www.khanacademy.org/math/algebra2/arithmetic-with-polynomials/long-division-of-polynomials/v/dividing-polynomials-1) explains the process.
* This [video](https://study.com/academy/lesson/how-to-divide-polynomials-with-long-division.html) explains the terms dividend, divisor, quotient and remainder while explaining the long division process.
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| Factor and remainder theorems(1 lesson) | * prove and apply the factor theorem and the remainder theorem for polynomials and hence solve simple polynomial equations (ACMSM089, ACMSM091)
 | **Remainder theorem*** If a polynomial is divided by then the remainder is . This can be demonstrated using the long division method as well as the functions method of substituting into the polynomial

ExampleDivide by and show that is equal to the remainder.**Factor theorem*** Given a polynomial , if then is a factor of the polynomial. The converse is also true: for a polynomial , if is a factor of the polynomial then
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| Roots and coefficients – quadratics (1 lesson) | * solve problems using the relationships between the roots and coefficients of quadratic, cubic and quartic equations **AAM**
	+ consider quadratic, cubic and quartic equations, and derive formulae as appropriate for the sums and products of roots in terms of the coefficients
 | **Roots and coefficients*** General quadratic equations can be written in the form If the roots are and then the quadratic equation can be written as and therefore the relationships between the coefficients and roots are:
* Students to solve problems using the relationships between the roots and coefficients.
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| Roots and coefficients – cubics and quartics (1 lesson) | * solve problems using the relationships between the roots and coefficients of quadratic, cubic and quartic equations **AAM**
	+ consider quadratic, cubic and quartic equations, and derive formulae as appropriate for the sums and products of roots in terms of the coefficients
 | * General cubic equations can be written in the form . If the roots are and then the cubic equation can be written as  and therefore the relationships between the coefficients and roots are:
* General quartic equations can be written in the form . If the roots are and then the quartic equation can be written as and therefore the relationships between the coefficients and roots are:
* Students to solve problems using the relationships between the roots and coefficients.
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| Determining multiplicity(1 lesson) | * determine the multiplicity of a root of a polynomial equation Critical and creative thinking icon
	+ prove that if a polynomial equation of the form has a root of multiplicity , then has a root of multiplicity
 | **Assumed knowledge*** Students should understand how to differentiate functions and therefore understand that is the derivative of the function/polynomial
* Students can use the factor theorem to determine factors of a polynomial expression

**Multiplicity*** Understand the definition of multiplicity of a root as the number of times a given polynomial equation has a root at a given point.
* Use the factor theorem and derivative of the polynomial to determine the multiplicity of an equation

**Example**Show that has a multiple zero. Find this zero and determine its multiplicity**Solution**The root at has multiplicity 2. |       |       |
| Graphing polynomials (1 lesson) | * graph a variety of polynomials and investigate the link between the root of a polynomial equation and the zero on the graph of the related polynomial function.
	+ examine the sign change of the function and shape of the graph either side of roots of varying multiplicity
 | **Investigating the shape of polynomials** * Teacher could lead an investigation using the desmos template [Investigating Polynomials](https://www.desmos.com/calculator/is3vfhfwmc) (or similar) to examine the sign change of the function and shape of the graph either side of roots of varying multiplicity.
* Show students that the curve of a polynomial of degree will fit points perfectly, or alternatively any polynomial of degree can be defined by any points that lie on the polynomial, **i.e.** a linear graph can be defined by any two points, a quadratic graph can be defined by any three points, etc.
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.