 Year 11 mathematics extension 1

| ME-F1.3 Inverse functions | Unit duration |
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| The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities. This topic provides the means to more fully understand the behaviour of functions, extending to include inequalities, absolute values and inverse functions. A knowledge of functions enables students to discover connections between algebraic and graphical representations, to determine solutions of equations and to model theoretical or real-life situations involving algebra. The study of functions is important in developing students’ ability to find, recognise and use connections, to communicate concisely and precisely, to use algebraic techniques and manipulations to describe and solve problems, and to predict future outcomes in areas such as finance, economics and weather. | 1.5 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to further explore functions in a variety of contexts including: reciprocal and inverse functions, manipulating graphs of functions, and parametric representation of functions. The study of inequalities is an application of functions and enables students to express domains and ranges as inequalities. Students develop proficiency in methods to identify solutions to equations both algebraically and graphically. The study of inverse functions is important in higher Mathematics and the calculus of these is studied later in the course. The study of parameters sets foundations for later work on projectiles. | A student:* uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses ME11-1
* manipulates algebraic expressions and graphical functions to solve problems ME11-2
* uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts ME11-6
* communicates making comprehensive use of mathematical language, notation, diagrams and graphs ME11-7
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| Prerequisite knowledge | Assessment strategies |
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| Students should have studied the concepts explored in MA-F1. | * Students could work in pairs to investigate a variety of functions or relations, restrict their domains to make them one-to-one, find the inverse functions and plot the two functions on the same set of axes to illustrate and explain the reflection property, with and without graphing software.
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All outcomes referred to in this unit come from [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| domain | The domain of a function is the set of 𝑥 values of for which the function is defined. Also known as the ‘input’ of a function.  |
| function | A function 𝑓 is a rule that associates each element 𝑥 in a set 𝑆 with a unique element (𝑥) from a set 𝑇. The set 𝑆 is called the domain of 𝑓 and the set 𝑇 is called the co-domain of 𝑓. The subset of 𝑇 consisting of those elements of 𝑇 which occur as values of the function is called the range of 𝑓. The functions most commonly encountered in elementary mathematics are real functions of a real variable, for which both the domain and co-domain are subsets of the real numbers. If we write 𝑦=(𝑥), then we say that 𝑥 is the independent variable and 𝑦 is the dependent variable.  |
| range (of function) **** | The range of a function is the set of values of the dependent variable for which the function is defined. |
| relation | A relation is a rule that associates elements in a set to elements in a set , where some element of maps to more than one element in .Informally, relations do not conform to the vertical line test for functions. |

Lesson sequence

| Lesson sequence | ContentStudents learn to: | Suggested teaching strategies and resources  | Date and Initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Introduction to inverse functions and relations(2 lessons) | * define the inverse relation of a function to be the relation obtained by reversing all the ordered pairs of the function
* examine and use the reflection property of the graph of a function and the graph of its inverse (ACMSM096)
	+ understand why the graph of the inverse relation is obtained by reflecting the graph of the function in the line
	+ using the fact that this reflection exchanges horizontal and vertical lines, recognise that the horizontal line test can be used to determine whether the inverse relation of a function is again a function
* write the rule or rules for the inverse relation by exchanging and in the function rules, including any restrictions, and solve for , if possible
 | **Introduction to inverse functions and relations*** Define the inverse relation of a function to be the relation obtained by reversing all the ordered pairs of the function
* Introduce the idea that the inverse of a function/relation can be found by interchanging the x and y coordinates.
* Students to work in pairs to plot a set of ordered pairs in one colour and then interchange the x and y coordinates and plot those ordered pairs in a different colour. Students should record their observations and compare their sets of pairs and observations with two other groups. Discuss the reflection property observed through these sets of points through . Note that this property will apply when plotting functions and relations from equations too and students should be asked to verify this as they progress through the topic.
* Reiterate the difference between a function and a relation. Students need to understand that it will only be an inverse function if the original function is one-to-one, which means that the original function has no ordered pairs with the same y coordinate and it therefore passes the horizontal line test. If it is not a one-to-one function, its inverse will be a relation. For example, a parabola can only a one-to-one function if its domain is restricted.
* Introduce that the notation used for the inverse of a function is represented by .

**Finding inverse functions and relations algebraically*** Note that students will need a solid understanding of algebraic techniques in order to rearrange equations.
* Demonstrate algebraically that if, in a function, the x and y values are switched, and then rearranging the equation to make y the subject, you have formed the equation of the inverse relation.

**Example*** Students should determine the rules for the inverse function or relation of several examples.
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| Domain and range of inverse functions(1 lesson) | * when the inverse relation is a function, use the notation and identify the relationships between the domains and ranges of and
* when the inverse relation is not a function, restrict the domain to obtain new functions that are one-to-one, and compare the effectiveness of different restrictions
 | **Domain and range*** Recall and revise the interval notation that was introduced in MA-F1.2 as it will still be new to students.
* Consider the domain and range of a function and make predictions about what the domain and range of its inverse will be.
* Establish that
	+ Domain of will be the range of
	+ Range of will be the domain of
* It has already been stated that if a function does not pass the horizontal line test, it will be an inverse relation not a function. The domain of the original function must be restricted to make it a one-to-one function, in order for its inverse to be a function.

**Example*** + is not a one-to-one function
	+ Restrict the domain to (-∞, 0] or [0, ∞), noting that either of these would restrict the domain to be a one-to-one function.
	+ Since it is now a one-to-one function, the inverse function can be found.
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| Inverse functions problems(1 lesson) | * solve problems based on the relationship between a function and its inverse function using algebraic or graphical techniques **AAM**
 | **Solving problems involving inverse functions*** Students should investigate a variety of functions or relations, restrict their domains to make them one-to-one, find the inverse functions and plot the two functions on the same set of axes to illustrate and explain the reflection property, with and without graphing software.
* Students should also explore further properties of inverse graphs, including that the composite function of the original function and its inverse will give you :
* Demonstrate the restriction of some values in graphs such as parabolas, hyperbolas and semi-circles in order to obtain a function that is one-to-one.
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in comments, feedback, additional resources used sections.