 Year 12 Mathematics Extension 1

| ME-C3 Applications of calculus | Unit duration |
| --- | --- |
| The topic Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It involves the development of analytic and numeric integration techniques and the use of these techniques in solving problems.The study of calculus is important in developing students’ knowledge, understanding and capacity to operate with and model situations involving change, and to use algebraic and graphical techniques to describe and solve problems and to predict future outcomes with relevance to, for example science, engineering, finance, economics and the construction industry. | 9 lessons |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to develop an understanding of applications of calculus in a practical context, including the more accessible kinds of differential equations and volumes of solids of revolution, to solve problems.Students develop an awareness and understanding of the use of differential equations which arise when the rate of change in one quantity with respect to another can be expressed in mathematical form. The study of differential equations has important applications in science, engineering, finance, economics and broader applications in mathematics. | A student:* applies techniques involving proof or calculus to model and solve problems ME12-1
* uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution ME12-4
* chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
* evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7
 |

| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| The material in this topic builds on content from MA-C2 Differential calculus, MA-C4 Integral calculus, ME-C1 Rates of change and ME-C2 Further calculus skills. | * ‘How can mathematics help predict trends?’is an investigative assignment which challenges students to apply their understanding of differential equations to develop models to explain the trending patterns of internet sensations or phenomena. Students will use Google Trends to source information regarding the interest in the chosen topic, for example, Candy Crush, and model the number of hits over time. ‘Design an ergonomic drinking vessel’ is an investigative assignment in which students will demonstrate their skills and understanding of integration techniques to design a drinking vessel given certain restrictions regarding the volume of material and the capacity of the vessel, shown as a volume.
 |

All outcomes referred to in this unit come from [Mathematics Extension 1](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Syllabus
© NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2017

Glossary of terms

| Term | Description |
| --- | --- |
| bounded by | The phrase “bounded by” is used in conjunction with a curve or an axis to define the boundary of a region or area. |
| differential equation | A differential equation is any equation containing the derivative of an unknown function. |
| direction field | A direction field (or slope field) is a graphical representation of the tangent lines to the solutions of a first-order differential equation. |
| integrand | An integrand is a function that is to be integrated. |
| logistic equation | The logistic equation is the differential equation where and are constants. Thus: if or ,  |
| separable differential equation | A separable differential equation can be rearranged in the form where the expressions relating to *x* and *y* can be separated prior to integration. |

Lesson sequence

| Lesson sequence | ContentStudents learn to: | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Calculating areas of regions between curves(2 lessons) | **C3.1 Further area and volumes of solids of revolution*** calculate area of regions between curves determined by functions (ACMSM124)
 | **Note:** For solutions to the exemplar questions from the NESA topic guidance, see me-c3-nesa-exemplar-question-solutions.DOCX. This contains questions and solutions from throughout the topic.**Key ideas for area of regions between curves*** The definite integral, with respect to x, for a given function is represented geometrically as the area bounded by the curve and the x-axis between the limits. Reference should be made to changing the integral with respect to y and its geometric representation as the area bounded by the curve and the y-axis.
* The area of the region between two curves is difference between the definite integrals defined by the two functions, and , when

 for * The intersection points of curves can be identified by solving the Cartesian equations for each curve simultaneously.
* The points of intersection define how the region should be partitioned into regions where and regions where
* Calculate the area for each region using

**Calculating areas of regions between curves*** Initiate this topic by establishing the key concept of the area under a curve as the integral of that function between a lower and upper limit.
* Discuss how definite integrals for regions below the -axis are negative.
* Introduce the idea of regions being created by two curves. **Resource**: identifying-the-regions-between-curves.DOCX
* Concentrate on a simple region where , for example and for , and lead students towards the concept of the area of the region being the difference between the definite integrals (think composite shapes). Which integral is subtracted from which? What happens if this is the other way around? Consider what happens when one of the functions is below the x-axis, for example

 and for Now consider cases where is not always greater than or equal to . Use the example above, and , but extend the domain to . * Introduce examples, which include quadratic and linear functions, where the points of intersection need to be found. For example, the area of the curve bounded by the functions , and the lines and
* Teacher resource detailing the techniques needed to calculate the area between curves with examples and interactive Geogebra applets. **Resource**: area-between-curves.DOCX
* Student activity for applying the area of a region between curves to estimate the area of an irregular block of land. **Resource**: estimating-the-area-of-a-section-of-land.DOCX
 |  |  |
| Calculating volumes of solids of revolution(2 lessons) | * sketch, with and without the use of technology, the graph of a solid of revolution whose boundary is formed by rotating an arc of a function about the -axis or -axis **AAM** Critical and creative thinking icon  Information and communication technology capability icon
* calculate the volume of a solid of revolution formed by rotating a region in the plane about the -axis or -axis, with and without the use of technology (ACMSM125) **AAM** Critical and creative thinking icon  Information and communication technology capability icon
* determine the volumes of solids of revolution that are formed by rotating the region between two curves about either the -axis or -axis in both real-life and abstract contexts **AAM** Critical and creative thinking icon
 | **Calculating volume of solids of revolution*** The cheese cutter idea: Build on the idea of an integral being a sum of infinitely thin strips of width and height . For solids of revolution consider the sum of infinitely thin strips being rotated by 360º about the-axis, much like a cheese cutter wire.
* Consider the infinitely thin disc (cylinder) cut by the infinitely thin strip and use with and
* Form the integral or , as it may be shown in some texts, where and are the lower and upper limits.
* Discuss the consistency of integrals, i.e. if an integral is defined with respect to , i.e. , then the integral expression and the limits must reference or values for for it to be consistent. An integral must be consistent to be correctly integrated. An inconsistent integral contains references to and and cannot be integrated correctly.
* Teacher to lead the development of the equivalent integral when a function is rotated about the -axis between a and b.

 or * Teacher resource detailing the techniques needed to calculate the volume of solids of revolution with examples and interactive Geogebra applets.

**Resource**: solids-of-revolution.DOCX |  |  |
| Slope fields showing informal solutions to differential equations(2 lessons) | **C3.2: Differential equations*** recognise that an equation involving a derivative is called a differential equation
* recognise that solutions to differential equations are functions and that these solutions may not be unique
* sketch the graph of a particular solution given a direction field and initial conditions
* form a direction field (slope field) from simple first-order differential equations
* recognise the shape of a direction field from several alternatives given the form of a differential equation, and vice versa
* sketch several possible solution curves on a given direction field
 | **Introducing differential equations*** A differential equation is an equation that contains a derivative.
* Differential equations can contain first, second, third etc. derivatives or a mixture of each.
* A first-order differential equation only contains a first derivative.

There are three types of first-order differential equations to consider: , and * Discuss the issue of constants being eliminated by differentiation, and multiple functions forming the same differential equation through differentiation.

**Creating a slope field*** Use square dot paper to produce a Cartesian number plane. Generate a direction or slope field by investigating the gradients for points on the Cartesian number plane. For example, for the differential equation , calculate the gradients for all integer points, i.e. At , . Represent this gradient as a short downwards sloping (steep) arrow at the point . Repeat for all integer points. The resulting slope field produces a map showing a family of functions which all satisfy the differential equation given.
* [Slope fields](https://www.geogebra.org/m/YyNAdjN4#material/jk9FucDB): This Geogebra app creates a slope field for the differential equation supplied. Move the red point on the curve to investigate possible curves of functions.
* An alternate [slope field](https://www.desmos.com/calculator/p7vd3cdmei) resource in desmos.
 |  |  |
| Solve first order differential equations(3 lessons) | * solve simple first-order differential equations (ACMSM130)
* solve differential equations of the form d y over d x equals f of x
* solve differential equations of the form d y over d x equals g of y
* solve differential equations of the form d y over d x equals f of x g of y using separation of variables
* recognise the features of a first-order linear differential equation and that exponential growth and decay models are first-order linear differential equations, with known solutions
* model and solve differential equations including the logistic equation that will arise in situations where rates are involved, for example in chemistry, biology and economics (ACMSM132) AAM Critical and creative thinking icon
 | **Solving first order differential equations*** The basic idea for solving a differential equation is that it will require integrating and substituting in known values (mostly initial conditions), using indefinite or definite integral techniques, to generate a solution which is an equation linking and .
* [Solving ordinary differential equations](https://mathinsight.org/ordinary_differential_equation_introduction_examples), this website contains explanations and examples for solving ordinary or simple differential equations through indefinite integral methods with initial conditions.
* [Solving separable differential equations](https://math.libretexts.org/Bookshelves/Calculus/Book%3A_Active_Calculus_%28Boelkins_et_al.%29/7%3A_Differential_Equations/7.4%3A_Separable_Differential_Equations), this website provides an explanation and examples for solving separable differential equations
* [Modelling with differential equations](https://nrich.maths.org/11052), this website provides some examples how differential equations can represent real life scenarios. There are examples for exponential growth and the logistic equation
* The following are teacher support resources outlining the technique of solving a first order differential equation in the forms ,

and with annotated examples* **Resources**: case-1-solving-simple-differential-equations.DOCX, case-2-solving-simple-differential-equations.DOCX, case-3-solving-simple-differential-equations.DOCX, solving-simple-differential-equations.DOCX
* Student modelling activity which applies exponential and logistical models to the life cycle of the compact disc (CD) using sales history from the USA.

**Resource:** the-life-and-times-of-the-humble-cd.DOCX and sales-history-for-cds.XLSX* Student activity that generates logistic models for internet trends like ITunes

**Resource:** modelling-music-trends.DOCX* HSC Mathematics Extension 2 2008 Question 5a:

A model for the population, , of elephants in Serengeti National Park iswhere is the time in years from today.1. Show that satisfies the differential equation
2. What is the population today?
3. What does the model predict that the eventual population will be?
4. What is the annual percentage rate of growth?
* Solutions:
1. As , ,
2. At *, ,*
 |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.