Mathematics Extension 1

# ME-V1 Introduction to vectors

## Table of contents

[Mathematics Extension 1 1](#_Toc47078905)

[ME-V1 Introduction to vectors 1](#_Toc47078906)

[Table of contents 2](#_Toc47078907)

[Outcomes 4](#_Toc47078908)

[Content 4](#_Toc47078909)

[Supplementary resources 7](#_Toc47078910)

[Department of Education resources 7](#_Toc47078911)

[NESA resources 7](#_Toc47078912)

[Jonathan Kim Sing videos 7](#_Toc47078913)

[Examination-style questions 8](#_Toc47078914)

[Sample question 1 8](#_Toc47078915)

[Sample question 2 8](#_Toc47078916)

[Sample question 3 9](#_Toc47078917)

[Sample question 4 9](#_Toc47078918)

[Sample question 5 10](#_Toc47078919)

[Sample question 6 10](#_Toc47078920)

[Sample question 7 11](#_Toc47078921)

[Sample question 8 12](#_Toc47078922)

[Sample question 9 13](#_Toc47078923)

[Sample question 10 13](#_Toc47078924)

[Sample question 11 14](#_Toc47078925)

[Sample question 12 15](#_Toc47078926)

[Sample question 13 16](#_Toc47078927)

[Sample question 14 17](#_Toc47078928)

[Sample question 15 18](#_Toc47078929)

[Sample question 16 19](#_Toc47078930)

[Sample question 17 20](#_Toc47078931)

[Solutions 21](#_Toc47078932)

[Sample question 1 21](#_Toc47078933)

[Sample question 2 21](#_Toc47078934)

[Sample question 3 22](#_Toc47078935)

[Sample question 4 22](#_Toc47078936)

[Sample question 5 23](#_Toc47078937)

[Sample question 6 23](#_Toc47078938)

[Sample question 7 24](#_Toc47078939)

[Sample question 8 24](#_Toc47078940)

[Sample question 9 24](#_Toc47078941)

[Sample question 10 25](#_Toc47078942)

[Sample question 11 25](#_Toc47078943)

[Sample question 12 26](#_Toc47078944)

[Sample question 13 27](#_Toc47078945)

[Sample question 14 28](#_Toc47078946)

[Sample question 15 29](#_Toc47078947)

[Sample question 16 30](#_Toc47078948)

[Sample question 17 30](#_Toc47078949)

**Disclaimer**

This document is to be used to supplement the support teachers are offering students undertaking HSC Mathematics courses. Questions can be printed off for students individually, with or without solutions, or as an entire booklet. Questions have been sourced from various states across Australia and the source of each question has been referenced. Permission to use these resources was provided in June 2020. Solutions for each of the questions can be found at the end of the document.

**Outcomes**

All outcomes referred to in this booklet are from [Mathematics Extension 1 Syllabus](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) © 2017 NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales.

## Outcomes

**A student:**

* applies concepts and techniques involving vectors and projectiles to solve problems ME12-2
* chooses and uses appropriate technology to solve problems in a range of contexts ME12-6
* evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms ME12-7

## Content

**V1.1: Introduction to vectors**

Students:

* define a vector as a quantity having both magnitude and direction, and examine examples of vectors, including displacement and velocity (ACMSM010)
	+ explain the distinction between a position vector and a displacement (relative) vector
* define and use a variety of notations and representations for vectors in two dimensions (ACMSM014)
	+ use standard notations for vectors, for example: , and
	+ represent vectors graphically in two dimensions as directed line segments
	+ define unit vectors as vectors of magnitude 1, and the standard two-dimensional perpendicular unit vectors  and 
	+ express and use vectors in two dimensions in a variety of forms, including component form, ordered pairs and column vector notation
* perform addition and subtraction of vectors and multiplication of a vector by a scalar algebraically and geometrically, and interpret these operations in geometric terms **AAM**
	+ graphically represent a scalar multiple of a vector (ACMSM012)
	+ use the triangle law and the parallelogram law to find the sum and difference of two vectors
	+ define and use addition and subtraction of vectors in component form (ACMSM017)
	+ define and use multiplication by a scalar of a vector in component form (ACMSM018)

**V1.2: Further operations with vectors**

Students:

* define, calculate and use the magnitude of a vector in two dimensions and use the notation  for the magnitude of a vector 
	+ prove that the magnitude of a vector, , can be found using: 
	+ identify the magnitude of a displacement vector as being the distance between the points and
	+ convert a non-zero vector  into a unit vector  by dividing by its length: 
* define and use the direction of a vector in two dimensions
* define, calculate and use the scalar (dot) product of two vectors  and  **AAM**
	+ apply the scalar product, , to vectors expressed in component form, where
	+ use the expression for the scalar (dot) product,  where is the angle between vectors  and  to solve problems
	+ demonstrate the equivalence,  and use this relationship to solve problems
	+ establish and use the formula 
	+ calculate the angle between two vectors using the scalar (dot) product of two vectors in two dimensions
* examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)
* define and use the projection of one vector onto another (ACMSM022)
* solve problems involving displacement, force and velocity involving vector concepts in two dimensions (ACMSM023) **AAM**
* prove geometric results and construct proofs involving vectors in two dimensions including to proving that: **AAM**
	+ the diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)
	+ the midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)
	+ the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides (ACMSM041)

## Supplementary resources

### Department of Education resources

#### Units of work

* [ME-V1 Introduction to vectors](https://education.nsw.gov.au/teaching-and-learning/curriculum/key-learning-areas/mathematics/stage-6/mathematics-extension-1#Year8)
* [ME V1 vectors example solutions](https://education.nsw.gov.au/teaching-and-learning/curriculum/key-learning-areas/mathematics/stage-6/mathematics-extension-1#Year10)

#### HSC Hub videos

* [Determining the projection of one vector on to another Q12b NESA sample examination](https://hschub.nsw.edu.au/mathematics-items/introduction-to-vectors-question-12b)
* [Finding the angle between vectors Q1 NESA sample examination](https://hschub.nsw.edu.au/mathematics-items/mathematics-extension-1-introduction-to-vectors)
* [Using vector arithmetic Q2 NESA sample examination](https://hschub.nsw.edu.au/mathematics-items/introduction-to-vectors-question-2)

### NESA resources

* [Mathematics Extension 1 – Sample examination materials (2020)](https://educationstandards.nsw.edu.au/wps/wcm/connect/0ca974bc-60ce-4be4-9410-6e2351ca8049/mathematics-ext-1-sample-examination-materials-2020.pdf?MOD=AJPERES&CVID=)

### Jonathan Kim Sing videos

* [Vectors (Y12 Extension 1)](https://www.youtube.com/playlist?list=PLnPz4TGRkQDuLsxrQuR5e45Tk2jnjEW9K)

## Examination-style questions

### Sample question 1

Consider the vectors given by and , where .

If the acute angle between is 30°, then equals

Source: **Question 11** [©VCAA 2018 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 2

A body has displacement of metres at a particular time. The body moves with constant velocity and two seconds later its displacement is metres.

The velocity, in , of the body is

Source: **Question 15** [©VCAA 2017 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 3

A force of magnitude 4 newtons acts in the north-easterly direction and another force of 3 newtons acts in the easterly direction.

The magnitude, in newtons, of the resultant of these two forces is

Source: **Question 15** [©VCAA 2009 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 4

Consider the three forces

, and

The magnitude of the sum of these three forces is equal to

1. the magnitude of
2. the magnitude of
3. the magnitude of
4. the magnitude of
5. the magnitude of

Source: **Question 11** [©VCAA 2011 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 5

A particle is acted on by two forces, one of 6 newtons acting due south, the other of 4 newtons acting in the direction .

The magnitude of the resultant force, in newtons, acting on the particle is

Source: **Question 14** [©VCAA 2012 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 6

Forces of magnitude 5 N, 7 N and Q N act on a particle that is in equilibrium, as shown in the diagram below.



The magnitude of , in newtons, can be found by evaluating

Source: **Question 16** [©VCAA 2013 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 7

A body on a horizontal smooth plane is acted upon by four forces, , , and , as shown.

The force acts in a northerly direction and the force acts in a westerly direction.



Given that , , and , the motion of the body is such that it

1. is in equilibrium.
2. moves to the west.
3. moves to the north.
4. moves in the direction 30° south of west.
5. moves to the east.

Source: **Question 18** [©VCAA 2014 VCE Specialist mathematics written examination 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx)

### Sample question 8

**Question 3** (8 marks)

Figure 2 shows the quadrilateral , where , , and .

The points , , , and are the midpoints of the sides , , , and respectively.



Figure 2

1. Find the following vectors in terms of , , and .

**(1 mark)**

**(1 mark)**

**(2 marks)**

1. Explain why EFGH is a parallelogram.

**(2 marks)**

Source: **Sample question 3** [©South Australian Certificate of Education VCE Specialist Mathematics 2019](https://www.sace.sa.edu.au/web/specialist-mathematics/stage-2/support-materials/chief-assessors-report-exams)

### Sample question 9

1. Figure 1 shows rhombus with and

****

**Figure 1**

Find in terms of and .

**(1 mark)**

Find in terms of and .

**(1 mark)**

Show that , hence prove that the diagonals of the rhombus are perpendicular, giving reasons.

**(4 marks)**

Source: **Sample question 3** [©South Australian Certificate of Education VCE Specialist Mathematics 2017](https://www.sace.sa.edu.au/web/specialist-mathematics/stage-2/support-materials/chief-assessors-report-exams)

### Sample question 10

A particle moves in the cartesian plane with position vector where and are functions of . If its velocity vector is , find the acceleration vector of the particle in terms of the position vector .

**(3 marks)**

Source: **Question 9** [©South Australian Certificate of Education VCE Specialist Mathematics 2007](https://www.sace.sa.edu.au/web/specialist-mathematics/stage-2/support-materials/chief-assessors-report-exams)

### Sample question 11

The point O is on a sloping plane that forms an angle of to the horizontal. A particle is projected from the point . The particle hits a point on the sloping plane as shown in the diagram.



The equation of the line *OA* is . The equations of motion of the particle are

where *t* is the time in seconds after projection. Do NOT prove these equations.

Find the distance between the point of projection and the point where the particle hits the sloping plane.

**(2 marks)**

What is the size of the acute angle that the path of the particle makes with the sloping plane as the particle hits the point ?

**(3 marks)**

Source: **Question 13d** [©NSW Education Standards Authority HSC Mathematics Extension 1 2019](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/resources/hsc-exam-papers/hsc-exam-paper-detail/2019/mathematics-extension-1-2019-hsc-exam-pack)

### Sample question 12

An object is projected from the origin with an initial velocity of V at an angle q to the horizontal. The equations of motion of the object are

**(Do not prove these)**

Show that when the object is projected at an angle , the horizontal range is

**(2 marks)**

Show that when the object is projected at an angle , the horizontal range is also

**(1 mark)**

The object is projected with initial velocity to reach a horizontal distance , which is less than the maximum possible horizontal range. There are two angles at which the object can be projected in order to travel that horizontal distance before landing.

Let these angles be and , where .

Let be the maximum height reached by the object when projected at the angle to the horizontal.

Let be the maximum height reached by the object when projected at the angle to the horizontal.



Show that the average of the two heights, , depends only on and .

**(3 marks)**

Source: **Question 13c** © [NSW Education Standards Authority HSC Mathematics Extension 1 2018](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/resources/hsc-exam-papers/hsc-exam-paper-detail/2018/mathematics-extension-1-2018-hsc-exam-pack)

### Sample question 13

A golfer hits a golf ball with initial speed ms-1 at an angle to the horizontal. The golf ball is hit from one side of a lake and must have a horizontal range of m or more to avoid landing in the lake.



Neglecting the effects of air resistance, the equations describing the motion of the ball are

where is the time in seconds after the ball is hit and is the acceleration due to gravity in ms−2. **Do NOT prove these equations.**

Show that the horizontal range of the golf ball is metres

**(2 marks)**

Show that if then the horizontal range of the ball is less than m.

**(1 mark)**

It is now given that and that the horizontal range of the ball is m or more.

Show that

**(2 marks)**

Find the greatest height the ball can achieve.

**(2 marks)**

Source: **Question 13c** [©NSW Education Standards Authority HSC Mathematics Extension 1 2017](https://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2017/mathematics-extension-1-2017-hsc-exam-pack)

### Sample question 14

The trajectory of a projectile fired with speed ms–1 at an angle to the horizontal is represented by the parametric equations

 and

where is the time in seconds.

Prove that the greatest height reached by the projectile is .

**(2 marks)**

A ball is thrown from a point m above the horizontal ground. It is thrown with speed ms–1 at an angle of to the horizontal. At its highest point the ball hits a wall, as shown in the diagram.



Show that the ball hits the wall at a height of m above the ground.

**(2 marks)**

The ball then rebounds horizontally from the wall with speed ms–1. You may assume that the acceleration due to gravity is ms–2.

How long does it take the ball to reach the ground after it rebounds from the wall?

**(2 marks)**

How far from the wall is the ball when it hits the ground?

**(1 mark)**

Source: **Question 13b** [©NSW Education Standards Authority HSC Mathematics Extension 1 2016](https://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2016/mathematics-extension-1-2016-hsc-exam-pack)

### Sample question 15

A projectile is fired from the origin O with initial velocity ms−1 at an angle to the horizontal. The equations of motion are given by

, . (Do NOT prove this.)



Show that the horizontal range of the projectile is

**(2 marks)**

A particular projectile is fired so that

Find the angle that this projectile makes with the horizontal when

**(2 marks)**

State whether this projectile is travelling upwards or downwards when . Justify your answer.

**(1 mark)**

Source: **Question 14a** [©NSW Education Standards Authority HSC Mathematics Extension 1 2015](https://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2015/mathematics-extension-1-2015-hsc-exam-pack)

### Sample question 16

The diagram below shows a quadrilateral . Points , , and are midpoints of the intervals , , and respectively.



Use vector methods to show .

**(1 mark)**

Hence prove that is a parallelogram.

**(2 marks)**

Source: [NSW Educations Standards Authority, Mathematics Extension 1 Stage 6 syllabus,](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) Page 51

### Sample question 17

**This question has been designed by the Mathematics Curriculum Support team. It illustrates an application of the scalar product.**

The diagram below shows a triangle , where and .



Show that vector is perpendicular to .

**(1 mark)**

Show that the scalar projection of on to is given by

**(2 marks)**

Hence find the area of the triangle in terms of , , and .

**(1 mark)**

Find the area of the triangle , where , and

**(2 marks)**

## Solutions

### Sample question 1

 and

Answer = c)

### Sample question 2

A body has displacement of metres at a particular time. The body moves with constant velocity and two seconds later its displacement is metres.

The velocity, in , of the body is

Therefore

Answer = b)

### Sample question 3



Answer = c)

### Sample question 4

Answer = e) the magnitude of

### Sample question 5



Answer = b)

### Sample question 6



Answer = a)

### Sample question 7



Resolving horizontally

Resultant N

Resolving vertically

Resultant

Answer = e) moves to the east.

### Sample question 8

1. i.
2. and , therefore

Similarly and , therefore and is a parallelogram

### Sample question 9

1. i.

Rhombus, therefore and .

 and are the diagonals of the rhombus and are perpendicular as

### Sample question 10

,

 and

Acceleration

### Sample question 11

 or

 m

1. and



,

Therefore the angle with the sloping plane is equal to

Source: **Question 13d** [©NSW Education Standards Authority HSC Mathematics Extension 1 marking guidelines 2019](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/resources/hsc-exam-papers/hsc-exam-paper-detail/2019/mathematics-extension-1-2019-hsc-exam-pack)

### Sample question 12

, as

, as

Therefore the range, at angle , is

1. Range,
2. , ,

Therefore and but

Therefore which depend on and only.

Source: **Question 13c** [©NSW Education Standards Authority HSC Mathematics Extension 1 marking guidelines 2018](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/resources/hsc-exam-papers/hsc-exam-paper-detail/2018/mathematics-extension-1-2018-hsc-exam-pack)

### Sample question 13

, as

1. If then Range

and Range as

1. If then range and where , therefore

 and

Source: **Question 13c** [©NSW Education Standards Authority HSC Mathematics Extension 1 marking guidelines 2017](https://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2017/mathematics-extension-1-2017-hsc-exam-pack)

### Sample question 14

1. , max height when

Therefore ,

Max height, , when

Therefore

, maximum height is

1. Max height

, as

, as

Ball hits the ground at

, seconds as

Therefore the ball hits the ground 2.5 seconds after hitting the wall

1. , taking left of the wall as positive.

, as

 metres.

The ball lands 25 metres from the wall.

Source: **Question 13b** [©NSW Education Standards Authority HSC Mathematics Extension 1 marking guidelines 2016](https://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2016/mathematics-extension-1-2016-hsc-exam-pack)

### Sample question 15

1. When

 as

Range, when seconds

 as

1. and

Substitute and

, ,

The projectile makes an angle or to the positive -axis.

1. The angle is obtuse, therefore it is descending.

Source: **Question 14a** [©NSW Education Standards Authority HSC Mathematics Extension 1 marking guidelines 2015](https://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2015/mathematics-extension-1-2015-hsc-exam-pack)

### Sample question 16

1. Let and

 and , therefore

1. Let and

 and , therefore and .

, therefore .

Therefore is a parallelogram

### Sample question 17

1. therefore is perpendicular to
2. Scalar projection of on to
3. Area of Triangle Scalar projection of on to
4. and

Area units2