Mathematics Extension 1

# ME-S1 The Binomial Distribution

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**Disclaimer**

This document is to be used to supplement the support teachers are offering students undertaking HSC Mathematics courses. Questions can be printed off for students individually, with or without solutions, or as an entire booklet. Questions have been sourced from various states across Australia and the source of each question has been referenced. Permission to use these resources was provided in June 2020. Solutions for each of the questions can be found at the end of the document.

**Outcomes**

All outcomes referred to in this booklet are from [Mathematics Extension 1 Syllabus](https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017) © 2017 NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales.

## Syllabus outcomes

The exam style questions presented in this document refer to the following outcomes and syllabus content.

### Outcomes

A student:

* **ME12-5** applies appropriate statistical processes to present, analyse and interpret data
* **ME12-6** chooses and uses appropriate technology to solve problems in a range of contexts
* **ME12-7** evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

### Content

**1.1: Bernoulli and binomial distributions**

Students:

* use a Bernoulli random variable as a model for two-outcome situations(ACMMM143)
* identify contexts suitable for modelling by Bernoulli random variables(ACMMM144)
* use Bernoulli random variables and their associated probabilities to solve practical problems(ACMMM146)
* understand and apply the formulae for the mean,  and variance,  of the Bernoulli distribution with parameter , and defined as the number of successes(ACMMM145)
* understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of ‘successes’ in  independent Bernoulli trials, with the same probability of success  in each trial(ACMMM147)
* calculate the expected frequencies of the various possible outcomes from a series of Bernoulli trials
* use binomial distributions and their associated probabilities to solve practical problems(ACMMM150)
* identify contexts suitable for modelling by binomial random variable(ACMMM148)
* identify the binomial parameter pp as the probability of success
* understand and use the notation  to indicate that the random variable X is distributed binomially with parameters n and p
* apply the formulae for probabilities associated with the binomial distribution with parameters  and  and understand the meaning of  as the number of ways in which an outcome with  successes can occur
* understand and apply the formulae for the mean, , and the variance, , of a binomial distribution with parameters nn and pp

**S1.2: Normal approximation for the sample proportion**

Students:

* use appropriate graphs to explore the behaviour of the sample proportion on collected or supplied data
* understand the concept of the sample proportion  as a random variable whose value varies between samples(ACMMM174)
* explore the behaviour of the sample proportion using simulated data
* examine the approximate normality of the distribution of  for large samples(ACMMM175)
* understand and use the normal approximation to the distribution of the sample proportion and its limitations

## Supplementary resources

### Department of Education resources

#### Units of work

* [ME S1 The binomial distribution](https://education.nsw.gov.au/teaching-and-learning/curriculum/key-learning-areas/mathematics/stage-6/mathematics-extension-1#Year8)

#### HSC Hub

* [Binomial distribution Q12a NESA sample examination paper](https://hschub.nsw.edu.au/mathematics-items/binomial-distribution)

### NESA resources

* [Mathematics Extension 1 – Sample examination materials (2020)](https://educationstandards.nsw.edu.au/wps/wcm/connect/0ca974bc-60ce-4be4-9410-6e2351ca8049/mathematics-ext-1-sample-examination-materials-2020.pdf?MOD=AJPERES&CVID=)

### WOOTUBE

* [The Binomial Distribution](https://www.youtube.com/playlist?list=PL5KkMZvBpo5Bcz-V51UHtlg_eBW-PtQ7_)

## Exam style questions

### Sample question 1

**Question 16**

For a random sample of five Australians, is the random variable that represents the proportion who live in a capital city.

Given that , then , correct to four decimal places, is

1. 0.0453
2. 0.3209
3. 0.4609
4. 0.5390
5. 0.7901

Source: [© VCAA 2017 Mathematical Methods exam paper 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 2

**Question 18**

Let X be a discrete random variable with binomial distribution . The mean and standard deviation of this distribution are equal.

Given that , the smallest number of trials, , such that is

1. 37
2. 49
3. 98
4. 99
5. 101

Source: [© VCAA 2017 Mathematical Methods exam paper 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 3

**Question 17**

Inside a container there are one million coloured building blocks. It is know that 20% of the blocks are red. A sample of 16 blocks is taken from the container. For samples of 16 blocks, is the random variable of the distribution of sample proportions of red blocks (Do not use a normal approximation)

 is closest to

1. 0.6482
2. 0.8593
3. 0.7543
4. 0.6542
5. 0.3211

Source: [© VCAA 2016 Mathematical Methods exam paper 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 4

**Question 10**

The binomial random variable, , has and

 is equal to

Source: [© VCAA 2015 Mathematical Methods exam paper 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 5

**Question 4 (16 marks)**

Doctors are studying the resting heart rate of adults in two neighbouring towns: Mathsland and Statsville. Resting heart rate is measured in beats per minute (bpm).

The resting heart rate of adults in Mathsland is known to be normally distributed with a mean of 68 bpm and a standard deviation of 8 bpm.

1. Find the probability that a randomly selected Mathsland adult has a resting heart rate between 60 bpm and 90 bpm. Give your answer correct to three decimal places**. (1 mark)**

The doctors consider a person to have a slow heart rate if the person’s resting heart rate is less than 60 bpm. The probability that a randomly chosen Mathsland adult has a slow heart rate is 0.1587

It is know that 29% of Mathsland adults play sport regularly.

It is also known that 9% of Mathsland adults play sport regularly and have a slow heart rate.

Let S be the event that a randomly selected Mathsland adult plays sport regularly and let H be the event that a randomly selected Mathsland adult has a slow heart rate.

1. i. Find P(H\S) correct to three decimal places **(1 mark)**

ii. Are the events H and S independent? Justify your answer. **(1 mark)**

1. i. Find the probability that a random sample of 16 Mathsland adults will contain exactly one person with a slow heart rate. Give your answer correct to three decimal places **(2 marks)**

ii. For random samples of 16 Mathsland adults, is the random variable that represents the proportion of people who have a slow heart rate.
 Find the probability that is greater than 10%, correct to three decimal places
 **(2 marks)**

iii For random samples of n Mathsland adults, is the random variable that represents the proportion of people who have a slow heart rate.

 Find the least value of n for which **(2 marks)**

[Source: © VCAA 2018 Mathematical Methods exam paper 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 6

**Question 6 (3 marks)**

Fred owns a company that produces thousands of pegs each day. He randomly selects 41 pegs that are produced on one day and finds eight faulty pegs.

1. What is the proportion of faulty pegs in this sample? **(1 mark)**

Pegs are packed each day in boxes. Each box holds 12 pegs. Let be the random variable that represents the proportion of faulty pegs in a box.

The actual proportion of faulty pegs produced by the company each day is

1. Find Express your answer in the form are positive rational numbers and n is a positive integer. **(2 marks)**

[Source: © VCAA 2019 Mathematical Methods exam paper 1](https://vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdfhttps%3A/vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 7

**Question 4 (2 marks)**

In a large population of fish, the proportion of angel fish is

Let be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population.

Find the smallest integer value of n such that the standard deviation of is less than or equal to

Source: [© VCAA 2017 Mathematical Methods exam paper 1](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 8

**Question 3 (16 marks)**

A school has a class set of 22 new laptops kept in a recharging trolley. Provided each laptop is correctly plugged into the trolley after use, its battery recharges.

On a particular day, a class of 22 students uses the laptops. All laptop batteries are fully charged at the start of the lesson. Each student uses and returns exactly one laptop. The probability that a student does **not** correctly plug their laptop into the trolley at the end of the day is 10%. The correctness of any student’s plugging-in is independent of any other student’s correctness.

1. Determine the probability that at least one of the laptops is **not** correctly plugged into the trolley at the end of the lesson. Give your answer correct to four decimal places. **(2 marks)**
2. A teacher observes that at least one of the returned laptops is not correctly plugged into the trolley.

Given this, find the probability that fewer than five laptops are **not** correctly plugged in. Give your answer correct to four decimal places. **(2 marks)**

The time for which a laptop will work without recharging (the battery life) is normally distributed, with a mean of three hours and 10 minutes and standard deviation of six minutes. Suppose that the laptops remain out of the recharging trolley for three hours.

1. For any one laptop, find the probability that it will stop working by the end of these three hours. Give your answer correct to four decimal places**.(2 marks)**

A supplier of laptops decides to take a sample of 100 new laptops from a number of different schools. For samples of size 100 from the population of laptops with a mean battery life of three hours and 10 minutes and standard deviation of six minutes, is the random variable of the distribution of sample proportions of laptops with a battery life of less than three hours.

1. Find the probability that Give your answer correct to three decimal places. Do no use a normal approximation. **(3 marks)**

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It is known that when laptops have been used regularly in a school for six months, their battery life is still normally distributed but the mean battery life drops to three hours. It is also know that only 12% of such laptops work for more than three hours and 10 minutes.

1. Find the standard deviation for the normal distribution that applies to the battery life of laptops that have been used regularly in a school for six months, correct to four decimal places. **(2 marks)**

The laptop supplier collects a sample of 100 laptops that have been used for six months from a number of different schools and tests their battery life. The laptop supplier wishes to estimate the proportion of such laptops with a battery life of less than three hours.

1. Suppose the supplier tests the battery life of the laptops one at a time.

Find the probability that the first laptop found to have a battery life of less than three hours is the third one. **(1 mark)**

Source: [© VCAA 2016 Mathematical Methods exam paper 2](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 9

**Question 3 (11 marks)**

Mani is a fruit grower. After his oranges have been picked, they are sorted by a machine, according to size. Oranges classified as **medium** are sold to fruit shops and the remainder are made into orange juice.

The distribution of the diameter, in centimetres, of medium oranges is modelled by a continuous random variable, *X*, with probability density function

1. **i.** Find the probability that a randomly selected medium orange has a diameter

greater than 7 cm. **(2 marks)**

**ii.** Mani randomly selects three medium oranges.
Find the probability that exactly one of the oranges has a diameter greater than 7 cm. Express the answer in the form , where a and b are positive integers **(2 marks)**

1. Not relevant to NSW syllabus

For oranges classified as **large**, the quantity of juice obtained from each orange is a normally distributed random variable with a mean of 74 mL and a standard deviation of 9mL.

1. What is the probability, correct to three decimal places, that a randomly selected large orange produces less than 85 mL of juice, given that it produces more than 74 mL of juice? **(2 marks)**

Mani also grows lemons, which are sold to a food factory. When a truckload of lemons arrives at the food factory, the manager randomly selects and weighs four lemons from the load. If one or more of these lemons is underweight, the load is rejected. Otherwise it is accepted.

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It is known that 3% of Mani’s lemons are underweight.

1. **i.** Find the probability that a particular load of lemons will be rejected. Express the answer correct to four decimal places. **(2 marks)**

**ii.** Suppose that instead of selecting only four lemons, *n* lemons are selected at random from a particular load.

Find the smallest integer value of *n* such that the probability of at least one lemon being underweight exceeds 0.5 **(2 marks)**

Source: [© VCAA 2015 Mathematical Methods exam paper (CAS2)](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 10

**Question 4 (14 marks)**

Patricia is a gardener and she owns a garden nursery. She grows and sells basil plants and coriander plants.

The heights, in centimetres, of the basil plants that Patricia is selling are distributed normally with a mean of 14 cm and a standard deviation of 4 cm. There are 2000 basil plants in the nursery.

1. Patricia classifies the tallest 10 per cent of her basil plants as super.
What is the minimum height of a super basil plant, correct to the nearest millimetre? **(1 mark)**

Patricia decides that some of her basil plants are not growing quickly enough, so she plans to move them to a special greenhouse. She will move the basil plants that are less than 9 cm in height.

1. How many basil plants will Patricia move to the greenhouse, correct to the nearest whole number? **(2 marks)**

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The heights of the coriander plants, *x* centimetres, follow the probability density function

1. Not relevant to NSW syllabus

Patricia thinks that the smallest 15 per cent of her coriander plants should be given a new type of plant food.

1. Find the maximum height, correct to the nearest millimetre, of a coriander plant if it is to be given the new type of plant food. **(2 marks)**

Patricia also grows and sells tomato plants that she classifies as either tall or regular. She finds that 20 per cent of her tomato plants are tall.

A customer, Jack, selects *n* tomato plants at random.

1. Let *q* be the probability that at least one of Jack’s *n* tomato plants is tall.
Find the minimum value of *n* so that *q* is greater than 0.95. **(2 marks)**

In another section of the nursery, a craftsman makes plant pots. The pots are classified as smooth or rough.

The craftsman finishes each pot before starting on the next. Over a period of time, it is found that if one plant pot is smooth, the probability that the next one is smooth is 0.7, while if one plant pot is rough, the probability that the next one is rough is *p*,
where 0 < *p* < 1. The value of *p* stays fixed for a week at a time, but can vary from week to week. The first pot made each week is always a smooth pot.

1. **i.** Find, in terms of *p*, the probability that the **third** pot made in a given week is smooth. **(2 marks)**

**ii.** In one particular week, the probability that the thirdpot made is smooth is 0.61.
Calculate the value of *p* in this week. **(2 marks)**

Source: [© VCAA 2014 Mathematical Methods exam paper (CAS2)](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 11

**Question 6**

1. A box contains 4 green marbles and 6 black marbles. A marble is randomly selected from the box; its colour noted, and then put back in the box. If X is the random variable that represents the number of green marbles selected from the box in 12 trials, calculate P (X ≤ 5).
2. A six-sided die is loaded so that the outcome of rolling an even number compared to the rolling of an odd number is in the ratio 3:2. The die is rolled 100 times.

The binomial variable X represents the number of times an even number appears on the uppermost face of the die when it is rolled. Using the normal approximation for a binomial event, determine P (55 < X ≤ 70).

Source: [© QCAA 2018 Mathematics B exam paper one](https://www.qcaa.qld.edu.au/senior/see/subject-resources/mathematical-methods)

### Sample question 12

**Question 6**

1. A box contains six blue marbles and nine red marbles. A marble is randomly drawn from the box, its colour noted and then the marble is put back in the box before another marble is drawn.

i. If two marbles are drawn from the box as described, what is the probability of obtaining two blue marbles?

ii. If X is the random variable that represents the number of blue marbles drawn from the box in 10 trials, calculate

1. A plant nursery has two sites where seedlings are grown. The costs at each site are the same, but the seedlings are of different varieties.

At the first nursery site, 848 seedlings are produced in a month. The mean height of the seedlings is 51.0 mm and the standard deviation is 5.36 mm. At the second site, 936 seedlings are grown in the same time period with a mean height of 52.0 mm and a standard deviation of 6.5 mm. Seedlings with a height between 44.0 mm and 66.0 mm are saleable. All seedlings are sold and all sell for the same price.

If the heights of the seedlings are normally distributed, determine if the claim that the first nursery site is more profitable is reasonable. Show full working.

Source: [© QCAA 2017 Mathematics B exam paper one](https://www.qcaa.qld.edu.au/senior/see/subject-resources/mathematical-methods)

### Sample question 13

**Question 18 (9 marks)**

A building has five alarms configured in such a way that the system functions if at least two of the alarms work. The probability that an alarm fails overnight is 0.05. Let the random variable X denote the number of alarms that fail overnight.

1. State the distribution of X. **(2 marks)**
2. What assumptions are required for the distribution in part (a) to be valid **(2 marks)**
3. What is the probability that the alarm system fails overnight? **(2 marks)**

One of the alarms is removed in the evening for maintenance and is not replaced

1. What is the probability that the alarm system still works in the morning? **(3 marks)**

Source: [© WA SCSA 2019 Mathematical Methods calculator assumed paper](https://senior-secondary.scsa.wa.edu.au/syllabus-and-support-materials/mathematics/mathematics-methods)

### Sample question 14

**Question 12 (19 marks)**

The manager of the mail distribution centre in an organisation estimates that the weight, x (kg), of parcels that are posted is normally distributed, with mean 3 kg and standard deviation 1 kg.

1. What percentage of parcels weigh more than 3.7 kg? **(2 Marks)**
2. Twenty parcels are received for posting. What is the probability that at least half of them weigh more than 3.7 kg? **(3 marks)**

The cost of postage, ($)y, depends on the weight of a parcel as follows:

* A cost of $5 for parcels 1 kg or less
* An additional variable cost of $1.50 for every kilogram or part thereof above 1 kg to a maximum of 4 kg
* A cost of $12 for parcels above 4 kg
1. Complete the probability distribution table for Y. **(4 marks)**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  | $5 |  |  |  |  |
|  |  |  |  |  |  |

1. Find the mean cost of postage per parcel (2 marks)
2. Calculate the standard deviation of the cost of postage per parcel (3 marks)
3. If the cost of postage s increased by 20% and a surcharge of $1 is added for all parcels, what will be the mean and standard deviation of the new cost? (3 marks)
4. Show one reason why the given normal distribution is not a good model for the weight of the parcels **(2 marks)**

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### Sample question 15

**Question 17 (14 marks)**

Tina believes that approximately 60% of the mangoes she produces on her farm are large. She takes a random sample of 500 mangoes from a day’s picking.

1. Assuming Tina is correct and 60% of the mangoes her farm produces are large, what is the approximate probability distribution of the sample proportion of large mangoes in her sample? **(3 marks)**
2. What is the probability that the sample proportion of large mangoes is les then 0.58? **(2 marks)**
3. Not relevant to NSW syllabus

A random sample of 500 contains 250 large mangoes

1. On the basis of this data, estimate the proportion of large mangoes produced on the farm **(1 mark)**
2. – g) Not relevant to NSW syllabus

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### Sample question 16

**Question 1 (9 marks)**

A bag contains 1 red marble and 4 green marbles. A single marble is drawn from the bag. The random variable Y is defined as the number of green marbles drawn from the bag.

1. Complete the probability distribution for Y shown below. **(2 marks)**

|  |  |  |
| --- | --- | --- |
|  | 0 | 1 |
|  |  |  |

1. State the distribution of Y **(1 mark)**
2. Determine the mean and standard deviation of the distribution **(2 marks)**

The above process is repeated five times, with the marble being replaced every time. The random variable X is defined as the number of green marbles drawn from the bag in five attempts.

1. State the distribution of X, including its parameters **(2 marks)**
2. Evaluate the probability of selecting exactly two green marbles **(2 marks)**

Source: [© WA SCSA 2018 Mathematical Methods calculator free exam paper](https://senior-secondary.scsa.wa.edu.au/further-resources/past-atar-course-exams/mathematics-methods-past-atar-course-exams)

### Sample question 17

**Question 19 (12 marks)**

A global financial institution transfers a large aggregate data file every evening from offices around the world to its Hong Kong head office. Once the file is received it must be processed in the company’s data warehouse. The time T required to process a file is normally distributed with a mean of 90 minutes and a standard deviation of 15 minutes.

1. An evening is selected at random. What is the probability that it takes more than two hours to process the file? **(2 marks)**
2. What is the probability that the process takes more than two hours on tow out of five days in a week? **(3 marks)**

The company is considering outsourcing the processing of the files.

1. i. A quotation for this job from an IT company is give in the table below.
 Complete the table **(1 mark)**

|  |  |  |  |
| --- | --- | --- | --- |
| Job duration (minutes) |  |  |  |
| Probability |  |  |  |
| Cost Y ($) | 200 | 600 | 1200 |

ii. What is the mean cost? **(2 marks)**

iii. Calculate the standard deviation of the cost **(2 marks)**

iv. In the following year, the cost (currently $Y) will increase due to inflation and also the introduction of an additional fixed cost, so the new cost $N is given by:
 In terms of a and/or b, state the mean cost in the following year and the standard deviation of the cost in the following year **(2 marks)**

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### Sample question 18

**Question 14 (9 marks)**

The simulation of a loaded (unfair) five-sided die rolled 60 times is recorded with the following results.



1. Calculate the proportion of prime numbers recorded in this simulation **(2 marks)**
2. Determine the mean and standard deviation for the sample proportion of prime numbers in 60 tosses, using the results above. **(2 marks)**

This simulation of 60 rolls of the die is performed another 200 times, with the proportion of prime numbers recorded each time and graphed.

1. Not relevant to the NSW syllabus
2. Comment briefly on the key features of this graph. **(2 marks)**

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### Sample question 19

**Question 16 (10 marks)**

An automated milk bottling machine fills bottles uniformly to between 247 mm and 255 ml. The label on the bottle states that it holds 250 ml.

1. Determine the probability that a bottle selected randomly from the conveyor belt of this machine contains less than the labelled amount. **(3 marks)**
2. Calculate the mean (and standard deviation – not within the scope of NSW course) of the amount of milk in the bottles
**(4 marks)**

A worker selects bottles from the conveyer belt, one at a time.

1. Determine the probability that it takes the selection of 15 bottles before five bottles containing less than the labelled amount have been selected **(3 marks)**

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### Sample question 20

**Question 14 (10 marks)**

Trucks carrying iron ore for the Croc Rock mining company arrive at a weighing station. The service time *T* per truck is defined to be the time elapsed from the moment a truck enters the station zone, including the time to be positioned and then weighed, up to the time it leaves the zone.

It is known that the population mean *μ*(*T* ) = 80 seconds and the population standard deviation *σ*(*T* ) = 20 seconds.

At the Croc Rock weighing station, 100 trucks are weighed.

1. State the (approximate) distribution of the sample mean service time per truck for the 100 trucks. **(3 marks)**
2. What is the probability that the sample mean service time will be more than 83 seconds? **(2 marks)**

Suppose that more than 100 trucks were weighed at the Croc Rock weighing station.

1. How would this affect your answer to part (b)? Explain without recalculation.
**(2 marks)**

It is desired that the probability that the sample mean service time will be between 80 seconds and 82 seconds is greater than 40%.

1. Determine the minimum number of trucks that will need to be weighed**. (3 marks)**

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### Sample question 21

**Question 12**

The lifetime of an electronic device is distributed as an exponential random variable with mean years and standard deviation years. A random sample of 50 of these devices is selected. Tam, a graduate electronics engineer, is interested in the mean lifetime of these 50 devices.

1. State the distribution of the sample mean lifetime Justify your answer **(3 marks)**
2. Determine the probability that the sample mean lifetime is between 15 and 25 years **(2 marks)**

Jai, the chief engineer, informs Tam that the lifetimes may not be exponentially distributed but could be some more complicated distribution, yet still having mean years and standard deviation years.

1. If Jai is correct, will your answer to part (b) change? Explain. **(2 marks)**

A different random sample of size n of these devices was selected. Repeated sampling with this sample size shows that there is a 3% chance of obtaining a sample mean greater than 25 years.

1. Determine the value of n. **(4 marks)**

Source: [© WA SCSA 2018 Mathematics Specialist calculator assumed exam](https://senior-secondary.scsa.wa.edu.au/further-resources/past-atar-course-exams/mathematics-specialist-past-atar-course-exams)

### Sample question 22

**Question 9 (6 marks)**

The time *T* in minutes that a particular flight arrives later than its scheduled time is uniformly distributed with . The population mean is and the population variance is .

A sample of 30 arrival times is taken and the sample mean is calculated.

1. Determine correct to 2 decimal places. **(3 marks)**
2. If a large number of samples, each with 30 arrival times, is taken, sketch the likely distribution of the sample mean below.

In the diagram indicate or refer to the calculation from part (a). **(3 marks)** 

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## Solutions

### Sample question 1

**Question 16**

Source: [© VCAA 2017 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 2

**Question 18**

Source: [© VCAA 2017 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 3

p=0.2 and n=16

Correct = A

Source: [© VCAA 2016 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 4

**Question 10**

 (1)

 (2)

From (1)

(2) becomes

Rearranging,

Substituting into (1)

Correct = D

Source: [© VCAA 2015 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 5

**Question 4a**

, correct to three decimal places

This question was answered well.

**Question 4bi.**

 correct to three decimal places

This question was answered reasonably well. Some students gave their answer as 0.31. A common mistake was or giving an answer greater than 1

**Question 4bii.**

No, the events are not independent,

0.310…0.1587, 0.090.1587x0.29=0.046…

A mathematical explanation was required. Some students confused mutually exclusive events with independent events. A common mistake was P(H\S)=P(S)

**Question 4ci.**

 correct to three decimal places.

This question was reasonably well done. A method was required to get full marks. Stating the correct n and p value was sufficient. Some students gave their answer as 0.19.

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**Question 4cii.**

 correct to three decimal places

Some students used the normal approximation to the binomial distribution. There was poor use of variables, for example,

**Question 4ciii.**

This question was not answered well. Many students appeared to be confused by the terminology

1 - P(X=0) < 0.01 was often evaluated, giving n=27. Trial and error was an acceptable method.

Source: [© VCAA 2018 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 6

**Question 6a**

This question was done well

**Question 6b**

Let X represent the number of faculty pegs in a box.

Most students recognised this as a binomial distribution; however, few managed to correctly find the two component expressions. Even fewer successfully managed to manipulate these expressions to the format specified by the question. Another common error was to apply the standard deviation formula.

Source: [© VCAA 2019 Mathematical Methods Exam 1 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 7

The smallest integer value is 1875.

Most students identified the correct formula; however, many were unable to correctly transpose the inequality to solve for n or to correctly manipulate the arithmetic involving rational numbers. Some students had poor use of notation work, in that they did not extend the square root sign to include n.

Source: [© VCAA 2017 Mathematical Methods Exam 1 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 8

**Question 3a**

 correct to four decimal places

This question was answered well. Most students recognised that it was binomial and gave the correct n and p values. Some used P(X>1) instead of P(X1)

**Question 3b**

 correct to four decimal places

Many student recognised that this was a conditional probability question but had the incorrect numerator or denominator. Some included 5 in their calculations, getting 0.9798. Others rounded too soon and gave 0.9312 as the answer.

**Question 3c**

 correct to four decimal places.

Some students thought 3 hours and 10 minutes was 3.1 hours and 6 minutes was 0.6 hours. Others had the standard deviation as 10 minutes. Some gave the answer without showing any working. Students are reminded that some working must be shown for questions with more than one mark. P(Y>180)=0.9522 was a common incorrect response

**Question 3d**

 correct to three decimal places

Most students used the conditional probability formula but tried to use the normal distribution rather than the binomial distribution.

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**Question 3e**

Students made some rounding errors

**Question 3f**

Source: [© VCAA 2016 Mathematical Methods exam paper 2 report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 9

**Question 3ai.**

 or 0.6875

Some students omitted the dx or had incorrect terminals such as , or . Others gave the answer without showing any working.

**Question 3aii.**

Many students were able to identify the binomial distribution with the correct n an p values. A common incorrect answer was

**Question 3b.**

Not relevant to NSW syllabus

**Question 3c.**

 correct to three decimal places

Most students were able to recognise that the problem involved conditional probability.

{continued on next page}

**Question 3di.**

 correct to four decimal places

Some students wrote 3% as 0.3. Others had the incorrect value for n, using

Some gave the answer without showing any working, whilst others attempted to use the normal distribution.

**Question 3dii.**

Some students rounded their answer to 22. Others did not state the minimum value, leaving their answer as N>22.7566. Some students used the trial and rror methods and this was acceptable. Some students did not show any working.

Source: [© VCAA 2015 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 10

**Question 4a.**

cm or 191 mm, correct to the nearest millimetre

Many students thought 100mm=0.1 cm, giving their final answer as 1913 mm. Others had incorrect units, such as 19.1 mm.

**Question 4b.**

**Question 4d.**

Solve

Some students attempted to use the normal distribution to answer this question. Some had incorrect units of conversions.

**Question 4e.**

Many students did not know how to use the binomial distribution and others used the inequality sign incorrectly. Many different approaches could have been used. Many different approaches were used, including trial and error.

**Question 4fi.**

0.7 x 0.7 + 0.3(1 - p) = 0.79 - 0.3p

Brackets were sometimes omitted, giving the incorrect answer

0.7 x 0.7 + 0.3 x 1 - p = 0.79 - p

Source: [© VCAA 2014 Mathematical Methods Exam 2 Report](https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Mathematical-Methods.aspx)

### Sample question 11

**Question 6a**

**Question 6b**

Mean

Var(X)

Source: [© QCAA 2017 Maths B exam report](https://www.qcaa.qld.edu.au/senior/see/subject-resources/mathematical-methods)

### Sample question 12

**Question 6ai.**

P(Blue and Blue) =

**Question 6aii.**

**Question 6b.**

Site 1:

(

Site 2:

more seeds are sold at the second site, so the claim is not reasonable

Source: [© QCAA 2018 Maths B exam report](https://www.qcaa.qld.edu.au/senior/see/subject-resources/mathematical-methods)

### Sample question 13

**Question 18a.**

**Question 18b.**

1. The alarms fail independent of each other.

2. The probability that an alarm fails is constant/unchanging/same for all alarms

**Question 18c.**

**Question 18d.**

Let the random variable Y denote the number of alarms that fail out of 4.

Then

1 - 0.00048 = 0.99952

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### Sample question 14

**Question 12a.**

24.2% are greater than 3.7 kg

**Question 12b.**

Let the random variable M denote the number of parcels that weigh more than 3.7 kg

Then

0.01095 (students will need Binomial tables to answer this question)

**Question 12c.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  | $5 | **$6.50** | **$8** | **$9.50** | **$12** |
|  | **0.0228** | **0.136** | **0.341** | **0.341** | **0.159** |

**Question 12d.**

That is, $8.87 is the mean cost of postage per parcel

**Question 12e.**

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**Question 12f.**

The mean will increase by 20% to 1.2 x 8.874535 + 1 = 11.64944

The standard deviation increases by 20% to 1.2 x 1.747086 = 2.096504

**Question 12g.**

There is a non-zero (small) probability that the weight can be negative, which is not possible.

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### Sample question 15

**Question 17a.**

The sample will be approximately Normally distributed with

mean of

Standard deviation = = 0.02191

**Question 17b.**

**Question 17d.**

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### Sample question 16

**Question 1a.**

|  |  |  |
| --- | --- | --- |
|  | 0 | 1 |
|  |  |  |

**Question 1b.**

It is a Bernoulli distribution.

**Question 1c.**

**Question 1d.**

**Question 1e.**

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### Sample question 17

**Question 19a.**

**Question 19b.**

Let the random variable X denote the number of days out of 5 that the process takes more than 2 hours. Then

**Question 19ci.**

|  |  |  |  |
| --- | --- | --- | --- |
| Job duration (minutes) |  |  |  |
| Probability | **0.0228** | **0.9545** | **0.0288** |
| Cost Y ($) | 200 | 600 | 1200 |

**Question 19cii.**

Mean cost = 200 x 0.0228 + 600 x 0.9545 + 1200 x 0.0288 = $604.55

**Question 19ciii.**

E(Y2) = 2002 x 0.0228 + 6002 x 0.9545 + 12002 x 0.0288 = 377290

Var(Y) = 377290 – 604.552 = 11809.36668

**Question 19civ.**

New mean = 604.55a + b

New SD = 108.67a

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### Sample question 18

**Question 14a.**

Prime numbers are 2, 3 and 5.

**Question 14b.**

**Question 14d.**

Graph takes the shape of a binomial distribution.

Approaches the shape of a normal distribution for large values of n.

Distribution is centred on 0.58.

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### Sample question 19

**Question 16a.**

(255 – 247) x h = 1



**Question 16b.**

Mean =

Standard deviation – not within scope of NSW syllabus

**Question 16c.**

Use binomial distribution for first 14 selections

Source: [(c) WA SCSA 2016 Mathematics Methods calculator assumed exam marking key](https://senior-secondary.scsa.wa.edu.au/further-resources/past-atar-course-exams/mathematics-methods-past-atar-course-exams)

### Sample question 20

**Question 14a.**

is approximately normally distributed as the sample size n=100>30

**Question 14b.**

**Question 14c.**

Since for n>100 would result in then the mean time of 83 minutes would be a more extreme sample mean in the normal distribution.

Hence there would be a lower probability

i.e. the answer to part (b) would be lower

**Question 14d.**

Require

i.e.

Hence Solving gives

 At least 165 trucks are required to be weighed

Source: [(c) WA SCSA 2019 Mathematics Specialist calculator assumed exam marking key](https://senior-secondary.scsa.wa.edu.au/further-resources/past-atar-course-exams/mathematics-specialist-past-atar-course-exams)

### Sample question 21

**Question 12a.**

 is approximately normally distributed as sample size n=50>30

**Question 12b.**

**Question 12c.**

There will be no change to the answers for the part (b). Since sample size n=50>30 then the distribution of sample means is still approximately normal.

**Question 12d.**

Given where

If

 i.e

Solving gives n=56.598…

i.e; the different sample size was 57 (still n>30)

Source: [(c) WA SCSA 2018 Mathematics Specialist calculator assumed exam marking key](https://senior-secondary.scsa.wa.edu.au/further-resources/past-atar-course-exams/mathematics-specialist-past-atar-course-exams)

### Sample question 22

**Question 9a.**

The sample mean will be normally distributed.

**Question 9b.**

There is approximately 71% of the total area under the curve for



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