 The colour of all rabbits

‘All rabbits are the same colour’

The mathematical induction proof given below proves that all rabbits are the same colour.

Question 1

Complete the cloze passage using the word bank. Terms can be used more than once.

Word bank – $r\_{1}$, same, $r\_{k+1}$, set B, one more, assume, n, one, natural, set A.

‘If X is a set of rabbits, then all rabbits in X are the same colour.’

Step 1

If we can prove the above statement to be true for every number n, then it will follow that all rabbits are the same colour, and X is a set consisting of only rabbit($r\_{1}$), then the statement that all rabbits in X are the same colour is true.

Step 2

If $k$ is a natural number and there are $k$ rabbits in set X, that all rabbits are the same colour.

$$X =\left\{r\_{1}\right. , r\_{2}, r\_{3}, r\_{4}…\left.r\_{k}\right\}$$

Let X at this stage be A so that $ A =\left\{r\_{1}\right. , r\_{2}, r\_{3}, r\_{4}…\left.r\_{k}\right\}$ where all rabbits are the same colour.

Step 3

To prove the statement to be right for all rabbits, we have to prove that the statement will be true if we add more rabbit ( ) of the same colour as the first one ( )

For example: to prove $X =\left\{r\_{1}\right. , r\_{2}, r\_{3}, r\_{4}…r\_{k}, \left.r\_{k+1}\right\}$ where all the rabbits in set X are the colour.

Now let’s remove the first rabbit and rename this new set B so that $B =\left\{r\_{2}\right. r\_{3}, r\_{4}…r\_{k}, \left.r\_{k+1}\right\}$

We can say that consists of rabbits of same colour because set B has all rabbits from (except the first one) which has already been already proven to be same colour and rabbit of the same colour as .

Conclusion

Hence it is proved by mathematical induction that the hypothesis is true for $n=k+1.$

That means ‘all rabbits are the same colour in set X’.

Question 2

Identify the initial statement, first case/base case and the inductive step from the proof.

Initial statement

First case/base case

Inductive step

Question 3

Why do you think this proof is not appropriate to prove this statement? Justify using the false step which made it correct.