 Finding the mistakes

Finding the missing or incorrect step

Question 1

Prove by induction

$$7+14+21+……………………+7k= \frac{7k\left(k+1\right)+3}{2} for integral k\geq 1.$$

Initial statement

For example: $7+14+21+……….+7n= \frac{7n\left(n+1\right)+3}{2}$ for all integers $n\geq 1.$

Step 1

As given, the statement is true for $n=k.$

For example:

$7+14+21+……….+7k= \frac{7k\left(k+1\right)+3}{2}$ for all integrals $k\geq 1.$

Step 2

Now test if the statement is true for $n=k+1$ for all integrals $k+1\geq 1.$

For example: to prove that

$$7+14+21+……….+7k+7\left(k+1\right)= \frac{7\left(k+1\right)\left(k+1+1\right)+3}{2}$$

For all integrals $k+1\geq 1.$

LHS = $7+14+21+……….+7k+7(k+1)$

 *=* $\frac{7k\left(k+1\right)+3}{2}+ $ $7(k+1)$ because $7+14+21+……….+7k= \frac{7k\left(k+1\right)+3}{2}$

 = $\frac{7k\left(k+1\right)+3+(2×7\left(k+1\right))}{2}$

 = $\frac{7k\left(k+1\right)+\left(14\left(k+1\right)\right)+3}{2}$

 =$\frac{7\left(k+1\right)(k+2)+3}{2}$

RHS = $\frac{7\left(k+1\right)\left(k+1+1\right)+3}{2}$

 =$\frac{7(k+1)\left(k+2\right)+3}{2}$

$$LHS = RHS $$

Hence proved.

Answer

Question 2

Prove that that $1+3+3^{2}+3^{3}+ …………………..+ 3^{n-1}= \frac{1}{4}\left(3^{n}-1\right)+\frac{1}{2}$ for all $n\geq 1.$

Step 1 – test for $n=1$

LHS = $3^{1-1}=1$

RHLHS = S = $\frac{1}{4}\left(3^{1}-1\right)+\frac{1}{2}$

 =$\frac{1}{4}×2+\frac{1}{2}$

 = 1

LHS = RHS hence proved.

Step 2

Hence assume true for $n=k.$

For example: $1+3+3^{2}+3^{3}+ …………………..+ 3^{k-1}= \frac{1}{4}\left(3^{k}-1\right)+\frac{1}{2}$ for all $k\geq 1$

Therefore, the initial statement is proved.

Answer

Question 3

Prove that $4+8+12+ ………………………+4n=2n(n+1)-2$ for all $n\geq 1.$

Step 1

Assume it to be true for $n=k$ and rewrite the statement.

For example: $4+8+12+ ………………………+4k=2k\left(k+1\right)-2$ for all $k\geq 1.$

Step 2

Now prove that the statement is true for $n=k+1.$

For example: prove that $4+8+12+ ………………………+4k+4(k+1)=2(k+1)(k+1+1)-2$

for all $k+1\geq 1.$

LHS = $4+8+12+ ………………………+4k+4\left(k+1\right)$

 =$2k(k+1)-2+4\left(k+1\right) \left( because 4+8+12+ ………………………+4k=2k(k+1)-2\right)$

 = $2k\left(k+1\right)+4\left(k+1\right)-2$

 =$ 2\left(k+1\right)\left(k+2\right)-2$

RHS = $2\left(k+1\right)\left(k+1+1\right)-2$

 = $2(k+1)(k+2)-2$

LHS = RHS

Hence proved.

Answer