 Applications of exponential growth and decay

Activity 1: The world’s population

Background information

In 2010 the UN estimated a continuous growth rate of world population at 1.162% and the total population to be approximately 6 895 889 000.

It projected that the population growth rate would change significantly over time; assuming a constant growth rate is unrealistic.

Let’s assume the population can be modelled using an exponential growth model. Let P be the world population at time t years after 2010. With continuous growth rate 1.162% we have dP/dt =0.01162P

[Source: A guide for teachers – Year 11 and 12](http://www.amsi.org.au/ESA_Senior_Years/SeniorTopic3/3_md/SeniorTopic3e.html%23history_1)

1. Show that P = P0e0.01162t satisfies this differential equation and find the value of P0.
2. Use this equation to predict the world’s population at various years and record the information in the table below:

| Year | t (years after 2010) | P (nearest thousand) |
| --- | --- | --- |
| 2010 | 0 | 6 895 889 000 |
| 2020 |  |  |
| 2030 |  |  |
| 2030 |  |  |
| 2060 |  |  |
| 2100 |  |  |

1. Use the equation to predict when the population will be 25 billion.
2. Sketch this exponential growth model on a Cartesian plane showing appropriate features and labels.
3. Is this a realistic model? Why or why not?

‘Assuming exponential growth, the world's population almost doubles in 50 years, and approaches 20 billion by 2100. However, all of the UN's projections were lower than this. For instance, the UN's 'medium fertility' projection gives a total world population of about half of our estimate, 10.9 billion, by 2100. A model of exponential growth is unrealistic.’

[Source: A guide for teachers – Year 11 and 12](http://www.amsi.org.au/ESA_Senior_Years/SeniorTopic3/3_md/SeniorTopic3e.html%23history_1)

Activity 2: Amazon’s net sale revenue

The following table provides Amazon’s revenue from 2004 until 2015.

| Year | t (years after 2004) | Revenue (Billion US dollars) |
| --- | --- | --- |
| 2004 | 0 | 6.92 |
| 2005 | 1 | 8.49 |
| 2006 | 2 | 10.71 |
| 2007 | 3 | 14.84 |
| 2008 | 4 | 19.17 |
| 2009 | 5 | 24.51 |
| 2010 | 6 | 34.2 |
| 2011 | 7 | 48.08 |
| 2012 | 8 | 61.09 |
| 2013 | 9 | 74.45 |
| 2014 | 10 | 88.99 |
| 2015 | 11 | 107.01 |

[Source: The Statistics Portal](https://www.statista.com/statistics/266282/annual-net-revenue-of-amazoncom/)

1. Open Resource: exponential-growth-revenue-modelling.GGB.
2. Plot each of the points from the table above.
3. Use the sliders to fit an exponential curve to the points. Note: You can also refer to resource 17 for an alternate method of determining k.
4. Use the y-intercept and an additional point on the curve to find an exponential equation approximating the relationship between Amazon’s revenue to the years after 2004:

R = R0ekt

Note: You can also refer to resource 17 for an alternate method of determining k.

1. What is the equivalent annualised percentage increase in Amazon’s revenue?
2. Use the equation to predict:
   1. Amazon’s revenue in 2016
   2. The rate of change in Amazon’s revenue in 2016 (dR/dt)
   3. Amazon’s revenue in 2017
   4. The rate of change in Amazon’s revenue in 2017
   5. When Amazon’s sales revenue will exceed 200 billion USD
   6. When dR/dt = 50
3. Compare your predicted revenue in 2016 and 2017 with the [actual figures.](https://www.statista.com/statistics/266282/annual-net-revenue-of-amazoncom/)

Activity 3: Google’s revenue

The following table provides Google’s revenue from 2002 until 2015.

| Year | t (years after 2002) | Revenue (Billion US dollars) |
| --- | --- | --- |
| 2003 | 0 | 0.4 |
| 2002 | 1 | 1.5 |
| 2004 | 2 | 3.2 |
| 2005 | 3 | 6.1 |
| 2006 | 4 | 10.6 |
| 2007 | 5 | 16.6 |
| 2008 | 6 | 21.8 |
| 2009 | 7 | 23.7 |
| 2010 | 8 | 29.3 |
| 2011 | 9 | 37.9 |
| 2012 | 10 | 50.18 |
| 2013 | 11 | 55.51 |
| 2014 | 12 | 65.67 |
| 2015 | 13 | 74.54 |

[Source: The Statistics Portal](https://www.statista.com/statistics/266206/googles-annual-global-revenue/)

1. Open Resource: exponential-growth-revenue-modelling.GGB
2. Plot each of the points from the table above.
3. Use the sliders to fit an exponential curve to the points. Note: You can also refer to resource 17 for an alternate method of determining k.
4. Use the y-intercept and an additional point on the curve to find an exponential equation approximating the relationship between Google’s revenue to the years after 2004:

R = R0ekt

1. What is the equivalent annualised percentage increase in Google’s revenue?
2. Use the equation to predict:
   1. Googles’s revenue in 2016
   2. The rate of change in Google’s revenue in 2016 (dR/dt)
   3. Google’s revenue in 2017
   4. The rate of change in Google’s revenue in 2016
   5. When Google’s sales revenue will exceed 200 billion USD
   6. When dR/dt = 25
3. Compare your predicted revenue in 2016 and 2017 with the [actual figures.](https://www.statista.com/statistics/266206/googles-annual-global-revenue/)

Activity 4: Exponential population growth of rabbits in Australia

Background information

‘The proliferation of rabbits after their introduction to Australia is a famous example of population growth. In 1859, a southern Australian farmer homesick for England imported two dozen (24) wild English rabbits and set them free on his land. Within six years, Thomas Austin's 24 rabbits had multiplied to 22 million!’

[Source from petefalzone.com](http://www.petefalzone.com/handouts/exp-growth-rabbits-australia.pdf)

1. Using an exponential growth model P = P0ekt, find the values of P0 and k and hence the equation which relates the population of rabbits and the number of years after 1859.
2. Use this equation to predict Australia’s population of rabbits at various years and record the information in the table below:

| Year | t (years after 1859) | P (nearest thousand) |
| --- | --- | --- |
| 1859 | 0 | 24 |
| 1865 | 6 | 22 000 000 |
| 1900 |  |  |
| 1920 |  |  |
| 1930 |  |  |
| 2018 |  |  |

1. Use the equation to predict when the population will be 25 billion.

Other Information:

By the 1930s the rabbit population was estimated to be 750 million.

[Source from petetalzone.com](Source:%20http://www.petefalzone.com/handouts/exp-growth-rabbits-australia.pdf%20)

Other sources put the population at 10 billion in the 1920s.

[Article from ABC](Source:%20http://www.abc.net.au/science/articles/2009/04/08/2538860.htm%20)

The current population of rabbit’s in Australia is predicted to be 200 million.

[The rabbit problem – Rabbit Free Australia](source:%20http://www.rabbitfreeaustralia.com.au/rabbits/the-rabbit-problem/)

1. Is our model realistic? Why or why not? Also consider:

[NSW National Parks and Wildlife Service (NPWS) rabbit factsheet](http://www.environment.nsw.gov.au/pestsweeds/RabbitFactsheet.htm)

[Australia’s battle with the bunny](file:///\\DETNSW.WIN\DATA\LL-SE\Accessibility%20documents%20-%20content%20check\Mathematics\New%20resources%20to%20check\me-c1.2\Australia's%20battle%20with%20the%20bunny)

Activity 5: Decay of carbon-14

Carbon-14 is a radioactive isotope of carbon that is present in the earth’s atmosphere in extremely low concentrations. It is naturally produced in the atmosphere by cosmic rays (and also artificially by nuclear weapons). Suppose we have a sample of a substance containing some carbon-14.

Let m be the mass of carbon-14 in nanograms after t years. It turns out that, if the sample is isolated, then m and t approximately satisfy the differential equation dm/dt = -0.000121m.

[Source: A guide to teachers – Year 11 and 12](https://www.amsi.org.au/ESA_Senior_Years/PDF/GrowthDecay3e.pdf)

1. Show that m = m0e-0.000121t satisfies this differential equation and find the value of m0 assuming we start with 200 nanograms of carbon-14.
2. Use this equation to predict the mass of carbon-14 remaining after the given time periods:

| t (years) | m (ng to 3 decimal places) |
| --- | --- |
| 0 | 200.000 |
| 11 |  |
| 1000 |  |
| 10000 |  |

1. Use the equation to predict when the mass remaining will be 10 ng.

Activity 6: Las Baulas Leatherback Turtles

Leatherback sea turtle nests have been monitored and protected since the 1988/89 nesting season in Parque Nacional Marino Las Baulas, Costa Rica.

The data shown below is the number of nesting leatherback sea turtles during each season.

| Nesting season | t (years after 1988/89) | T (nesting turtles) |
| --- | --- | --- |
| 1988/89 | 0 | 1504 |
| 1989/90 | 1 | 1474 |
| 1990/91 | 2 | 732 |
| 1991/92 | 3 | 847 |
| 1992/93 | 4 | 1000 |
| 1993/94 | 5 | 195 |
| 1994/95 | 6 | 569 |
| 1995/96 | 7 | 421 |
| 1996/97 | 8 | 140 |
| 1997/98 | 9 | 234 |
| 1998/99 | 10 | 126 |
| 1999/2000 | 11 | 246 |
| 2000/01 | 12 | 417 |
| 2001/02 | 13 | 79 |
| 2002/03 | 14 | 68 |
| 2003/04 | 15 | 188 |
| 2004/05 | 16 | 54 |
| 2005/06 | 17 | 124 |
| 2006/07 | 18 | 76 |
| 2007/08 | 19 | 90 |
| 2008/09 | 20 | 32 |
| 2009/10 | 21 | 49 |

[Source: Earthwatch 2017 Annual Field Report](http://earthwatch.org/FieldReports/earthwatch-field-report-costa-rican-sea-turtles-2017.pdf)

1. Open Resource: exponential-decay-leatherback-modelling.GGB
2. Plot each of the points from the table above.
3. Use the sliders to fit an exponential curve to the points. Note: You can also refer to resource 17 for an alternate method of determining k.
4. Use the y-intercept and an additional point on the curve to find the exponential equation relating the number of nesting leatherback sea turtle to the years after the 1988/89 nesting season::  
   T = T0ekt
5. What is the equivalent annualised percentage decrease in the number of nesting sea turtles?
6. Use the equation to predict:
   1. The number of nesting sea turtles in 2015/16
   2. The number of nesting sea turtles in 2016/17
   3. The rate of change in the number of nesting sea turtles in 2016/17 (dT/dt)
   4. When the number of nesting sea turtles will fall below 10.
   5. When the rate of change in the number of nesting sea turtles was -60.
7. Compare your predictions for the number of nesting sea turtles in 2015/16 and 2016/17 with the [actual figures.](http://earthwatch.org/FieldReports/earthwatch-field-report-costa-rican-sea-turtles-2017.pdf)

Activity 7: The stopping distance of a car

In this activity you will examine the stopping distance of cars in wet and dry weather.

You are to collect the data from the [Queensland Government.](https://www.qld.gov.au/transport/safety/road-safety/driving-safely/stopping-distances/graph)

1. Open Resource: stopping-distance-modelling.GGB
2. Plot the data for the total stopping distance in dry weather.
3. Use the sliders to fit an exponential curve to the points. Note: You can also refer to resource 17 for an alternate method of determining k.
4. Use the y-intercept and an additional point on the curve to find the exponential equation relating the stopping distance of the car and the speed it is travelling. Record the equation in the table below.  
   D = D0ekS
5. Repeat the process for the total stopping distance in wet weather.

| Stopping distance | Exponential equation |
| --- | --- |
| In dry weather |  |
| In wet weather |  |

1. Use your equations to answer the following:

Sections of the Northern Territory have a speed limit of 130km/h. For this speed, find

* 1. The stopping distance in dry weather
  2. The stopping distance in wet weather
  3. The difference in stopping distance in wet and dry weather

Parts of the autobahn in Germany have no posted speed limit. Find the speed of the fasted road car in km/h. If the car was travelling at its maximum speed, predict:

* 1. The stopping distance in dry weather
  2. The stopping distance in wet weather

Activity 8: How many bounces?

You will need a ruler (tape measure), ball and recording device for this activity.

Finding the equation of the model:

1. Set up the tape or ruler vertically against a wall.
2. Drop the ball from a height in from of the tape.
3. Use a video recorder to record the ball as it bounces and eventually stops.
4. Record the heights of each bounce in the table below:

| Bounce | Height (cm) | ln[H/H0] |
| --- | --- | --- |
| 0 | (Initial height) |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |

Note: Take the height to be the distance from bottom of ball to floor.

1. Assuming the bounce height of the ball follows the simple exponential decay model, H = H0ekB then

ln(H/H0) = kB

Plot ln(H/H0) verse B using a scatterplot

1. Add a line of best fit (with y-intercept zero) and display the equation.
2. Use the gradient of the line of best fit to find the bounce factor ‘k’ for the exponential decay model.

If required, refer to Resource: another-way-to-determine-k.DOCX pages 1 and 2 for an example of steps 6 to 8.

1. Complete the equation H = H0ekB with the correct values of H0 and k.
2. In Geogebra, plot the height verse the bounce and add the exponential decay equation to ensure the model fits appropriately.
3. Decide on a new height (>1m) to release the same ball. Dropping it onto the same surface.
   1. Use an equation (with a new H0) to predict how many bounces it will take before the ball will not reach your knee.
   2. Conduct this experiment and compare the results with the prediction.