Mathematics: Advanced  
Conditional probability transcript

(Duration 6 minute 08 seconds)

This is the HSC Hub Mathematics Curriculum Support for the New South Wales Department of Education. My name is Jackie Blue.

This question has been adapted from an exemplar question from the NESA year 11 topic guide to Statistical Analysis in the Mathematics Advanced course. This question looks at Conditional probability. We acknowledge that there may be different approaches, methods or techniques to answering it. We encourage you to discuss and share these with each other.

The first three parts of this question explore two stage probability. The fourth part of the question involves conditional probability. Click pause now to read through the question and then we'll work through the solution together.

A bag contains two red balls, one black ball and one white ball. Andrew selects one ball from the bag and keeps it hidden. He then selects a second ball, also keeping it hidden.

Part 1. Draw a tree diagram to show all the possible outcomes.

Part 2. Find the probability that both the selected balls are red.

Part 3. Find the probability that at least one of the selected balls is red.

Part 4. Andrew reveals that one of the selected balls is red. What is the probability that the ball that is still hidden is also red?

Part one asked us to draw a tree diagram to show all the possible outcomes, a bag contains two red balls, one black ball and one white ball. So the first stage of branches will need two branches for red, and one each for black and for white.

Now, once Andrew selects the first ball, he keeps it hidden and then he selects a second ball. This means that in the second stage of the event, each option will have three branches .So if a red ball is selected first, we'll have one red, one black and one white ball remaining. If a black ball is selected first, we'll have two red and one white ball remaining. And if a white ball is selected first, we'll have two red balls and one black ball remaining.

Part two asks us to find the probability that both the selected balls are red. If we use the tree diagram, we can see that there are 12 outcomes in the sample space. I've listed those here for you on the screen.

RR

RB

RW

RR

RB

RW

BR

BR

BW

WR

WR

WB

Of the 12 outcomes in the sample space, two of them are red red, that's two-twelfths which gives us one-sixth.Part three asks us to find the probability that at least one of the selected balls is red.At least one means one or more. If we use the tree diagram, we can see that there are 12 outcomes in the sample space and 10 of those include at least one red ball, that's ten twelves or five sixth.

Let's get into part four, part four says, Andrew reveals that one of the selected balls is red. What is the probability that the ball that is still hidden is also red? Well, this is a conditional probability question because we know already that one of the balls is definitely red.And knowing this means that instead of our probability being calculated as a fraction of the whole sample space, we're reducing the size of the sample space to include only the combinations that have at least one red ball. This means removing “black, white” as an option. And it also means removing “white, black”, and this reduces the total number of possible outcomes to 10. From this reduced sample space, we need to look at the number of outcomes where the other ball is also red. So we're looking for red red, or two red balls. We can see here that this happens twice. That gives us two tenths or one fifth.

An alternative method is using the conditional probability formula which can be found in the probability section of your reference sheet and is highlighted on the bottom left hand side of the screen. It says that the probability of A given B is equal to the probability of both A and B, the intersection of A and B, as a fraction of the probability of B occurring. Now in this particular question, that notation might look a little confusing because event A and B both include a red ball.

So let's break down the language when we use this notation. On the left hand side, we have the probability that the hidden ball is red, given that we know one of the balls is red. This is equal to the right hand side which is the probability that both of the balls are red. So two red balls as a fraction of the probability that at least one of the balls is red,because we know it already. You might notice at this point that the numerator is actually what we calculated in part two. And the denominator is what we calculated in part three. And if we notice that, it is just a matter of substituting the previous answers into the formula. It's definitely good practice to consider whether the earlier parts of the question can help you in a later part. So in this case, it gives us one-sixth on five-sixths, which gives us one-fifth.

This is the HSC hub for the New South Wales Department of Education.

End of transcript