 Year 12 Mathematics Advanced

| MA-T3 Trigonometric functions and graphs | Unit duration |
| --- | --- |
| The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations.A knowledge of trigonometric functions enables the solving of practical problems involving the manipulation of trigonometric expressions to model behaviour of naturally occurring periodic phenomena such as waves and signals and to predict future outcomes.Study of trigonometric functions is important in developing students’ understanding of periodic functions. Utilising the properties of periodic functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals | 3 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to explore the key features of the graphs of trigonometric functions and to understand and use basic transformations to solve trigonometric equations.Students develop an understanding of the way that graphs of trigonometric functions change when the functions are altered in a systematic way. This is important in understanding how mathematical models of real-world phenomena can be developed. | A student:* uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1
* applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs MA12-5
* chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
* constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10
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| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| Students should have studied Year 11 Trigonometry content, MA-T1, Trigonometry and measures of angles, and MA-T2, Trigonometric functions and identities, as well as the Year 12 subtopic, MA-F2, Graphing techniques. | Can mathematics predict periodic phenomena? **Resource:** ma-t3-assignment-can-mathematics-predict-periodic-phenonema.DOCX |

All outcomes referred to in this unit come from [Mathematics Advanced](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| amplitude | The amplitude of a wave function is the height from the horizontal centre line to the peak (or to the trough) of the graph of the function. Alternatively, it is half the distance between the maximum and minimum values. |
| dilation **** | A dilation stretches or compresses the graph of a function. This could happen either in the x or y direction or both. |
| domain | The domain of a function is the set of x values of $y=f(x)$ for which the function is defined. Also known as the ‘input’ of a function. |
| function | A function is a special relation where each input is mapped to a single output.Note: Different inputs could relate to the same output.Common notation includes $f(x)$. In this case, $f$ is a rule that maps each input $x$, to a unique output $f(x)$. If $y=f(x)$, then we say that $x$ is the independent variable and $y$ is the dependent variable. The output is dependent on the input.These diagrams illustrate the concept of a function.  The first shows each input mapped to a different output, which is a function.  The second shows each input mapped to an output. Some different inputs give the same output but this is still a function.  The third shows some input values mapped to more than one output value. This is called a relation. It is not a function.  |
| odd function **** | Algebraically, a function is odd if $f\left(-x\right)=-f\left(x\right),$ for all values of x in the domain.An odd function has point symmetry about the origin. |
| period | The period of a trigonometric function is the smallest interval for which the function repeats itself. |
| phase | When a trigonometric function is translated horizontally, the phase (or phase shift) is the magnitude of this translation. |
| range (of function) **** | The range of a function is the set of values of the dependent variable for which the function is defined. |
| sketch | A sketch is an approximate representation of a graph, including labelled axes, intercepts and any other important relevant features. Compared to the corresponding graph, a sketch should be recognisably similar but does not need to be precise. |

| **Lesson sequence** | **Content** | **Suggested teaching strategies and resources**  | **Date and initial** | **Comments, feedback, additional resources used** |
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| Revision of trigonometric graphs and transformations (Optional)(1 lesson) |  | **Revision of trigonometric graphs and transformations*** Students to review basic trigonometric graphs:

$$y=\cos(x), y=\sin(x) and y=\tan(x)$$Consider the concepts of amplitude (for sine and cosine graphs) and period ($2π$ for sine and cosine graphs, and π for tangent graph).* Students to review transformations of a graph. Consider a parabola or other non-linear function. Suggested methods include, but are not limited to:
	+ Graph using technology and compare the following.

$$y=x^{2}, y=2x^{2}, y=\frac{1}{2}x^{2}$$$$y=x^{2}, y=x^{2}+4, y=x^{2}-1$$$$y=x^{2}, y=\left(x-2\right)^{2}, y=\left(x+3\right)^{2}$$* + Investigate using graphing software applets. Sample applets include:
		- [Parabolic transformations in desmos.](https://www.desmos.com/calculator/gmq16ymdlw)
		- [Parabolic transformation in geogebra](https://www.geogebra.org/m/KU2zYScE)
 |  |  |
| Investigation of trigonometric graph transformations(2 or 3 lessons) | * examine and apply transformations to sketch functions of the form $y=kf(a(x+b))+c$, where $a,b,c$ and $k$ are constants, in a variety of contexts, where $f(x)$ is one of $sin x$, $cos x$ or $tan x$, stating the domain and range when appropriate
* use technology or otherwise to examine the effect on the graphs of changing the amplitude (where appropriate), $y=kf(x)$, the period, $y=f(ax)$, the phase, $y=f(x+b)$, and the vertical shift, $y=f(x)+c$
 | **Investigation of trigonometric graph transformations*** Using technology or otherwise, students examine and discover the effect on a trigonometric graph when changing the values of $a, b,c $and$ k$. Format: $$f(x)=ksin[a(x+b)]+c$$
* Teachers can use inbuilt sliders in an applet to investigate. Sample applets include:
	+ [Transformations of sine in desmos](https://www.desmos.com/calculator/2tvamrilx0)
	+ [Transformations of cosine in desmos](https://www.desmos.com/calculator/pvxhrezwnb)
	+ [Transformations of tangent in desmos](https://www.desmos.com/calculator/fgo045alwk)
	+ [Geogebra by Mossback](https://www.geogebra.org/m/Usn3pDQp) (You may want to adjust the variable names to reflect the syllabus, $k, a, b and c)$
* Students should explore the graphs of sine and cosine before tangent.
	+ amplitude:
		- Students discover what happens when ‘$k$’ is adjusted. (Teachers can refer to a parabola $y=x^{2}$ and $y=4x^{2}$)
		- Students explore what happens when ‘$k$’ is negative.
		- Students define the value of ‘$k$’ as the amplitude.
		- Students to find the new range for functions.
	+ period:
		- Students discover what happens when ‘$a$’ is adjusted.
		- Students formally define the period of a trigonometric graph and link this to ‘$a$’.
		- Clarify that ‘a’ is not the period. For sine and cosine, period $=\frac{2π}{a}$ and for tangent, period $=\frac{π}{a}$.
	+ vertical shift:
		- Students discover what happens when ‘c’ is adjusted. (Teachers can refer to a parabola $y=x^{2}$ and $y=x^{2}+4$)
		- Students define the value of ‘$c$’ as the vertical shift of the function.
		- Students to find the new range for functions.
	+ phase:
		- Students discover what happens when ‘$b$’ is adjusted. (Teachers can refer to a parabola $y=x^{2}$ and $y=(x+4)^{2}$
		- Start with $a=\pm 1$.
		- When $a\ne \pm 1$, students factorise to find the value of the phase ‘$b$’.
		- Students to find the new domain of functions.
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| Describing and sketching transformational shifts in trigonometric graphs(2 or 3 lessons) | * examine and apply transformations to sketch functions of the form $y=kf(a(x+b))+c$, where $a,b,c$ and $k$ are constants, in a variety of contexts, where $f(x)$ is one of $sin x$, $cos x$ or $tan x$, stating the domain and range when appropriate
* use $k, a, b,c$ to describe transformational shifts and sketch graphs
 | **Describing and sketching trigonometric graph transformations*** Students apply their knowledge of transformations to sketch and describe transformation shifts of trigonometric functions of the form$y=kf(a(x+b))+c$, where $a, b, c$ and $k $are constants.

Sample activities include:* + Transformations of trigonometric functions
		- Activity 1: Amplitude
		- Activity 2: Period
		- Activity 3: Vertical Shift
		- Activity 4: Phase
		- Activity 5: Composite Transformations
		- Activity 6: Composite Transformations – paired activity
		- Activity 7: Graph to an equation

**Resource:** transformations-of-trigonometric-functions.DOCX |  |  |
| Solve trigonometric equations (1 lesson) | * solve trigonometric equations involving functions of the form $kf(a(x+b))+c$, using technology or otherwise, within a specified domain **AAM**
 | **Solve trigonometric equations*** Solve trigonometric functions with a specified domain.
	+ Teachers to model solving trigonometric functions including domain adjustment.
	+ Example of domain adjustment:

Solve $sin2x=\frac{1}{2}$ in the domain $[0,2π]$$0\leq x\leq 2π$ implies $0\leq 2x\leq 4π$* Using technology
	+ To solve $sin2x=\frac{1}{2}$ in the domain $[0,2π]$

Graph $y=sin2x$ and $y=\frac{1}{2}$Read $x$ values of the points of intersections in the specified domain.* + Suggested technology:
		- Desmos**Note:** To display $π$ on the $x$-axis, under settings, set the$ x$-axis step to $π$ or $\frac{π}{2}$ or similar by typing ‘pi’ or ‘pi/2’.
		- [Geogebra applet](https://www.geogebra.org/m/AGUqDtY2)
 |  |  |
| Solve practical problems using trigonometric functions (1 lesson) | * use trigonometric functions of the form $kf(a(x+b))+c$ to model and/or solve practical problems involving periodic phenomena **AAM**
 | **Solve practical problems using trigonometric functions*** Past HSC Questions:
	+ [2013 Mathematics Q13(a), population of horses](http://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2013/mathematics-2013-hsc-exam-pack)
	+ [2009 Mathematics Q7(b), tides](http://educationstandards.nsw.edu.au/wps/portal/nesa/resource-finder/hsc-exam-papers/2009/mathematics-2009-hsc-exam-pack)
* Applications of periodic phenomena:
	+ Electric currents
	+ Radio signals
* Modelling opportunities based on the collection of raw data:
	+ Tides
	+ Pendulums

**Resources:** applications-of-periodic-phenomena.DOCX, modelling-of-periodic-phenomena.DOCX |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.