 Year 11 Mathematics Advanced

| MA-F1 Working with functions Paperclip icon | Unit duration |
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| The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities. A knowledge of functions enables students to discover, recognise and generalise connections between algebraic and graphical representations of the same expression and to describe interactions through the use of both dependent and independent variables. The study of functions is important in developing students’ ability to find connections and patterns, to communicate concisely and precisely, to use algebraic techniques and manipulations, to describe and solve problems, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather. | 6 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to introduce students to the concept of a function and develop their knowledge of functions and their respective graphs. Function notation is introduced, which is essential for describing the ideas of calculus. Students develop their use of mathematical language to describe functions, their properties and respective graphs while applying this knowledge to everyday problems and applications. In business and economics, for example revenue depends on the number of items sold, and expressing this relationship as a function allows the investigation of changes in revenue as sales change. Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | A student:* Uses algebraic and graphical techniques to solve, where appropriate, compare alternative solutions to problems MA11-1
* Uses the concepts of functions and relations to model, analyse and solve practical problems MA11-2
* Uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
* Provides reasoning to support conclusions which are appropriate to the context MA11-9
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| Prerequisite knowledge | Assessment strategies |
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| Students should have studied Stage 5.3 algebraic techniques, surds, index laws linear relationships and non-linear relationships.  | * Terminology quiz that focuses on the literacy skills needed to understand questions in this unit.
* Have students create their own composite functions questions based on functions previously learnt and partner with a friend to see if the questions work. Discuss why, why not.
* Students are to explore and investigate functions through DESMOS. This allows an element of investigative learning, as they change features on their equation, the form of the graph changes. Students understanding can be assessed by challenging them to describe any relationships or findings.
* Have students create their own functions questions based on linear, quadratic and cubic functions and partner with a friend to see if the questions work. Discuss why, why not?
* Students should use graphing software tools to investigate the important features of the various functions and relations explored in this unit and explain the effect that various terms in the equation have on the position or shape of the function or relation.
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All outcomes referred to in this unit come from [Mathematics Advanced](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| Composite functions | In a composite function, the output of one function becomes the input of a second function.More formally, the composite of and , acting on , can be written as with being performed first. |
| Domain | The domain of a function is the set of values of for which the function is defined. Also known as the ‘input’ of a function. |
| Odd function | Algebraically, a function is odd if , for all values of in the domain.An odd function has point symmetry about the origin. |
| Function (notation) | A function is a rule that associates each element in a set with a unique element from a set .The set is called the domain of and the set is called the co-domain of. The subset of consisting of those elements of which occur as values of the function is called the range of . The functions most commonly encountered in elementary mathematics are real functions of a real variable, for which both the domain and co-domain are subsets of the real numbers.If we use function notation to write , then we say that is the independent variable and is the dependent variable. |
| Interval notation | Interval notation is a notation for representing an interval by its endpoints. Parentheses and/or square brackets are used respectively to show whether the endpoints are excluded or included. |
| Even function | Algebraically, a function is even if , for all values of in the domain.An even function has line symmetry about the -axis. |
| One-to-one | In a one-to-one function, every element in the range of a function corresponds to exactly one element of the domain. |
| Range (of function) | The range of a function is the set of values of the dependent variable for which the function is defined. |
| Set language and notation | A set is a collection of distinct objects called elements.The language and notation used in the study of sets includes:A set is a collection of objects, for example ‘ is the set of the numbers 1, 3 and 5’ is written as .Each object is an element or member of a set, for example ‘1 is an element of set ’ is written as .The number of elements in set is written as , or .The empty set is the set with no members and is written as or .The universal set contains all elements involved in a particular problem. is a subset of if every member of is a member of and is written as , ie ‘ is a subset of ’. may also be equal to in this scenario, and we can therefore write .The complement of a set is the set of all elements in the universal set that are not in and is written as or .The intersection of sets and is the set of elements which are in both and and is written as , ie ‘ intersection’.The union of sets and is the set of elements which are in or or both and is written as , ie ‘ union ’. |
| Vertical line test | The vertical line test determines whether a relation or graph is a function. If a vertical line intersects or touches a graph at more than one point, then the graph is not a function. |
| Constant of variation (or proportionality) | The constant of variation (or proportionality) is the ratio between any two associated quantities. |
| Linear | Linear describes any function in the form .More informally, the points of a linear function plot a straight line on a Cartesian plane. |
| Parabola | A Parabola is a geometric shape where each point on the parabola is equidistant from the focus and the directrix.More informally, it describes the shape of the curve generated by (Quadratic) functions in the form . |
| Quadratic | Quadratic describes any function in the form , where , and are constants. |
| Turning point | A turning point describes a point on the curve where the function is neither increasing nor decreasing. The gradient of the curve at this point is zero. There are three types of turning point: a local maximum, a local minimum and a point of horizontal inflection. |
| Direct/inverse variation | Variation describes a one-to-one relationship between two quantities.There are two types of variation, direct and inverse.Two variables are in direct variation if one is a constant multiple of the other. This can be represented by the equation 𝑦=𝑘𝑥, where 𝑘 is the constant of variation (or proportionality). Also known as direct proportion, it produces a linear graph through the origin. Two variables are in inverse variation if the product of the variables is constant. This can be represented by the equation , where 𝑘 is the constant of variation (or proportionality). Also known as inverse proportion, it produces a hyperbola with the coordinate axes as asymptotes. |
| Vertex | The vertex describes the turning point of a parabola. |

Lesson sequence

| **Lesson sequence** | **Content****Students learn to:** | **Suggested teaching strategies and resources**  | **Date and initial** | **Comments, feedback, additional resources used** |
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| Functions, relations and notation(2 lessons) | **F1.2: Introduction to functions*** Define and use a function and a relation as mappings between sets, and as a rule or a formula that defines one variable quantity in terms of another
	+ define a relation as any set of ordered pairs of real numbers
	+ understand the formal definition of a function as a set of ordered pairs of real numbers such that no two ordered pairs have the same first component (or -component)
* Understand the concept of the graph of a function (ACMMM024)
* Identify types of functions and relations on a given domain, using a variety of methods
	+ know what is meant by one-to-one, one-to-many, many-to-one and many-to-many
	+ use the vertical line test to identify a function
	+ determine if a function is one-to-one (ACMSM094)
* recognise that solving the equation corresponds to finding the values of for which the graph of cuts the -axis (the -intercepts)
 | **Note to teacher:** Students who have covered the optional 5.3 substrands of Functions and Other Graphs in Stage 5 will be familiar with many elements of the content in F1.2.**Introduction to functions and relations*** Introduce the key terms that will be used throughout this topic:
	+ A relation is a set of ordered pairs where the variables and are related through some rule.
	+ A function is a special type of relation where for every input (independent variable) there is only one output (dependent variable) or for every value there is only one value.
	+ For the function , the variable is called the independent variable since it may be chosen freely within the domain of , while is called the dependent variable since its value depends on the value chosen for .
	+ A real function of a real variable assigns to each element of a given set of real numbers, exactly one real number , called the value of the function at . Thus is written as .
* Continuity and discontinuity are to be treated informally at this stage in this course.
* We say that is a function of for example - that .If this means that is the value of when . For example

IfWhen * Define the idea of one-to-one, one-to-many, many-to-one and many-to-many by using concrete materials in class, even the students themselves.
	+ One-to-one: Students pair up across a line on the ground
	+ One-to-many: Some students stand on their own, opposite them multiple students gather rather than one partner.
	+ Many-to-many: Students gather in small groups opposite each other
* A property of the graph of a function is that no two distinct points have the same -value. This means that every vertical line will cut the graph of a function in, at most, one point. If a vertical line cuts a graph in more than one point, the graph cannot represent a function. This is known as the vertical-line test. It may be of benefit to have a series of graphs on paper for students to experiment with the vertical line test. Rather than show them, give students the name of the test and see if they can extrapolate how to use it.
* Sketch the graph of a relation that is not a function.

**Introduction to intercepts** * Students should be familiar with the form of functions:
* Students should be aware that *c* represents the intercept of the graphs.
* It is also important to show the link between the content of this lesson and the previous algebraic concepts revised in F1.1, including solving quadratic equations through factorising, the quadratic formula and completing the square.
* Having previously been introduced to function notation, students can be shown that the solutions to these equations relate to the functions topic because they are theintercepts.
 |  |  |
| Domain and range(1 lesson) | * Use function notation, domain and range, independent and dependent variables (ACMMM023)
	+ understand and use interval notation as a way of representing domain and range, eg
 | **Introduction to the domain and range*** Introduce the idea of **domain** as the set of all real numbersfor which a function is defined.
* Introduce the idea of **range** as the set of real numbers (or ) given as varies for which a function is defined.
* The teaching of interval notation should be taught explicitly as it is a new feature in the syllabus and would not have been seen by students previously. For example, when talking about an interval between *a* and *b*:
	+ If the endpoints are included in the interval, square parentheses are used
	+ If the endpoints are not included in the interval, curved parentheses are used
	+ If a is included but b is not
	+ If b is included but a is not
* Students could complete a matching task to explore and compare various examples of interval notation. **Resource:** interval-notation-matching-task.DOCX
* When using interval notation for domain and range, some other considerations include:
	+ All real numbers (represented by ℝ), interval notation would be . Note that curved parentheses must be used as infinity cannot be reached.
	+ If the set has a lower bound of 1, and no upper bound, the interval notation would be .
* Find the domain and range of the functions:
	+ Domain: , range: .

Domain: , range: . |  |  |
| Working with linear functions (1 lesson) | **F1.3: Linear, quadratic and cubic functions*** Model, analyse and solve problems involving linear functions **AAM Paperclip icon**
	+ explain the geometrical significance of and in the equation
	+ derive the equation of a straight line passing through a fixed point ( and having a given gradient 𝑚 using the formula
* derive the equation of a straight line passing through two points and by first calculating its gradient using the formula
* recognise that solving the equation corresponds to finding the values of for which the graph of cuts the -axis (the -intercepts) **(F1.2)**
 | **Assumed knowledge*** Gradient Formula
* Parallel lines
* Perpendicular lines
* Gradient-Intercept form of an equation
* Application of the point-gradient formula

**Applications of linear functions formulae*** Students should already be familiar with the linear functions formulae from Stage 5. It will be important to review and apply these formulae.
* Review gradient-intercept form 𝑓(𝑥) = 𝑚𝑥 + 𝑐 using function notation if F1.3 is being taught prior to or with F1.2
* Review gradient formula and its use in determining the equation of the line passing through point (*x, y*), and passing through two points (𝑥1, 𝑦1) and (𝑥2, 𝑦2)
	+ 𝑦 − 𝑦1 = 𝑚(𝑥 − 𝑥1)
* Students should be exposed to a number of problems that require them to find the equation of a straight line given its gradient and y-intercept; one point and its gradient; or two points that lie on the line.
 |  |  |
| Solving problems involving linear functions(1 lesson) | * Model, analyse and solve problems involving linear functions **AAM Paperclip icon**
	+ understand and use the fact that parallel lines have the same gradient and that two lines with gradient 𝑚1 and 𝑚2 respectively are perpendicular if and only if
* find the equations of straight lines, including parallel and perpendicular lines, given sufficient information (ACMMM004)
 | **Parallel and perpendicular lines*** Explore and apply the gradient relationships for parallel and perpendicular lines.
	+ Parallel lines
	+ Perpendicular lines
* Students can use graphical software such as DESMOS to investigate a number of different straight line equations that satisfy these conditions to determine the gradient relationships for parallel and perpendicular lines. Teachers may scaffold this learning by providing them with a range of lines with different gradients to compare or students may choose their own sets of lines to compare.

**Further applications of linear functions** * Students should be exposed to a number of additional problems where different initial information is given. e.g. Find the equation of a straight line that passes through the point and is perpendicular to
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| Graphing parabolas (1 – 2 lessons) | * Model, analyse and solve problems involving quadratic functions **AAM Paperclip icon**
	+ recognise features of the graph of a quadratic, including its parabolic nature, turning point, axis of symmetry and intercepts (ACMMM007)
* find the vertex and intercepts of a quadratic graph by either factorising, completing the square or solving the quadratic equation as appropriate
* recognise that solving the equation corresponds to finding the values of for which the graph of cuts the -axis (the -intercepts) **(F1.2)**
 | **Features of the parabola*** Students should be familiar with factorising and solving quadratic equations using a variety of methods. These skills will form the foundation of exploring intercepts of a parabola.
* The **axis of symmetry** is found with the equation
* The **vertex** is found by substituting the axis of symmetry x-value into the equation of the parabola
* Discuss the vertex as a **turning point** of the parabola.

**Graphing the parabola*** Use the skills of factorising, finding the vertex, and finding the intercepts to graph various parabolas.
* Students may want to experiment with graphing technology to visualise the shape of the parabola. Informal discussion can also occur regarding transformations but this content will be covered formally in Year 12.
 |  |  |
|  | * Model, analyse and solve problems involving quadratic functions **AAM Paperclip icon**
	+ understand the role of the discriminant in relation to the position of the graph
* find the equation of a quadratic given sufficient information (ACMMM009)
 | **The discriminant*** Students have previously looked at factorising with the quadratic formula when solving quadratic equations in the form . It is important to link the discussion of the discriminant to this concept, so students understand how different elements of the course link together.
* Explicit teaching of the discriminant and its application is required: , including the following concepts:
	+ If , the quadratic equation has 2 real unequal (different) roots. If is a perfect square, the roots are rational and if it is not, the roots are irrational.
	+ If , the quadratic equation has 1 real or 2 equal roots.
	+ If , the quadratic equation has no real roots. If , the graph is concave down and negative definite, if , the graph is concave up and positive definite.
* Students could complete a familiarisation matching activity sorting cards into groups that include the picture of a graph, its parabolic equation, its discriminant value range and a description of the number and type of roots.

**The equation of the graph*** Students should also be exposed to problems that require them to find the equation of the parabola from a given with specific features graph.
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| Introducing cubic functions(1 lesson) | * Recognise cubic functions of the form: 𝑓(𝑥) = 𝑘𝑥3, 𝑓(𝑥) = 𝑘(𝑥 − 𝑏)3 + 𝑐 and 𝑓(𝑥) = 𝑘(𝑥 − 𝑎)(𝑥 − 𝑏)(𝑥 − 𝑐), where 𝑎, 𝑏, 𝑐 and 𝑘 are constants, from their equation and/or graph and identify important features of the graph
* recognise that solving the equation corresponds to finding the values of for which the graph of cuts the -axis (the -intercepts) **(F1.2)**
 | **Features of the cubic function*** as an odd function
* Make it clear to students that the basic cubic is flat at the origin (i.e. it is a horizontal point of inflexion). Students must make this clear when sketching all cubic functions.
* Connect the skills that students have developed through the study of quadratics that apply to cubic functions, i.e. the solutions to the equations become the intercepts.
* Students should use DESMOS or other graphing software to investigate transformations of the cubic curve of the form (𝑥) = (𝑥 − 𝑏)3 + 𝑐. The effects of:
	+ that dictates the steepness of the graph
	+ that dictates a shift left/right
	+ that dictates a shift up/down
* Students can also use DESMOS to explore cubics of the form (𝑥) = (𝑥 − 𝑎)(𝑥 − 𝑏)(𝑥 − 𝑐), which leads nicely into introducing the notion of polynomials in MA-F1.4.
 |  |  |
| Odd and even functions(1 lesson) | **F1.4: Further functions and relations*** given the graph of , sketch and and using reflections in the and -axes

**F1.2: Introduction to functions*** Define odd and even functions algebraically and recognise their geometric properties
 | **Investigating odd and even functions*** Introduce to students the ideas of

**Graphical transformations*** Discuss transformations of simple polynomials, hyperbolas and absolute value functions
* These transformations could be taught as each type of graph is explored. For example, when teaching absolute value graphs, the graphs of and and can be found at the same time.
* Graphing calculators, DESMOS or Geogebra can be used to show , , and on the same Cartesian plane
* Students to sketch each type of transformation for different types of graphs.
* Discuss the features of each, practice the substitution of values and allow students to notice the pattern when for some functions, for others , and for some, none of the three equal each other.
* Explain that for even functions and odd functions
* Show students images of various odd and even functions and discuss the geometrical features of line symmetry across the axis of symmetry for even graphs, and point symmetry around the origin for odd functions.
* A sound knowledge of odd and even functions assists in curve sketching, which is important in later topics.
* A function is even if for all values of in the domain. Its graph has line symmetry about the -axis.
* A function is odd if for all values of in the domain. Its graph has point symmetry about the origin.
* Students could investigate both the geometric and algebraic nature of odd and even functions.

**Examples*** Which of the following functions are even functions? Which are odd functions? Which are neither even nor odd? Justify your answers.
	+
	+
	+
	+
* For the functions in the list above, use the horizontal line test to determine which are one-to-one functions.
* A function has a domain of , and a range of . It is also known that is an even function. Draw a possible graph of .
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| Introduction to polynomials(2 lessons) | * define a real polynomial as the expression where and are real numbers
* identify the coefficients and the degree of a polynomial (ACMMM015)
* identify the shape and features of graphs of polynomial functions of any degree in factored form and sketch their graphs
* recognise that solving the equation corresponds to finding the values of for which the graph of cuts the -axis (the -intercepts) **(F1.2)**
 | **Introduction to and features of polynomials*** Make explicit that linear, quadratic and cubic functions are all types of polynomials
* Use [DESMOS](https://www.desmos.com/calculator) graphing calculator to show how the graphs change as the degree of a polynomial changes
* Provide students with clues or features for them to determine the polynomial
	+ What is the polynomial of degree 3 which has 4 as the coefficient of and , a coefficient of -8 for and no constant term?
	+ Write down a polynomial that has five terms, positive, even coefficients and a constant of -6.
* Sketch a variety of graphs by identifying x and y-intercepts from factored form
* Students to sketch ; and compare and contrast their features. **Resource:** sketching-polynomials.DOCX
 |  |  |
| Hyperbolas(1 lesson) | * recognise that functions of the form represent inverse variation, identify the hyperbolic shape of their graphs and identify their asymptotes AAM
 | **Investigating hyperbolic functions*** Asymptotes: values of *x* and *y* that cannot exist
	+ The line that the graph approaches but never touches
	+ Only address functions where andhere
* Briefly show students for comparison
* Use [DESMOS](https://www.desmos.com/calculator) graphing calculator to show the rate of increase and decrease changes as *k* changes. Use it to show how the graph approaches the horizontal and vertical asymptotes.
* **Note:** Further transformations of hyperbolas will be studied in MA-F2.
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| Direct and inverse variation(1 lesson) | **F1.3: Linear, quadratic and cubic functions*** Model, analyse and solve problems involving linear functions **AAM Paperclip icon**
* recognise that a direct variation relationship produces a straight-line graph

**F1.4: Further functions and relations*** recognise that functions of the form represent inverse variation, identify the hyperbolic shape of their graphs and identify their asymptotes **AAM** **Paperclip icon**  Information and communication technology capability icon
 | **Direct variation*** Direct variation relationships are in the form , whereis a constant of variation/proportionality.
	+ If increases, then increases
	+ If decreases, then decreases
	+ If is doubled, is doubled
	+ If is halved, is halved
* Direct variation has numerous applications to physical phenomena, including the relationship between distance and time, fuel consumption and distance travelled, wages earned and hours worked
* Students could investigate an instance of direct variation in the real world. They should collect their own set of values, find the constant of proportionality and graph the variation.

**Inverse variation** * Inverse variation where k is the constant of variation/proportionality:
	+ If increases, then decreases
	+ If decreases, then increases
 |  |  |
| Solving simultaneous equations(1 – 2 lessons) | * Solve practical problems involving a pair of simultaneous linear and/or quadratic functions algebraically and graphically, with or without the aid of technology; including determining and interpreting the break-even point of a simple business problem **AAM Paperclip icon**
	+ understand that solving 𝑓(𝑥) = 𝑘 corresponds to finding the values of 𝑥 for which the graph 𝑦 = 𝑓(𝑥) cuts the line 𝑦 = 𝑘
 | **Assumed knowledge*** Students should be familiar with solving simultaneous equations from Stage 5 and F1.1

**Introduction to simultaneous equations*** Introduce or reinforce the idea that a mathematical model is when you begin with a set of data and then find the rule that links the data together.

**Solving simultaneous equations graphically** * This area should have a focus on the practical applications of simultaneous equation problems including areas such as profit/loss calculations.
* As with direct variation, this is a good topic for students to investigate an instance of simultaneous equations/intersection of lines in the real world. They can collect their own sets of values, and graph/solve their resulting equations.

**Solving simultaneous equations algebraically*** Both the substitution and elimination methods should be taught to students with a focus on what method is appropriate to use in different situations. These methods should also be explicitly linked to the graphical solutions.
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| Absolute values(1 lesson) | * define the absolute value of a real number as the distance of the number from the origin on a number line without regard to its sign
* use and apply the notation || for the absolute value of the real number and the graph of y = |x| (ACMSM098)
* recognise the shape and features of the graph of and hence sketch the graph
 | **Investigating absolute value functions*** Use the explicit definition: the absolute value of a real number is the distance of the number from the origin on a number line without regard to its sign
* Perform calculations that involve the absolute sign with numerical values
* Considerations/class discussion:
	+ Vertex is found using
	+ The gradient dictates the steepness of the arms
	+ The graph should be symmetrical on either side of the vertex
* Show various absolute value graphs using various sources such as [Geogebra](https://www.geogebra.org/m/A3K6eH8z) or DESMOS
* Compare graphs of with using graphing software such as DESMOS, inserting sliders for a and b
 |  |  |
| Absolute value equations(1 lesson) | * solve simple absolute value equations of the form both algebraically and graphically
 | **Solving absolute value equations*** When solving algebraically, students must consider both cases and . Once this is solved, they should also make sure to check if the solutions exist by using substitution.
* When solving graphically, students should sketch the two graphs and , where the solution to x are the x-coordinates of the point(s) of intersection.
* Students to solve various equations both algebraically and graphically and compare and contrast the advantages and disadvantages of each method. **Resource:** absolute-value-equations.DOCX
* Provide students with an equation to solve and have them explain why they would choose one method over the other.
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| Deriving the equation of a circle(1 lesson) | * recognise features of the graphs of and , including their circular shapes, their centres and their radii (ACMMM020) Critical and creative thinking icon  Information and communication technology capability icon
	+ recognise that and are functions, identify the semicircular shape of their graphs and sketch them
	+ derive the equation of a circle, centre the origin, by considering Pythagoras’ theorem and recognise that a circle is not a function
* sketch circles given their equations and find the equation of a circle from its graph
 | **Establish the circle as a relationship, not a function*** Students need to discuss the reasons why the circle is not a function, using the vertical line test.
* Students need to establish that semi-circles in the form and are functions.

**Derive the equation of a circle*** Start by defining a general point in the first quadrant, where is a fixed length, ie)
* Using Pythagoras’ Theorem determine the relationship that defines , ie)
* Use this [Geogebra activity](https://www.geogebra.org/m/b4zstn4g) to define the locus of points defined by a fixed distance from the origin.

**Sketching Circle Relationships from an Equation*** Use this [Geogebra activity](https://www.geogebra.org/m/k9ubeyss) to investigate the effect of changing certain parameters in the equation has on the sketch of the circle.
* Students need to explain their findings and use it to sketch equations of the form

where students need to clearly show the centre of the circle and the extremities of the circle.* Students need to be able to interpret a sketch of a circle to determine its equation for example

A circle shown on a cartesian plane with radius equal to 4 and centred at the point (3, -1)By determining its radius from the extremities and its centre, giving  |  |  |
| Determining the equation of a circle by completing the square(1 lesson) | * recognise features of the graphs of and , including their circular shapes, their centres and their radii (ACMMM020) Critical and creative thinking icon  Information and communication technology capability icon
	+ form into the form
	+ , by completing the square
 | **Completing the square review*** Students need to review and build on the technique of completing the square, established during Stage 5. Staff may like to use this [activity from mathcentre.ac.uk](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-completingsquare2-2009-1.pdf).

**Using the complete the square technique to determine the standard equation for the circle*** Students may need to use the completing the square technique twice, for and variables, to determine the standard equation for the circle. This [activity from freemathhelp.com](https://www.freemathhelp.com/complete-square-circles.html) provides a scaffold and example to demonstrate this.
* Students need to be able to interpret the equation to find the centre and radius of the circle.
 |  |  |
| Further domain and range of functions(2 lessons) | * Define the sum, difference, product and quotient of functions and consider their domains and ranges where possible
 | **Note:** The sum, difference, product and quotient of functions in this section of the sub-topic are algebraic calculations rather than graphical representations. **Sum, difference, product and quotient of functions*** Students are first introduced to the notation of f(x) and g(x) being two separate functions. To introduce the idea of the addition and subtraction, give two simple functions such as: and noting that
* Note: Even though students are not required to graph their functions, giving a visual representation of these introductory functions may help some students understand the work.
* Once students have performed various operations on functions, they will then apply their previous knowledge of domain and range to the sum, difference, product and quotient of various functions.
* Students could investigate whether sums, differences, products and quotients of odd (or even) functions are themselves odd (or even).
 |  |  |
| Composite functions(2 lessons) | * define and use the composite function of functions and where appropriate
* identify the domain and range of a composite function
 | **Working with composite functions*** Work on composite functions could include finding the composite of the same function applied twice. This should be notated as .
* When introducing composite functions, it is again important to start with simple functions that they are familiar with the process. For example: If and then
* Students should be taught to make a judgement about whether the final answer should be expanded. For example:
	+ and

Therefore In this example, it is not necessary to expand due to the higher degree of the power.* Once students have performed various operations on functions, they will then apply their previous knowledge of domain and range to the new composite function.
 |  |  |

Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All information and communication technologies (ICT), literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.