 Modelling home loans

Consider a home loan of $450000 to be repaid over 25 years with an interest rate of 6% p.a. compounded monthly. This is 0.5% per month.

Define $B\_{n}$ as the balance owing after n periods and m as the monthly repayment.

**Example:**

1. **Calculate the repayment:**

$$B\_{0}=450000$$

$$B\_{1}=450000×1.005-m$$

$$B\_{2}=\left(450000×1.005-m\right)×1.005-m=450000×1.005^{2}-m(1.005+1)$$

$$B\_{3}=\left(450000×1.005^{2}-m\left(1.005+1\right)\right)×1.005-m=450000×1.005^{3}-m(1.005^{2}+1.005+1)$$

$$…$$

$$B\_{n}=450000×1.005^{n}-m(1.005^{n-1}+1.005^{n-1}+...+1.005^{2}+1.005+1)$$

or

$$B\_{n}=450000×1.005^{n}-m(1+1.005+1.005^{2}+...+1.005^{n-2}+1.005^{n-1})$$

To calculate the size of the repayment, recognise that the balance owing is $0 after 25 years or 300 months (n)

$$B\_{300}=0=450000×1.005^{300}-m(1+1.005+1.005^{2}+...+1.005^{300-2}+1.005^{300-1})$$

The brackets form a geometric series with a first term of 1 and a common ratio of 1.005.

$$0=450000×1.005^{300}-m×\frac{1(1.005^{300}-1)}{1.005-1}$$

$$m×\frac{1(1.005^{300}-1)}{1.005-1}=450000×1.005^{300}$$

$$m=450000×1.005^{300}÷\frac{1(1.005^{300}-1)}{1.005-1}$$

$$m=\$2899.36 (nearest cent)$$

1. **Calculate the total amount repaid:**

$$Total amount repaid = \$2899.36×300=\$869 808$$

1. **Calculate the total interest paid:**

$$Total interest paid = \$869 808-\$450 000=\$419 808$$