 Arithmetic sequences and series

The activities include ideas for questions and sample questions.

Part 1: The nth term of an arithmetic sequence

1. Find a specific term in the sequence given values for a and d.

Match the information to the value of the term:

|  |  |
| --- | --- |
| Find $T\_{5}$ given $a = 4, d = 8$ | 98 |
| Find $T\_{15}$ given $a = 3, d = 7$ | 19 |
| Find $T\_{8}$ given $a = 5, d = 2$ | 36 |
| Find $T\_{9}$ given $a = 26, d = 9$ | 98 |
| Find $T\_{21}$ given $a = 400, d = -15$ | 30 |
| Find $T\_{12}$ given $a = 151, d = -11$ | 100 |

Solution:

|  |  |
| --- | --- |
| Find $T\_{5}$ given $a = 4, d = 8$ | 36 |
| Find $T\_{15}$ given $a = 3, d = 7$ | 101 |
| Find $T\_{8}$ given $a = 5, d = 2$ | 19 |
| Find $T\_{9}$ given $a = 26, d = 9$ | 98 |
| Find $T\_{21}$ given $a = 400, d = -15$ | 100 |
| Find $T\_{12}$ given $a = 151, d = -11$ | 30 |

1. Given the finite arithmetic sequence or series, find the number of terms.
2. $5, 11, 17, …71$
3. $44, 42, 40, …, 12$
4. $4 + 7 + 10 + … + 130$
5. $99 + 93 + 87 + … + 39$
6. Write each of the arithmetic series from question 2 using sigma notation.
7. Given the two terms of the arithmetic series, find a and d.
8. $T\_{5}=11$ and $T\_{11}=35$
9. $T\_{3}=191$ and $T\_{20}=89$
10. $T\_{14}=11$ and $T\_{50}=101$
11. Find the first term in the sequence 3, 11, 19… that is more than 500
12. Find the first term in the sequence 300, 284, 268… that is less than 50

Part 2: The sum of an arithmetic sequence.

1. Find the sum of the first 20 terms of the sequence 7, 10, 13, …
2. Find the sum of the first 30 terms of the sequence 100, 97, 94,…
3. Evaluate $\sum\_{n=1}^{16}7+(n-1)×2$
4. Find the sum of:
5. $5, 11, 17, …71$
6. $99 + 93 + 87 + … + 39$
7. Evaluate the sum of the first 50 terms given $T\_{1}=11$ and $T\_{50}=109$
8. Evaluate the sum of the first 35 terms given $T\_{5}=11$ and $T\_{11}=35$

Note: Questions could be completed using either formulae for the sum of an arithmetic sequence.

Part 3: Problems involving arithmetic sequence

Samples problems include:

1. $1000 is invested in an investment account paying 4% p.a. simple interest.
2. Model the balance of the investment account using an arithmetic sequence.
3. What will the balance be in 20 years?
4. After how many years will the balance be $2000?
5. Model the investment balance using a spreadsheet, graph the balance verse time and use the graph to confirm the answers to parts b) and c).
6. A trust account is established. It has an initial balance of $10000. Every year $350 is withdrawn.
7. Model the balance of the account using an arithmetic sequence.
8. What is the balance after 8 years?
9. After how many years will the balance be zero?
10. Use a spreadsheet to graph the balance verse time?
11. Is the sum of the arithmetic sequence contextually relevant?
12. A student decides to start saving for a new phone which costs $1000. Currently the student has $140 in their piggy bank. The student decides to save $40 every week.
13. Model the balance of the piggy bank using an arithmetic sequence.
14. How much will the student have after 10 weeks?
15. After how many weeks will they be able to afford the new phone?
16. Could this be modelled by considering the balance still to be saved?
17. A child has 250 blocks and wishes to build a tower. The top row is to contain 1 block, the second top row 3 blocks, then 5 blocks etc.
18. Model the number of blocks in each row using an arithmetic series.
19. What is the tallest tower that the child can build?
20. How many blocks should be in the bottom row?
21. A school is setting up a hall for an end of year presentation. The first row contains 50 seats, the second row 53 seats, the third 56 seats and so on.
22. Model the number of seats in each row using an arithmetic series.
23. How many seats will be in the 13th row?
24. Which row would obtain 371 seats?
25. How many rows are required to seat 1800 people?
26. A ball is dropped from the world’s tallest building which is approximately 830m tall. During the first second the ball falls 4.9m, during the next 14.7m, then 24.5m and so on.
27. Model the distance fallen by the ball each second using an arithmetic sequence.
28. How many metres will the ball fall in the 10th second?
29. What will be the height of the ball be after 10 seconds?
30. After how many seconds will the ball strike the ground?