 Applications of integral calculus

Definite integrals

The study of calculus is important to many commercial fields including biomathematics, economics, engineering and the construction industry.

Application 1 – economics

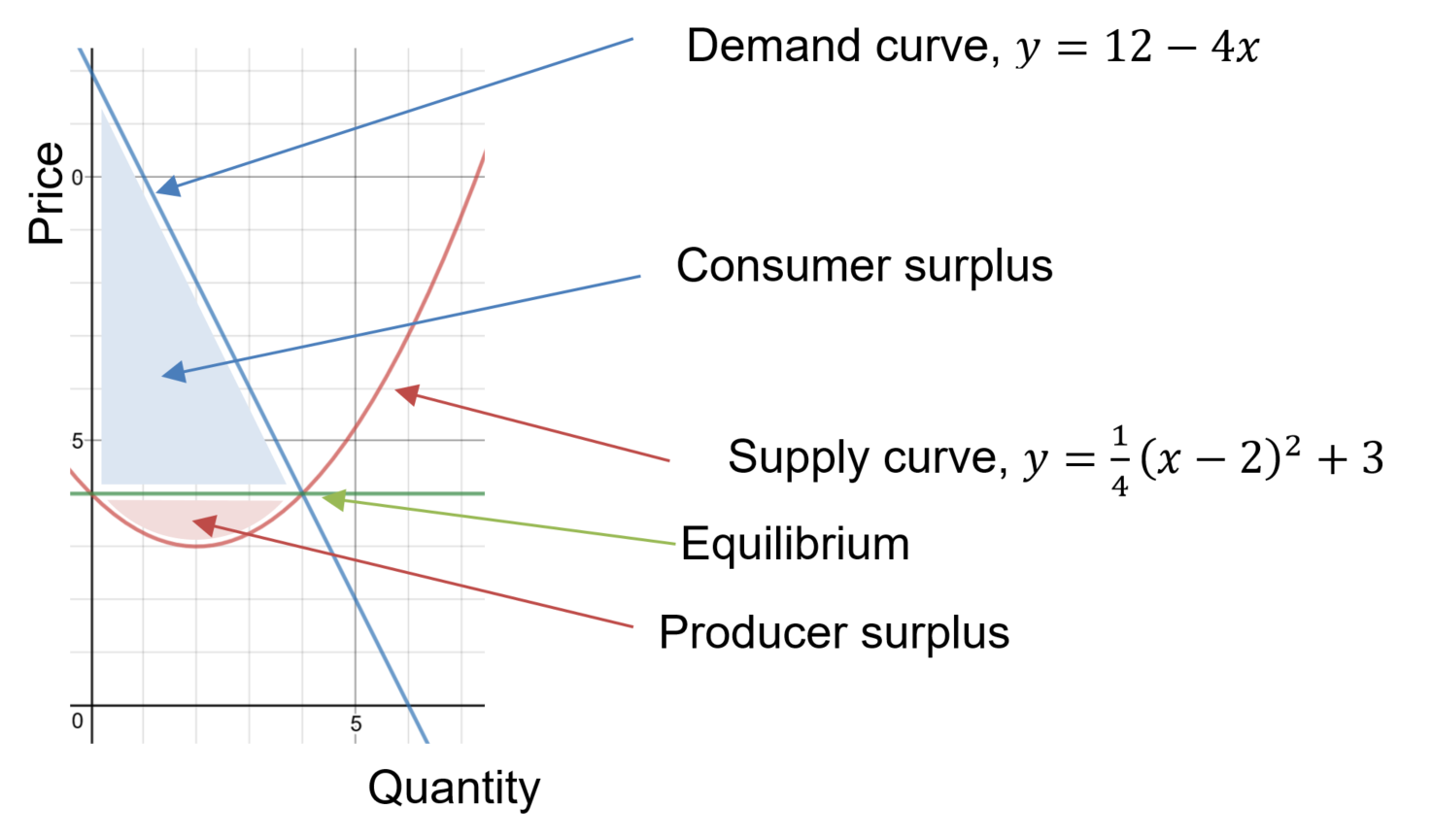
Producer and consumer surplus

A producer surplus is achieved when a producer sells a product for more than the minimum price they were willing to accept. For example, if a producer is willing to sell an item for but achieve a sale of , the producer surplus is .

A consumer surplus is achieve when a consumer purchases a product for less than the maximum price they were willing to pay. For example, if the consumer was willing to purchase an item for , but manages to purchase it for , the consumer surplus is .

The sum of the two surpluses is the overall economic surplus, in our scenario,

Assuming that a product is sold for an equilibrium, where supply and demand are equal, this can be graphically represented as



Activity: Calculate the size of the producer and consumer surpluses if the diagram shown.

Application 2 – average value of a function

The average value of a function can be calculated by:

An application of the average value of a function is the head injury criterion.

Activity: Calculate the average temperature for an exponential cooling model or average population for an exponential growth model.

Application 3 – work

The work done when a constant force is applied over a distance is given by

When the force required is not constant, such as when compressing a spring, integrals can be used to calculate the work done.

Hooke’s law for springs: The force required to compress a spring is directly proportional to the length of compression, x, from its normal length.

Sample questions are presented on [Interactive Mathematics](https://www.intmath.com/applications-integration/7-work-variable-force.php).

Application 4 – braking distance of a car

An average passenger car brakes produce a deceleration force of

The velocity (m/s) of a car, t seconds after the brakes are applied is given by

Where is the initial velocity.

Activities

* Assuming it takes an average driver 1.5 seconds to react, calculate the stopping distances of a standard car for a range of speeds. Check your answers at the [Queensland government, stopping distance](https://www.qld.gov.au/transport/safety/road-safety/driving-safely/stopping-distances) webpage.
* A racing car produces a braking force of approximately . Calculate the stopping distances of a racing car for a range of speeds.
* Refer to wet stopping distance shown on the Queensland government, stopping distance webpage. Calculate the average braking force during wet weather.