 Year 12 Mathematics Standard 1

| MS-M3 Right-angled triangles | Unit duration |
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| Measurement involves the application of knowledge, skills and understanding of numbers and geometry to quantify and solve problems in practical situations.Knowledge of measurement enables an understanding of basic daily situations involving rates and ratios, such as speed and the interpretation of maps and plans, effectively in a variety of situations.Study of measurement is important in developing students’ ability to solve problems related to two-dimensional and three-dimensional models and representations and to work effectively with a variety of rates and ratios. | 3 weeks |

| Subtopic focus | Outcomes |
| --- | --- |
| The principal focus of this subtopic is to solve problems involving right-angled triangles in a range of practical contexts using Pythagoras’ theorem and basic trigonometric ratios.Students develop their ability to justify mathematical thinking and to communicate solutions.Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students. | A student:* interprets the results of measurements and calculations and makes judgements about their reasonableness MS1-12-3
* analyses simple two-dimensional and three-dimensional models to solve practical problems MS1-12-4
* chooses and uses appropriate technology effectively and recognises appropriate times for such use MS1-12-9
* uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others MS1-12-10
* Related Life Skills outcomes: MALS6-3, MALS6-4, MALS6-13, MALS6-14
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| Prerequisite knowledge | Assessment strategies |
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| Students need to build on the prior knowledge of Pythagoras’ Theorem and trigonometric ratios developed in Stage 5 and the Year 11 topics of MS-M1 applications of Measurement and MS-M2 Working with Time. | Summative Assessment: Investigative Task |

All outcomes referred to in this unit come from [Mathematics Standard Stage 6](https://syllabus.nesa.nsw.edu.au/mathematics-standard-stage6/) Syllabus
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Glossary of terms

| Terms  | Description  |
| --- | --- |
| angle of depression  | When an observer looks at an object that is lower than 'the eye of the observer', the angle between the line of sight and the horizontal is called the angle of depression. C:\Users\mrodda\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\54253CA3.tmp  |
| angle of elevation  | When an observer looks at an object that is higher than 'the eye of the observer', the angle between the line of sight and the horizontal is called the angle of elevation. C:\Users\mrodda\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\D72101A9.tmp  |
| compass bearing  | Compass bearings are specified as angles either side of north or south. For example, a compass bearing of N500E is found by facing north and moving through an angle of 500 to the East.  |
| true bearing  | True bearings are measured in degrees clockwise from true north and are written with three digits being used to specify the direction. For example, the direction of north is specified 0000, east is specified as 0900, south is specified as 1800 and north-west is specified as 3150.   |

| **Sequence** | **Content** | **Suggested teaching strategies and resources**  | **Date and initial** | **Comments, feedback, additional resources used** |
| --- | --- | --- | --- | --- |
| Practising Pythagoras’ Theorem(2 lessons) | * Review the application of Pythagoras’ theorem to solve practical problems in two dimensions **AAM Paperclip icon** Critical and creative thinking icon Literacy icon
 | Review of Pythagoras’ Theorem* Teachers should begin by verifying students’ understanding of Pythagoras’ theorem. Teacher could use the [Pythagoras’ Theorem Calculator](https://www.geogebra.org/m/Nw7wsSD5) to generate right-angled triangles to practise Pythagoras’ theorem, with the option of showing fully worked solutions
* Student activity: Students practise solving Pythagoras problems by completing the [Pythagoras Treasure Hunt](https://www.tes.com/en-au/teaching-resource/differentiated-pythagoras-treasure-hunts-6296687)
* Problems are placed on the walls around the room using blu-tak
* Students begin at any problem and solve it
* They then find the answer on another poster around the room and then solve its problem etc.
* They should end up back where they started.
* Student activity: Students practise solving Pythagoras problems by completing the [Pythagoras Pile up](https://www.tes.com/en-au/teaching-resource/pythagoras-pile-up-11270615). A set of triangles are stacked on top of each other - the aim is to find the length of the top line using Pythagoras repeatedly.

Solving practical problems* Student activity: Students could be asked to find the diagonal length of a range of everyday objects, before verifying their answer through measurement. For example, ask the students to measure the diagonal length of their table or the teacher’s table.
* They should also be given the opportunity to work backwards and investigate the dimensions of a variety of objects, for example. What could be the dimensions of a 65-in (≈164 cm) television?
* Students may find some of the terminology encountered in word problems involving Pythagoras’ theorem difficult to interpret, for example, ‘foot of a ladder’, ‘inclined’, ‘guy wire’, ‘wire stay’, ‘vertical’, ‘horizontal’. Teachers should provide students with a variety of word problems and explain such terms explicitly.
* Student activity: Students could be given some word problems to work through as groups and encouraged to decode the problem (students will need to read a problem and reconstruct it in their own words to demonstrate understanding). They should then find the solution to the problem and share it using strategies such as think-pair-share, justifying their approach to the problem, when there is a disagreement.
* Student activity: Use the [Spider and Fly](https://www.youtube.com/watch?v=tfHpsW6LBtE) problem to review Pythagoras’ Theorem.
* Show students the start of the video and give them time to investigate before showing them the rest of the video which explains the solution. (Hint: students need to investigate the different ways they can create a net of the room)
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| Using trigonometric ratios(5 lessons) | * Review and extend the use of trigonometric ratios (sin, cos, tan) to solve practical problems **AAM** Paperclip icon
* Work with angles correct to the nearest degree and/or minute
 | **Review of Trigonometric Ratios – Finding the length of sides*** Student activity: The teacher should give students an opportunity to practise using triangle terminology by asking them to complete the information gap activity.
* In pairs, each student will be given a handout with four diagrams and four empty boxes. Students should not show their handout to their partner.
* They must take it in turns to describe their diagram to the partner to help them draw it exactly. In doing this, they must take into account the orientation of the triangle (For example, the right angle is at the top left corner) and guide their partners to label all the information in their handout**Resource:** information-gap-activity.DOCX
* Teachers should begin by reviewing students’ understanding of trigonometric ratios. The must allocate some time to teaching the following content before delving into practical problems:
* Labelling the three sides of a right-angled triangle as opposite, adjacent and hypotenuse with reference to a chosen angle.
* Writing trigonometric ratios for a given right-angled triangle and drawing a right-angled triangle for a given trigonometric ratio.
* Using a calculator to operate with trigonometric ratios.
* Students should learn to verify if their calculator is in the degrees mode and to change their calculator to degrees mode.
* Student activity: [Exploring Trigonometry](http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.TRIG&lesson=html/video_interactives/trigonometry/trigonometrySmall.html) is a video explaining how this mathematics concept is used at airports to ensure the safe landing of airplanes.
* Using an interactive component, students then practise labelling the sides of triangles using the terminology – opposite, adjacent and hypotenuse, before exploring the use of the trigonometric ratios: sine, cosine, and tangent. A trigonometry print activity is included.
* The teacher should review finding the length of unknown sides using trigonometric ratios. This should increase in difficulty as suggested:
* finding unknown sides in the numerator
* finding unknown sides in the denominator
* miscellaneous questions, for example multiple applications of trigonometric ratios

NESA exemplar question* What is the length of the side $MN$ in the following triangle, correct to two decimal places?

The diagram shows triangle LMN that is right-angled at L. NL is 12 cm long and angle M is 50 degrees.**Resource:** ms-m3-nesa-exemplar-question-solutions.DOCX**Review of Trigonometric Ratios – Finding the size of angles*** The teacher should then reintroduce students to degrees, minutes and seconds as units for measuring angles to varying degrees of accuracy and teach them rounding angles and calculator usage. Students should practice rounding angles (to the nearest degree and to the nearest minute only) and evaluating expressions involving trigonometric ratios and inverse trigonometric ratios.
* Students may find some of the terminology encountered in word problems involving trigonometry difficult to interpret, for example ‘base/foot of the mountain’, ‘directly overhead’, ‘pitch of a roof’, ‘inclination of a ladder’. Students must therefore solve a variety of worded problems to apply their knowledge of trigonometric ratios. The teachers should teach students to decode worded problems and to translate it into a labelled diagram.
* Student activity: [Trigonometry Pile Up](http://www.greatmathsteachingideas.com/2012/03/12/trigonometry-pile-up/) is a great practise activity for the right-angled trigonometric ratios and Pythagoras Theorem.
* Teachers could have different groups of students rounding off to different numbers of decimal places and then comparing their final answers to teach the importance of not rounding off too early.
* Student activity: Students practise labelling the sides of a triangle, choosing the appropriate ratio and then solving the problem by playing [Mini Golf](http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.TRIG&lesson=html/object_interactives/trigonometry/use_it.html)

**Solving practical Trigonometry problems*** Student activity: Students look at blood spatter to determine the angle of impact
* Teachers to read the article [bloodstain-pattern-analysis](https://science.howstuffworks.com/bloodstain-pattern-analysis3.htm) with students.
* Students work through the worksheet [blood spatter](https://www.yumpu.com/en/document/view/22869546/fsb-09-blood-spatter-worksheets-pdf-file-2632-kb/3)
* Student activity: Students can investigate the angle of the Leaning Tower of Pisa using the video [Applying Trigonometry - Leaning Tower](http://education.abc.net.au/home#!/media/154926/)
* Students watch the Leaning Tower video
* Draw the triangle made between the tower, the ground and the falling object. How far from the tower should the melons hit the ground? Use a trigonometric ratio to work out the distance to the nearest centimetre.
* Design a tower where the dropped object will fall exactly 100 m vertically and land exactly 10 m from the base of the tower. Include all measurements and angles.
* Student activity: Students investigate [Are lasers accurate enough to track space debris?](http://education.abc.net.au/home#!/media/86394/)
* Watch video about lasers and space debris
* Let's say the radar beam had an accuracy of 1 degree and a target was 200 km away. Two right-angled triangles could be drawn, each with a base of 200 km and an angle of 0.5 degrees. The spread of the beam at 200 km would be: 2 x 200 tan(0.5) = 3.4907 km
* Laser beams may have an accuracy of 0.002 degrees. What would the spread of the laser beam be (in metres) at 200 km?
* How accurate are you? Find a wall and draw a target on it in chalk. Stand 20 m away and throw a wet tennis ball at the target. Measure the horizontal and vertical distances of spread. Use trigonometry to find your own degree of accuracy horizontally and vertically. Ask others to do the same, and compare your accuracy.
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| Introduction to bearings(3 lessons) | * Understand various navigational methods
* Understand the difference between compass and true bearings
* Investigate navigational methods used by different cultures, including but not limited to those of Aboriginal and Torres Strait Islander Peoples Aboriginal and Torres Strait Islander histories and cultures icon Critical and creative thinking icon
 | **Introduction to bearings*** Students should be able to interpret a variety of phrases involving bearings. They should be taught explicitly how to identify the location from where a bearing is measured and to draw the centre of the compass rose at this location on a diagram.
* When practising questions that reference a path involving more than one bearing, students may need to be explicitly shown to look for words such as ‘after this’, ‘then’ and ‘changes direction’ that indicate a change of bearing. A new compass rose needs to be centred on the location of each change in direction.
* Teachers should begin by ensuring that students know the four cardinal directions (North, East, South, West) and the four intercardinal directions (North West, North East, South West, South East) in the correct order. Teachers should help students to recognise the angles between the directions and demonstrate that the intercardinal locations are located between cardinal directions.
* Teachers should then teach true bearings as the number of degrees in a clockwise direction from North.
* Student activity: Students would benefit from an outdoor learning activity where they measure the true bearings of important locations around the school using a magnetic compass.
* It may be useful to take a map of the local area and use it to identify the bearings between locations.
* Students should be allowed to practise a variety of questions where they find the true bearings from A to B and B to A.
* Student activity: Students could develop an orienteering course and/or complete an orienteering course around the school
* Student activity: Students could also measure the true bearing between important landmarks or places of interest using Google Earth.
* They will need to use the [ruler tool in ‘Google Earth’](https://www.youtube.com/watch?v=x8hzpMn-nNs) and look at the ‘heading’ feature
* Students will need assistance in deconstructing worded problems and would benefit from participating in an information gap activity where one student must describe a bearing diagram to another student in order to help them draw it.
* Teachers should then teach compass bearings as the number of degrees east or west of the north-south line. Students would benefit from practising a variety of questions on the board under step-by-step teacher guidance.

NESA Exemplar question* Students should have opportunities to translate a variety of phrases involving bearings into diagrams, for example:
* The bearing of Melbourne from Sydney is 230°.
* A plane flies to Melbourne on a bearing of 230°.
* A plane flies from Sydney to Melbourne on a bearing of 230°.
* A plane leaves Sydney and flies on a bearing of 230° to Melbourne.

Resource: ms-m3-nesa-exemplar-question-solutions.DOCXNavigation Methods* Teachers may also wish to discuss other methods of describing direction such as clock positions and port and starboard, which are used in aviation and maritime navigation.
* Student activity: Students can identify the use of Aboriginal navigational methods in the past using the article titled [How ancient Aboriginal star maps have shaped Australia’s highway network](https://theconversation.com/how-ancient-aboriginal-star-maps-have-shaped-australias-highway-network-55952) Additional Resources:
* [Songlines and navigation in Wardaman and other Aboriginal cultures](http://www.atnf.csiro.au/people/Ray.Norris/papers/n315.pdf)
* [Songlines: the Indigenous memory code](http://www.abc.net.au/radionational/programs/allinthemind/songlines-indigenous-memory-code/7581788)
* Teachers should then work with students to investigate navigational methods used by at least one other culture (for example, [Polynesian navigation](http://annex.exploratorium.edu/neverlost/#/home)). Teachers may wish to explore options for cross-curricular learning by collaborating with History, Science and Aboriginal Studies faculties.
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| Solving practical problems involving trigonometric ratios(2 lessons) | * Solve practical problems involving angles of elevation and depression and bearings **AAM Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon Literacy icon
* Convert between compass and true bearings, for example, convert N35°W into a true bearing
 | * Students should be explicitly taught the meaning of the phrases ‘angle of elevation’ and ‘angle of depression’ with the aid of diagrams.
* Student activity: Students could measure the heights of buildings or other structures using a clinometer. [Make a Clinometer](https://www.wikihow.com/Make-a-Clinometer)
* They should be asked to measure the angle of elevation and depression of a variety of objects from a partially elevated position.
* Students should be asked to observe any changes in the angle of elevation and depression as they walk closer to/further away from the base of the object.
* Student activity: To help students recognise that angle of elevation equals to the angle of depression, students should be grouped in pairs and asked to identify the bearing of each other from different locations.
* Students should then be given the opportunity to practise a variety of problems, guided by the teacher when deconstructing complex worded problems.
* At the end of this section, teachers can extend their capable students with the following challenge that requires them to apply their knowledge of trigonometric ratios and arc length: [Far Horizon](https://nrich.maths.org/2357)
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.