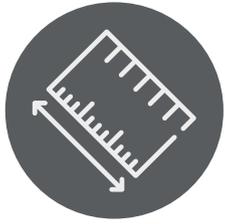


# Teaching Measurement

## STAGE 4

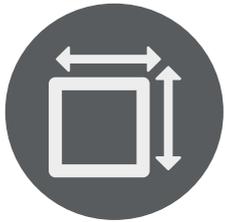


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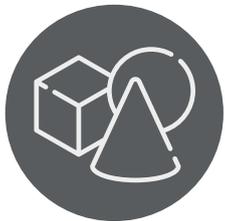
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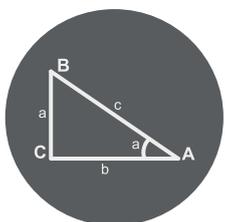
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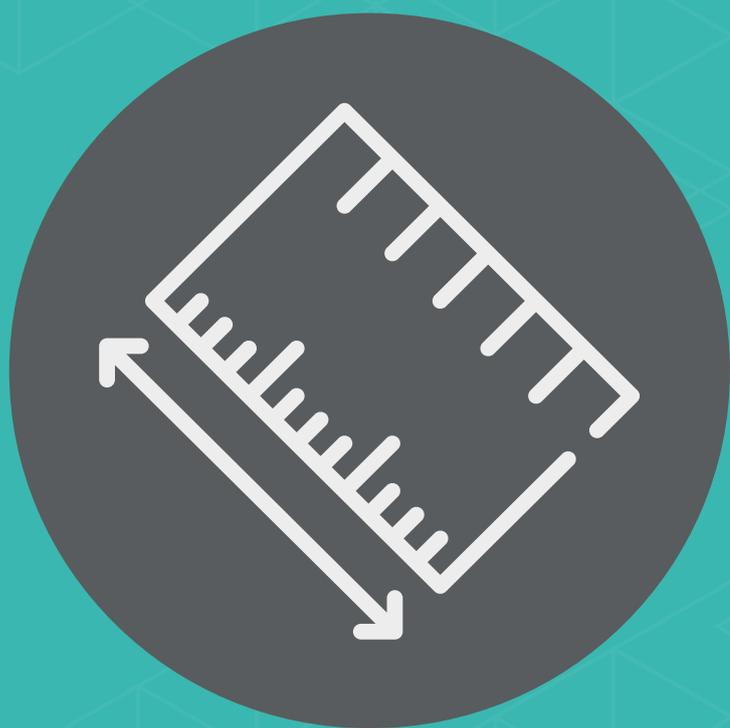


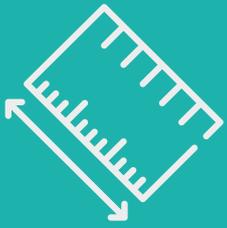
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# Teaching Measurement LENGTH





# LESSON 1

## Estimation and measuring devices

### Overview of lesson

Students will have the opportunity to practise their measurement skills using a range of measuring devices. This lesson reinforces the importance of estimation and introduces students to error in measurement.

### Background knowledge

Students should understand the concept of length being the measurement of words an object from one end to the other end.

---

### Language considerations

Students should be able to communicate using the following language: length, distance, kilometre, metre, centimetre, millimetre, measure, measuring device, ruler, tape measure, trundle wheel, and estimate.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA3-9MG selects and uses the appropriate unit and device to measure lengths and distances, calculates perimeters, and converts between units of length.

---

### Syllabus content

#### Students:

- selects and uses the appropriate unit and device to measure lengths and distances.
- describe how a length or distance was estimated and measured.
- question and explain why two students may obtain different measures for the same length, distance or perimeter.

## Resources needed

Enough measuring devices for each student (preferably the same number of each type)

- Trundle wheel
  - Retractable tape measure (at least 10 metres)
  - Sewing tape measure
  - Metre ruler
  - 30cm ruler
- 

## Teacher instructions

### Step 1

- Give each student a copy of the **Class Height Recording Worksheet**.
- A student comes up to the front of the class. Students record their name and an estimate of their height on the recording sheet. The student's height is measured, this measurement is recorded and the difference between the actual and estimate measurements is calculated.
- Repeat for other students and observe changes to the difference.
- Between taking students' heights, discuss with student's strategies to improve estimations.

Alternative: Use a Google Doc and have class enter their name and height estimate onto the one document (which can be projected). Students could also work in pairs to take the actual height measurement which can be added to the Google Doc.

---

### Step 2

- Ask students to measure their arm span and record it on **Class Height Recording Worksheet**.
  - Discuss with students the observations of the relationship between height and arm span.
- 

### Step 3

- Take students to an indoor or outdoor open space.
  - Students to stand side by side in a line, students step forward their estimate of 1 metre. Use a measuring tape to demonstrate 1 metre from the starting point, students to observe their position in relation to 1 metre - is it more or is it less?
  - Students return to starting point, students step forward their estimate of 5 metres. Use a measuring tape to demonstrate 5 metres from the starting point. Discuss with student's strategy for using the step they have just made to help them estimate 5 metres.
  - Students return to starting point, students use discussed strategy to step forward their estimate of 10 metres. Use a measuring tape to demonstrate 10 metres from the starting point. Discuss with students how their estimates have changed since the first step.
  - Parts of our bodies can be used as informal measures to assist in estimating actual lengths. The length of your step is called your stride. Discuss with student's other parts of the body that could be used to estimate length (e.g. arm span, hand span, foot length).
  - Students measure the following, hand span, foot length.
    - Hand span (<http://www.topendsports.com/testing/images/hand-span.jpg>)
    - Foot length ([http://www.shoetalk.co.nz/site/twinkle/images/Foot\\_Images/Foot\\_Length.gif](http://www.shoetalk.co.nz/site/twinkle/images/Foot_Images/Foot_Length.gif))
  - Students record their hand span and foot length on the **Class Height Recording Worksheet**.
-

## Step 4

- With measuring devices on display to the class, check student's prior knowledge of devices (name and method for usage).
- Students work in pairs to:
  - select (or assign them to) a measuring device.
  - choose an object or feature within the classroom that they could measure its length with that device.
  - measure the length of the object or feature.
- Students pair up with another pair that used the same measuring device.
  - Each pair measures the other pair's length and they compare measurements.
- Groups to present measuring device to class explaining how it is used and methods of using device to get an accurate measurement.
- Measurement relay: Students find one length outdoors (e.g. the length of a bench) to estimate (using one of their body measurements taken earlier), then measure with the same device they used in the classroom and calculate the difference between the estimate and the actual measurement. They then swap devices with another student until each student has made an estimation and measurement with each device. Students record their results on the **Measurement Relay Recording Sheet**.
- Discuss with students how the difference between the estimate and actual measurements changed as they completed the activity.

---

## Extension activity

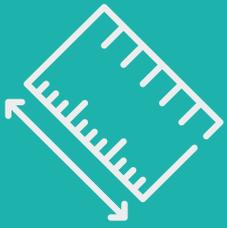
1. Students to calculate how many:
  - strides in 1 kilometre
  - hand spans in 1 metre
  - foot lengths in 1 metre
2. Exploring Leonardo da Vinci's Vitruvian Man.

---

## Measurement Relay Recording Sheet

Object	Estimate	Actual measurement	Difference





# LESSON 2

## Perimeter of simple plane shapes

### Overview of lesson

Students will investigate the process for calculating the perimeter of simple plane shapes starting with quadrilaterals that have a perimeter of 1 metre. Students will then apply their understanding to calculating the perimeter of triangles.

### Background knowledge

Plane shapes and their properties (especially quadrilaterals).

---

### Language considerations

Students should be able to communicate using the following language: dimensions, parallelogram, trapezium, rhombus, kite, length, width, breadth, base, opposite, adjacent and perimeter.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-12MG calculates the perimeters of plane shapes and the circumference of circles.

---

### Syllabus content

#### Students:

- find perimeters of parallelograms, trapeziums, rhombuses and kites.
- find the perimeters of a range of plane shapes, including parallelograms, trapeziums, rhombuses, kites and simple composite figures.
- compare perimeters of rectangles with the same area.
- solve problems involving the perimeters of plane shapes, e.g. find the dimensions of a rectangle, given its perimeter and the length of one side.

## Resources needed

- Dice
  - Masking tape
  - String (1 m per student)
  - A3 paper
- 

## Teacher instructions

### Step 1

- Students work in pairs with 1 metre of string.
  - With the 1 metre of string students construct each of the six quadrilaterals (square, rectangle, parallelogram, rhombus, kite, and trapezium). Measure the length of each side (in centimetres) and record the sum and total of the length of the sides beside a diagram of the quadrilateral.
  - Discuss the different combinations of lengths that emerged from the activity. Were there some shapes where more than one set of lengths was possible? Were there some shapes where only one set of lengths was possible?
  - The sum of the four sides is the perimeter (or the distance around the outside) of the shape. Discuss strategies for calculating the perimeter of shapes other than adding the four sides (have students consider a strategy for squares and rhombuses as well as parallelograms, rectangles and kites).
- 

### Step 2

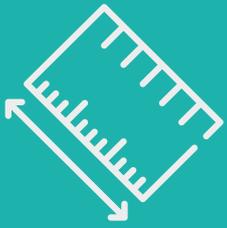
- Ask a student for the date of the month in which they were born. Use this as one dimension of a rectangle with a perimeter of 1 metre. Discuss with students a strategy for finding the other dimension.
  - Students work in pairs using the dates of the month in which they were born as one dimension of a rectangle with a perimeter of 1 metre to calculate the other dimension.
  - Challenge students to think about the number of pairs of whole number dimensions that are possible for a rectangle with a perimeter of 1 metre.
- 

### Step 3

- Students work in pairs with two dice. Students roll the two dice and use the two numbers to draw a rectangle with these dimensions. Students roll the dice two more times to create two more rectangles.
  - Students calculate the perimeter for each rectangle and record this underneath the rectangle.
  - Discuss with students how many different rectangles (pairs of dimensions) could be made by rolling two dice. Students investigate which pair would give the greatest perimeter.
- 

## Extension activity

- In step 3 have students roll three dice to construct triangles.



# LESSON 3

## Perimeter of composite shapes

### Overview of lesson

Students will construct and explore composite figures. They will be challenged to consider how they can combine shapes to maximise or minimise perimeter.

### Background knowledge

Students can calculate the perimeter of simple plane shapes.

---

### Language considerations

Students should be able to communicate using the following language: dimensions, parallelogram, trapezium, rhombus, kite, length, width, breadth, base, opposite and adjacent.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-12MG Calculates the perimeters of plane shapes and the circumference of circles.

---

### Syllabus content

#### Students:

- find perimeters of parallelograms, trapeziums, rhombuses and kites.
  - find the perimeters of a range of plane shapes, including parallelograms, trapeziums, rhombuses, kites and simple composite figures.
- 

### Resources needed

- Cardboard
- Grid paper printed on card

## Teacher instructions

### Step 1

- On the grid paper, students draw the following shapes, each with a perimeter of 12 cm:
    - Square (3 cm by 3 cm)
    - Rectangle (2 cm by 4 cm)
    - Parallelogram (2 cm on pair of parallel sides and 4 cm on the other)
    - Rhombus (3 cm along each side)
    - Right-angled, scalene triangle (3 cm, 4 cm, 5 cm)
  - Students write the length of each side inside the shape and cut the shapes out.
- 

### Step 2

- Students work in pairs and choose 3 shapes. They create and draw as many different composite shapes that use the 3 shapes they chose. The shapes must be placed together by matching up sides with the same length, i.e. there should be no overhang. Have students record their shapes on grid paper.
  - Discuss with students: If the perimeter of each shape is 12 cm, does that mean if we have used 3 shapes the perimeter will be 36 cm? Have students make a prediction and then check by calculating the perimeter of the composite shapes they have drawn on grid paper.
  - Challenge students to investigate how they can make a composite shape with the smallest perimeter using the 3 shapes they chose.
  - Challenge students to investigate how they can make a composite shape with the largest perimeter using the 3 shapes they chose.
- 

### Step 3

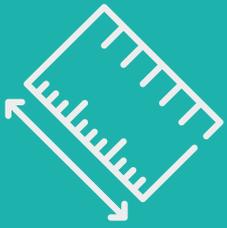
- Students work in pairs to construct two composite shapes on grid paper that each have a perimeter of 40 cm.
  - Students write the length of each side inside the shape and cut the shapes out.
  - Combining two pairs to form groups of 4, students use their 4 composite shapes to:
    - construct the composite shape with the largest perimeter, justifying with calculations.
    - construct the composite shape with the smallest perimeter, justifying with calculations.
- 

## Alternatives

- Before the lesson you could construct the shapes each with the perimeter of 12cm and photocopy the page for your students.
- 

## Extension activity

- In addition to the shapes in step 1, ask students to construct a kite, a trapezium and an equilateral triangle each with a perimeter of 12 cm.
- Have shapes overlap to allow the calculation of unknown sides.



# LESSON 4

## Irrational numbers

### Overview of lesson

Students will use everyday objects to investigate the relationship between the circumference and diameter of a circle. Students will use this relationship to derive a formula for finding the circumference of a circle given its radius or diameter.

### Background knowledge

Students should know the names of the parts of a circle.

---

### Language considerations

Students should be able to communicate using the following language: arc, tangent, chord, segment, centre, radius, diameter, circumference, sector, semicircle, and quadrant.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-12MG calculates the perimeters of plane shapes and the circumferences of circles.

---

### Syllabus content

#### Students:

- investigate the relationship between features of circles, such as the circumference, radius and diameter; use formulas to solve problems involving circumference.
- identify and name parts of a circle and related lines, including arc, tangent, chord, sector and segment.
- develop and use the formulas to find the circumference of circles in terms of the diameter or radius.
- use mental strategies to estimate the circumference of circles, using an approximate value of  $\pi$ , such as 3.14.
- find the diameter and/or radius of a circle, given its circumference (problem solving).

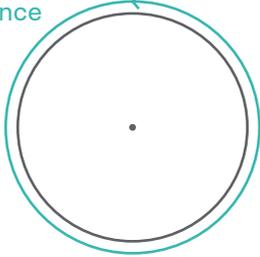
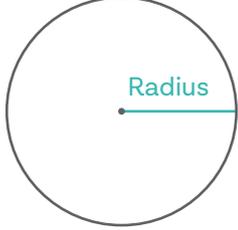
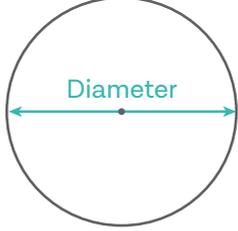
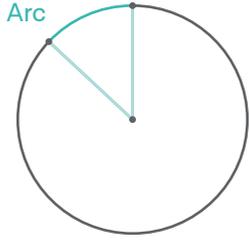
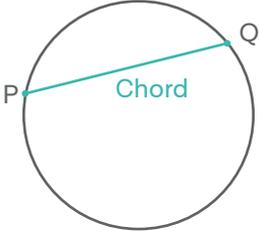
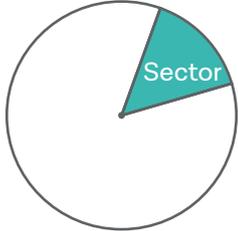
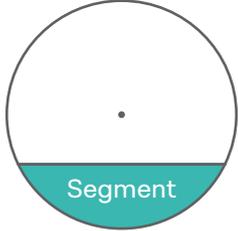
## Resources needed

- Paper plates, pipe cleaners, sticky tape
  - Parts of the circle
  - In the previous lesson, ask students to bring in any cylindrical containers/objects they might have around their home
- 

## Teacher instructions

### Step 1

- Each student has their own paper plate.
- Teacher names a part of the circle in words and students use highlighter or other bright colour to draw the part of the circle on their paper plate. Teachers check for understanding by asking students to hold up their paper plate and gives feedback where necessary. The following parts will need to be covered:

<p><b>Circumference:</b> the distance around the outside of the circle</p>	<p>Circumference</p> 
<p><b>Radius:</b> a line from the centre of a circle to its circumference</p>	
<p><b>Diameter:</b> a line passing from one side of the circle to the other through the centre of the circle</p>	
<p><b>Arc:</b> a part of the circumference of the circle</p>	
<p><b>Chord:</b> a line that touches the circumference of the circle at two points</p>	
<p><b>Sector:</b> the area between two radii</p>	
<p><b>Segment:</b> the area between a chord and the circumference</p>	

## Step 2

- For this task, students will need the cylindrical container/object they have brought from home or the cylindrical containers provided by the teachers.
  - Working in groups of 3-4, students measure and record the circumference and diameter of the circular bases of each cylinder. Students record their results in the **Circumference Investigation worksheet** (This could be done on a Google Doc).
  - Note: before beginning the activity discuss strategies for:
    - finding the centre of the circular base accurately to then measure the diameter, and
    - measuring the circumference of the circular bases by rolling the cylinder along a ruler, winding a piece of string around the cylinder and measuring the length on a ruler etc.
  - Once students have measured and recorded the circumference and the diameter have students complete the last column of the table.
  - Discuss the ratio of circumference to diameter i.e. similarities (it is a fixed ratio) and reasons for difference (error in measurement). We give this ratio the name pi, denoted by the symbol  $\pi$ .
- 

## Step 3

- Discuss with students:
    - we have just found out that  $\frac{\text{Circumference}}{\text{diameter}} = \pi$  which can be written simply as  $\frac{C}{d} = \pi$
    - how could this equation be rearranged so that  $C$  was the subject? This is the formula for finding the circumference of a circle.  $C = \pi d$
    - how can we write the diameter ( $d$ ) in terms of the radius ( $r$ )? i.e.  $d = 2r$ , which then gives us another formula for the circumference of the circle,  $C = 2\pi r$
- 

## Step 4

- Give students the **Page of Circles Worksheet**, students use an estimate of  $\pi$  (e.g. 3, 3.1,  $\frac{22}{7}$  depending on ability) to estimate the circumference before calculating. Record estimates and calculations in **Circumference recording sheet** (Encouraged students to check their answer by measuring the circumference practically e.g. with a piece of string).
  - Discuss with students when it might be appropriate to use an estimation for  $\pi$ .
- 

## Step 5 (extension)

- Discuss with students: we have discovered the formula for the circumference of a circle is  $C = \pi d$  or  $C = \pi r$  how could we rearrange these formulae to make  $d$  and  $r$  the subject of the formula respectively?
- 

## Alternatives

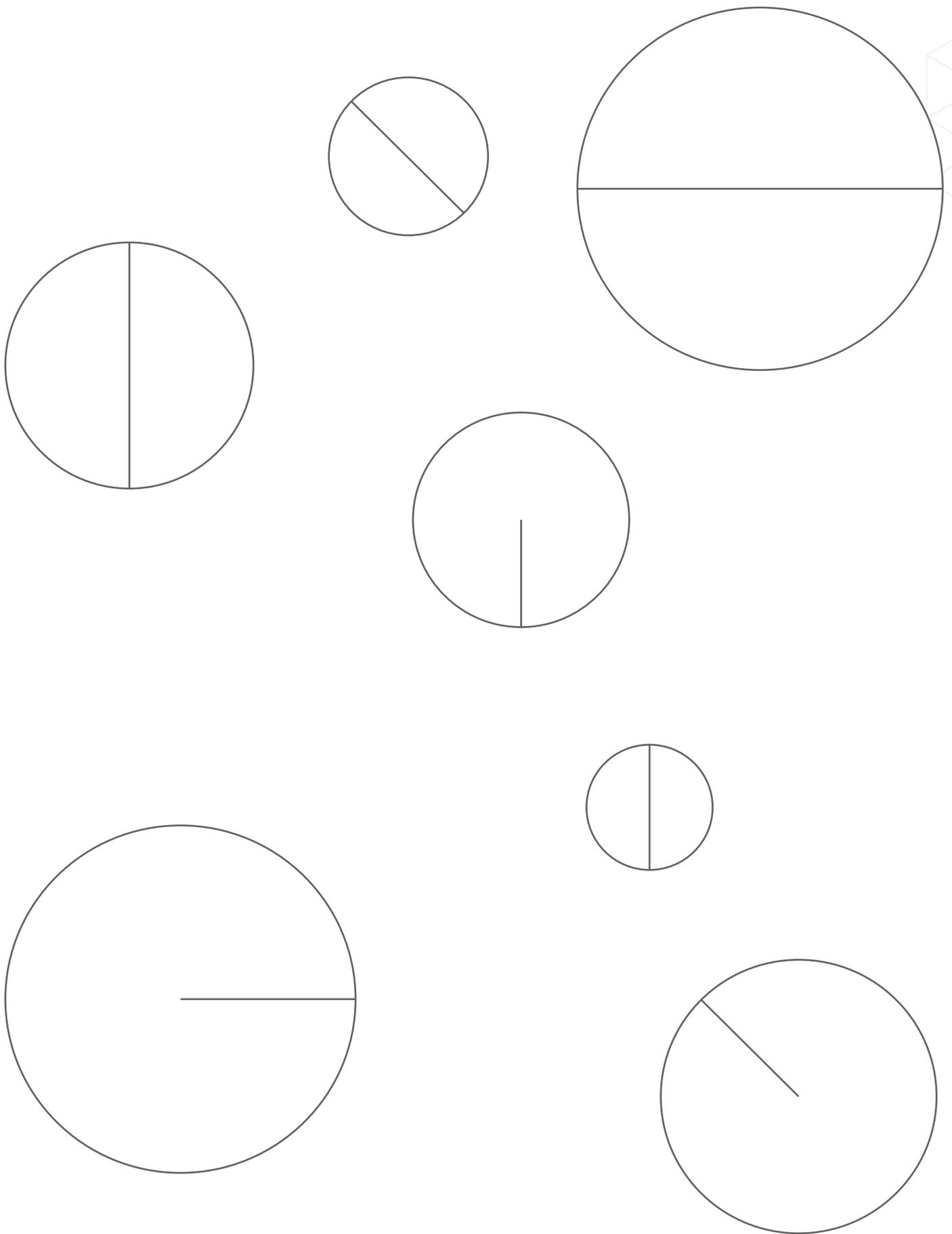
- Students could measure the circumference of coins instead of cylindrical container.
- 

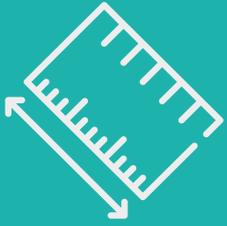
## Extension activity

- Students research different approximations for  $\pi$  and/or compare approximations by using them to calculate the circles on the recording sheet.



## Page of Circles





# LESSON 5

## Circumference of circles, quadrants and semicircles

### Overview of lesson

Students will investigate the parts of the circle that comprise the perimeters of semicircles and quadrants. Students will manipulate circles to determine the perimeter of the semicircles and quadrants.

### Background knowledge

Students should know how to find the circumference of whole circles.

---

### Language considerations

Students should be able to communicate using the following language: quadrants, semicircles, radius, perimeter and circumference.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-12MG calculates the perimeters of plane shapes and the circumferences of circles.

---

### Syllabus content

Students:

- find the perimeters of quadrants and semicircles.
  - find arc lengths and the perimeters of sectors.
  - solve a variety of practical problems involving circles and parts of circles, giving an exact answer in terms of  $\pi$  and an approximate answer using a calculator's approximation for  $\pi$ .
- 

### Resources needed

- Rotating circles (1 copy of each circle per student, preferably printed on cardboard and in different colours)
- Circle paper (2 pieces per student, different colours)
- Compasses

## Teacher instructions

### Step 1

- Brainstorm sectors (semicircles and quadrants) in real life.
  - Cut a radius in each rotating circle and slide one circle into the other along the cut radius. Place the circle with the angles marked facing up. Have students rotate one circle on another to show a quadrant and a semicircle. Students in pairs estimate sectors with a given angle at the centre using the angles facing away from the student.
- 

### Step 2

- Give each student 2 pieces of coloured circle paper. Ask students for strategies to find the centre of the circle (by folding).
  - Students use a highlighter to trace the circumference on both circles. Students use the appropriate formula to calculate the circumference of the circle.
  - For one of the circles, students cut it in half and discuss the fraction of the circumference that is remaining. Students use answer from previous point to calculate the length of the remaining portion of the circumference. Discuss with students what is missing that is necessary to find the perimeter. At each stage students develop a rule for finding the perimeter.
  - For one of the circles, students cut it in quarters and discuss the fraction of the circumference that is remaining. Students use answer from previous points to calculate the length of the remaining portion of the circumference. Discuss with students what is missing that is necessary to finding the perimeter. At each stage students develop a rule for finding the perimeter.
  - Students estimate the circumference of the circle and the perimeters of the semicircle and quadrant (in terms of  $\pi$ ) before measuring and calculating.
- 

### Step 3

- Students use a compass to construct three different circles. Students use a ruler to divide the circles into halves and quarters. For each circle, students calculate the perimeter of the semicircle and the perimeter of the quadrant.
- 

## Alternatives

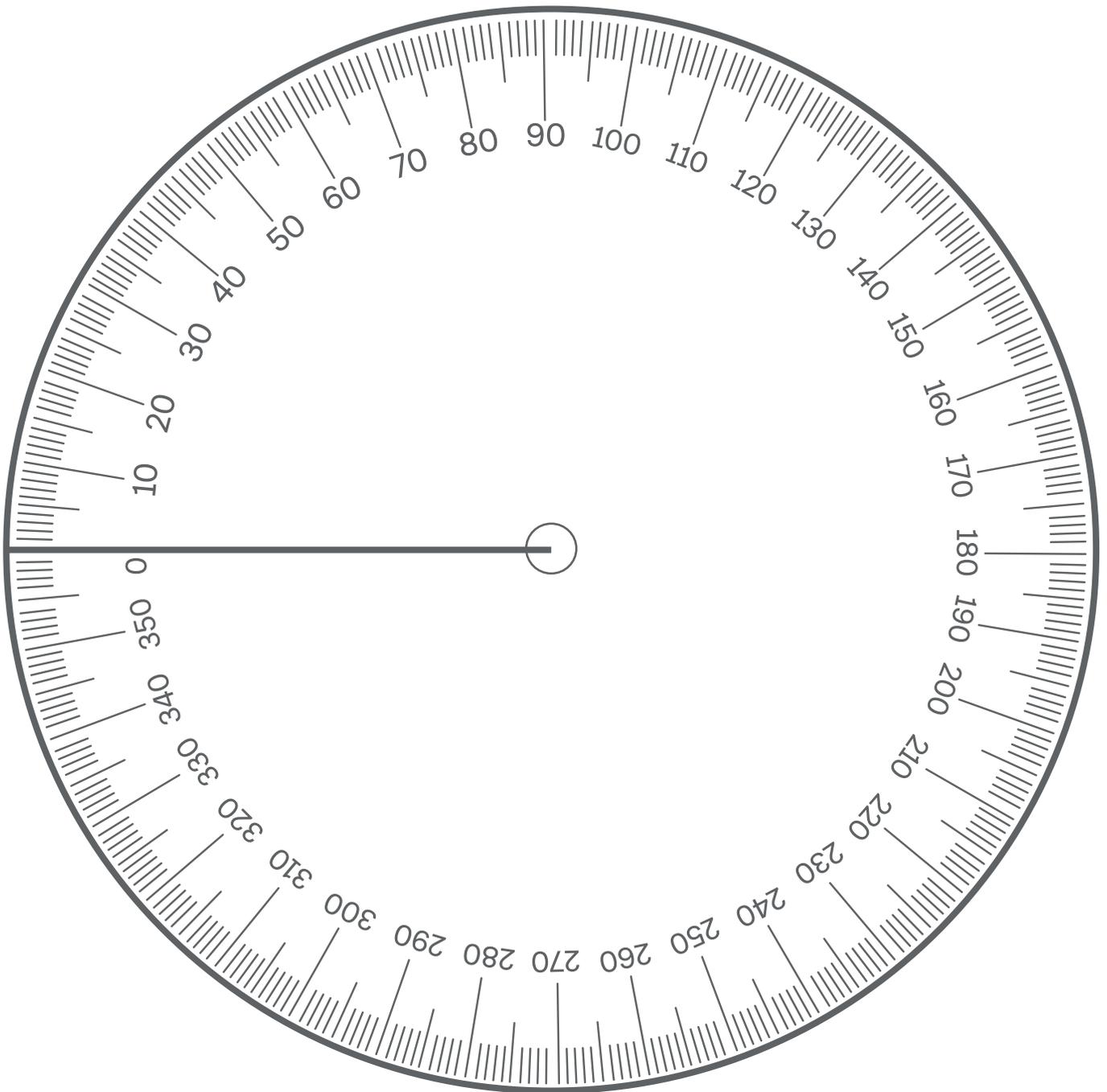
- Students use the rotating circles from step 1 to calculate the perimeter of the semicircle and quadrant instead of using a compass to construct own circles.
- 

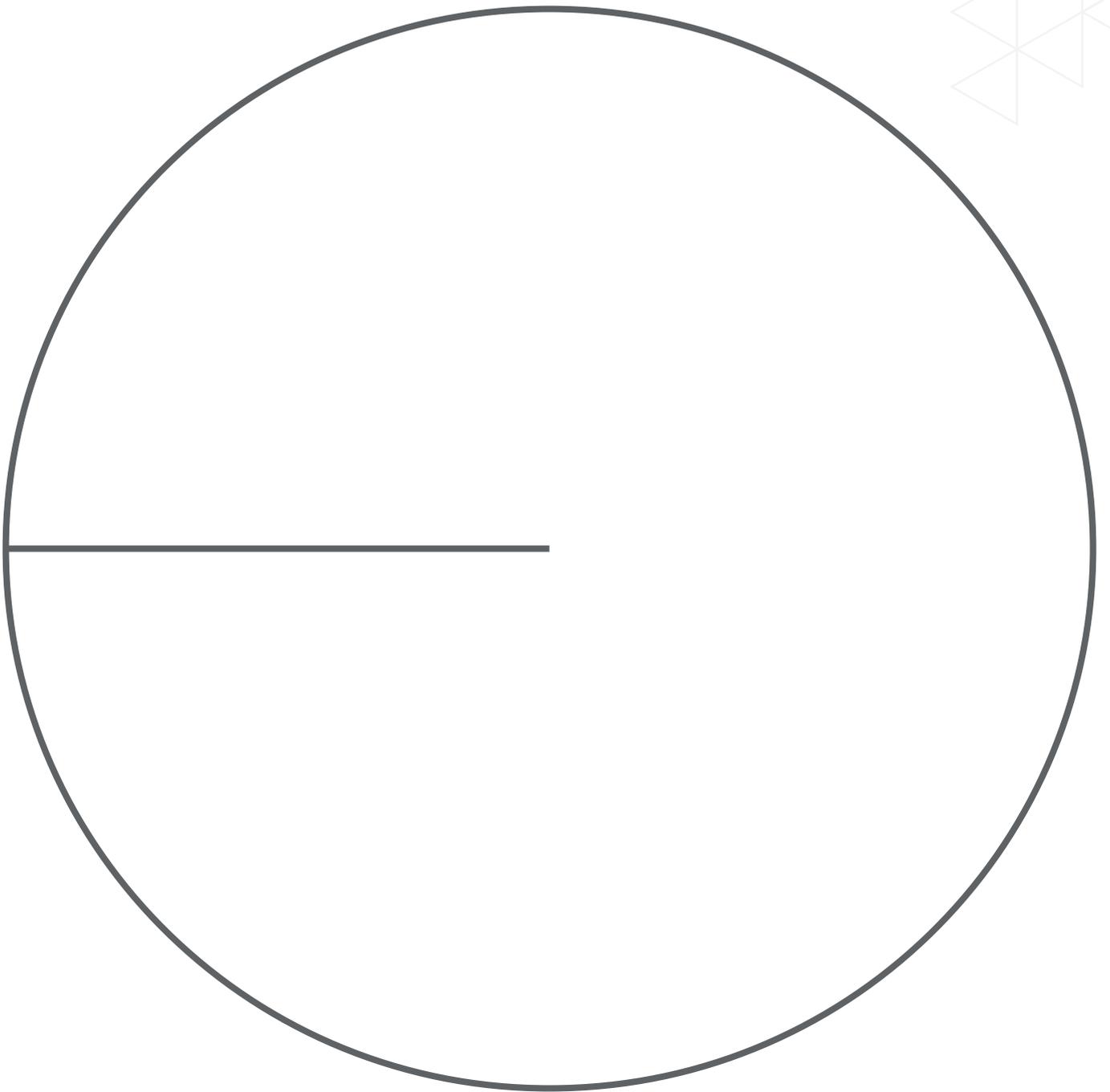
## Extension activity

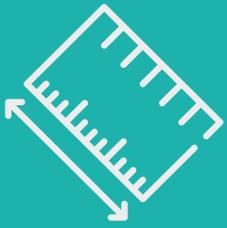
- Students develop a rule for finding the arc length of any sector by expressing a sector as a fraction of the whole circle. I.e. as students work through Step 3 develop

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$

## Rotating Circles







# LESSON 6

## Composite shapes involving semicircles and quadrants

### Overview of lesson

Students will explore the perimeter of composite shapes by investigating the properties of an Olympic running track. Students will have the opportunity to apply their understanding to calculating the perimeter of composite shapes in their school or create their own according to certain requirements.

### Background knowledge

Students should know how to find the perimeter of semicircles and quadrants and the process for finding the perimeter of composite shapes.

---

### Language considerations

Students should be able to communicate using the following language: length, distance, composite shape, semicircles, quadrants, diameter, radius, and arc.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-12MG calculates the perimeters of plane shapes and the circumferences of circles.

---

### Syllabus content

Students:

- find the perimeters of simple composite figures consisting of two shapes, including quadrants and semicircles.
- 

### Resources needed

- The 2 images of the racing track to project for students

## Teacher instructions

### Step 1

- Show students video of Cathy Freeman winning gold in the 400 m at the 2000 Sydney Olympics. She won with a time of 49.11 seconds. <https://www.youtube.com/watch?v=XTCnO5MXvol>.
- Ask students to calculate her speed in metres per second ( $400 \div 49.11 = 8.14$  m/s).

### Step 2

- Show students the image of the Olympic running track. Ask students what simple shapes have been combined to make the track. Ask students what information they would need to be able to find the perimeter of the inside track (the length of the straight and the radius/diameter of the circular end).
- Show students the image of the Olympic running track with dimensions and ask them how to calculate the perimeter. i.e.  $(2 \times 84.39) + (2 \times \pi \times 36.5) = 398.11$  metres.
- The running distance for the inside track is taken from 30 cm inside the edge of the track - which of the dimensions changes? - The length of the straight or the radius? (answer=radius) What is the new measurement? ( $36.5 + 0.3 = 36.8$ ). Students then recalculate i.e.  $(2 \times 84.39) + (2 \times \pi \times 36.8) = 400$  metres.
- There are 8 lanes in total and for lanes 2-8, have students calculate the running distance for each of the lanes 2-8 (you can ask students to round to the nearest whole number).

Lane	Radius	Straight edge	Running distance
1	36.8	84.39	400
2	38.02	84.39	
3	39.24	84.39	
4	40.46	84.39	
5	41.68	84.39	
6	42.9	84.39	
7	44.12	84.39	
8	45.34	84.39	454

### Step 3

- How does the International Athletics Federation make it fair so that each runner runs 400 metres regardless of the lane they are in? (answer=staggered the starting point)
- Who starts at the front? (the person in lane 8) By how much? (answer=approximately 54 metres)
- Have students calculate the distance of the starting points in each lane from the starting point for lane 1 using the worksheet **Olympic Running Track Calculation Sheet**.

## Alternatives

1. Take students outside to a feature in the school (e.g. a garden bed) that is a composite figure that includes semicircles and quadrants).
2. Calculate the perimeter of a running track that is marked out at your school or near your school. Students could check their calculations by using a trundle wheel to measure the distance from each lane. Students could also have their own race on the running track and compare their speed to Cathy Freeman's.

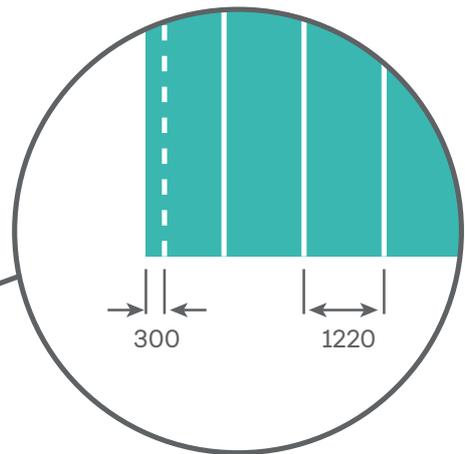
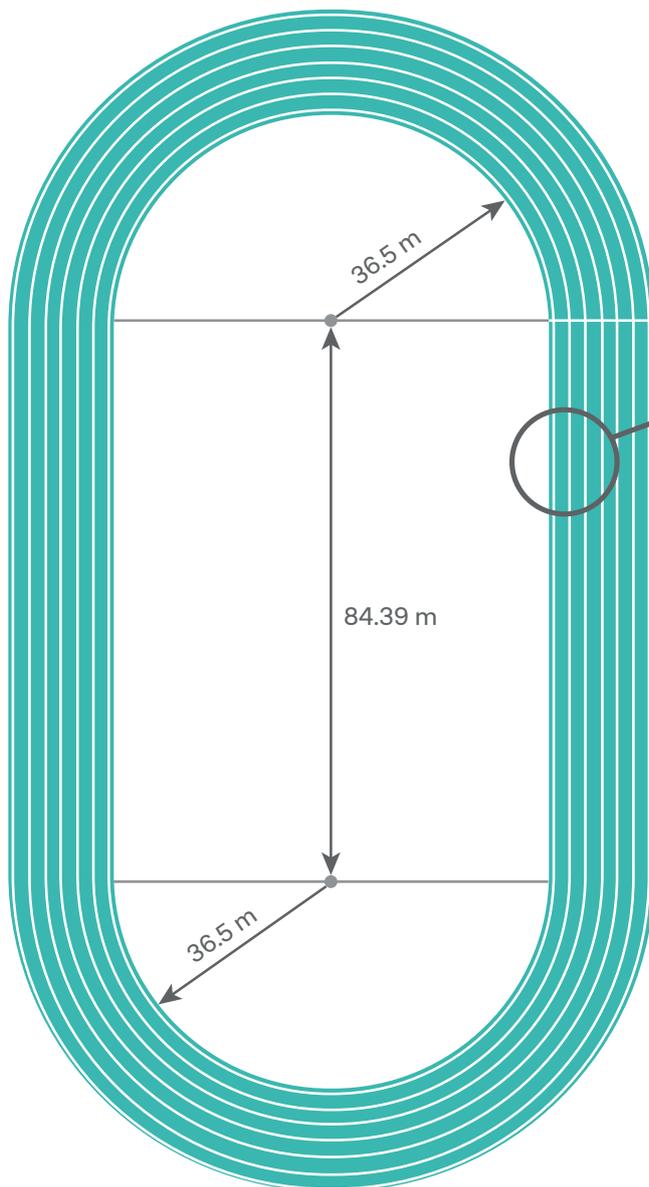
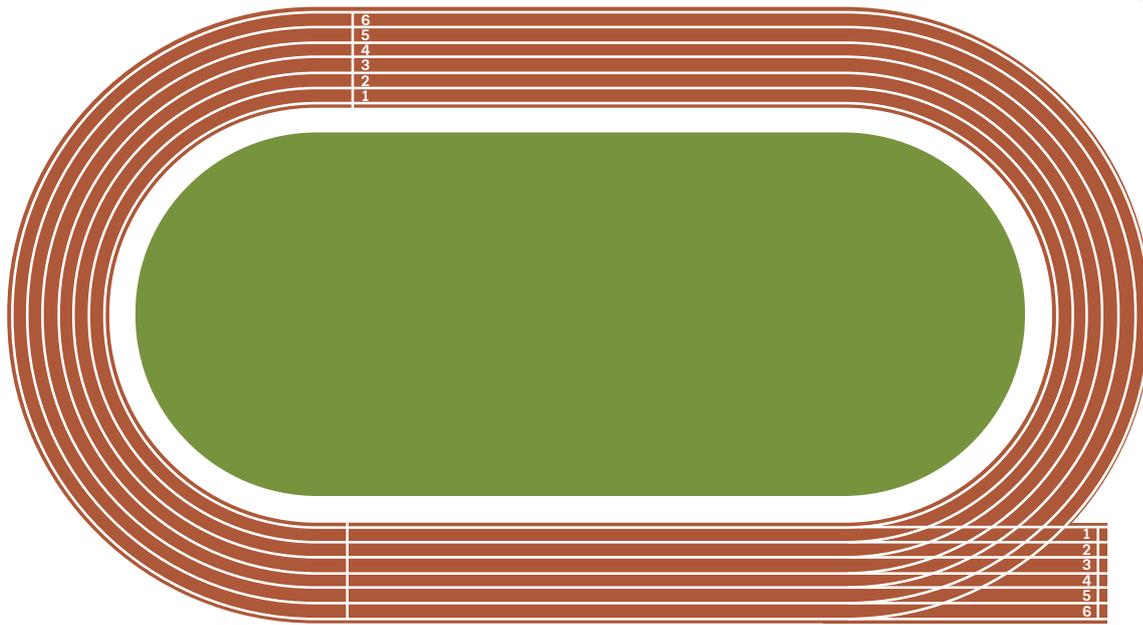
## Extension activity

- Students design a running track with other dimensions where the inside lane has a length of 400 metres.

## Olympic Running Track Calculation Sheet

Lane	Radius	Straight edge	Running distance	Distance from lane 1
1	36.8		400	
2	38.02			
3	39.24			
4	40.46			
5	41.68			
6	42.9			
7	44.12			
8	45.34			

# Olympic Running Track

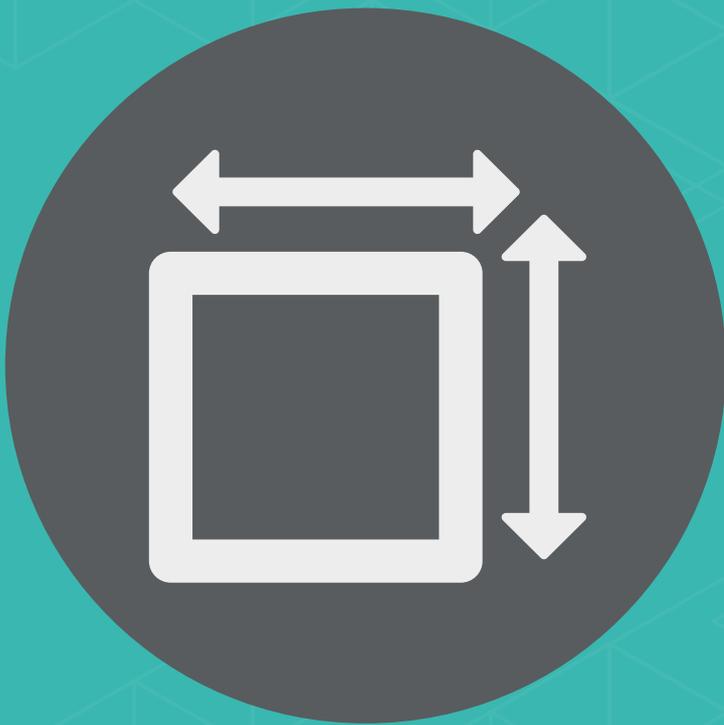


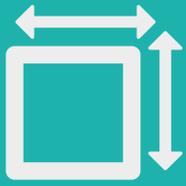
Running distance is measured 300 mm from inner edge.

Tracks are 1220 mm wide



# Teaching Measurement AREA





# LESSON 1

## Area of rectangles and squares

### Overview of lesson

Students derive and investigate the formula for area of a square and a rectangle through the use of practical activities.

### Background knowledge

Students should be able to:

- Draw closed shapes.
- Choose and use appropriate units for measuring.
- Use and understand correct vocabulary - area, shape, inside, outside, boundary, perimeter.
- Explain the relationship between length and breadth of rectangles.
- Use the length and breadth of rectangles to calculate area.

### Language considerations

- Students should be able to communicate using the following language: length, breadth, width.
- Teachers reinforce with students the use of the term 'breadth', 'length', 'side' in relation to finding the areas of rectangles and squares.
- The abbreviation  $m^2$  is read as 'square metre(s)' and not 'metre(s) squared' or 'metre(s) square'. Similarly, the abbreviation  $cm^2$  is read as 'square centimetre(s)' and not 'centimetre(s) squared' or 'centimetre(s) square'.
- When units are not provided in an area question, students should record the area in 'square units'.

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-13MG uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area.

### Syllabus content

Students:

- choose appropriate units of measurement for area and convert from one unit to another.
- establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.

## Resources needed

- Grid paper
  - Ruler
  - Pencils
- 

## Teacher instructions

### Step 1

- Students list the factors of 24 either in their books or collaboratively list and write on the board.
  - Have students construct as many rectangles as they can that cover 24 squares and label the length of the sides of the rectangles. This can be demonstrated on the board to reinforce understanding, depending on the ability of the class (grid paper may be used if needed).
  - Discuss as a class the length of the sides and relationship with the number 24 and the factors that were listed.
- 

### Step 2

- Students write a rule in words for calculating the area of a rectangle.
  - Show students how this translates to the formula  $A = lb$ 
    - Question: Does this rule work for a square?
    - Question: How could we write our formula in another way?  $A = s^2$
- 

### Step 3

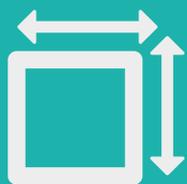
- Students look at the rectangles that they constructed at the beginning of the lesson and calculate the perimeter of each.
- 

## Extension Activities

- Provide students with a bigger number (e.g. 32, 48, 72) or let them choose their own number.
  - Students consider which number between 1 and 100 would give you the most number of rectangles if the sides had to be whole number lengths.
  - Students use the factors of 36 to determine how many different rectangles there are with whole number lengths.
- 

## Discussion

- Will all rectangles with an area of 24 units<sup>2</sup> have the same perimeter?
- Which has the largest perimeter?
- Which has the smallest perimeter?



# LESSON 2

## Working with square metres, square kilometres and hectares

### Overview of lesson

- Students become familiar with estimation and are able to correctly use a scale to accurately measure lengths on maps
- Correctly determine the size of a square metre, kilometre and hectare.

### Background knowledge

It is important from Stage 3 that students have established a real reference for the square kilometre and the hectare, e.g. locating an area of one square kilometre or an area of one hectare on a local map.

---

### Language considerations

- Students should be able to communicate using the following language: metre, kilometre square metre, square kilometre, hectare, distance and area.
  - The abbreviation  $m^2$  is read as 'square metre(s)' and not 'metre(s) squared' or 'metre(s) square'. Similarly, the abbreviation  $cm^2$  is read as 'square centimetre(s)' and not 'centimetre(s) squared' or 'centimetre(s) square'.
- 

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-13MG uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area.

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### Syllabus content

#### Students:

- choose appropriate units of measurement for area and convert from one unit to another.
- choose an appropriate unit to measure the areas of different shapes and surfaces, e.g. floor space, fields.
- use the areas of familiar surfaces to assist with the estimation of larger areas, e.g. the areas of courts and fields for sport.
- convert between metric units of area using  $1\text{ cm}^2 = 100\text{ mm}^2$ ,  $1\text{ m}^2 = 1\,000\,000\text{ mm}^2$ ,  $1\text{ ha} = 10\,000\text{ m}^2$ ,  $1\text{ km}^2 = 1\,000\,000\text{ m}^2 = 100\text{ ha}$ .

## Resources needed

- Grid paper
  - Aerial image printed prior to the lesson
  - Google Earth outline of school showing all borders
  - Newspapers and sticky tape
  - ICT booked prior to the lesson which has IWB/Data Projector/student computers etc.
  - 'City' name cards with 10 names on each prepared and printed
- 

## Teacher instructions

### Step 1

Working in either small groups or individually

- Give students the names of 10 cities on cards and have them sort them from the smallest area to the biggest area using the following link <http://www.citymayors.com/statistics/largest-cities-area-125.html>.

**Discussion:** Students discuss their findings/results as a class, using prompting questions such as; What is the city with the largest area? What is the city with the smallest area? Is this different to what you expected? Why or why not?

---

### Step 2

Working in small groups

- Students draw with chalk on the ground what they think 1 square metre looks like.
  - Give students newspaper and ask them to construct a square metre. Have students compare their estimate of a square metre with an actual square metre. How accurate were you?
  - Show students a square centimetre and have them work out how many square centimetres would fit inside a square metre.
  - Explain that a hectare is 10 000 m<sup>2</sup>, and discuss who might use a measurement such as hectares and for what purpose?
  - Students work out how many square metres in one square kilometre etc.
- 

### Step 3

- Before lesson, using Google Earth, Google Map or Six Maps (<https://maps.six.nsw.gov.au/> see attached instruction sheet) teacher will need to have prepared an appropriate aerial image of the local area, Sydney Harbour and Canberra for example and provide one copy per student.
- Students estimate and draw a length of 1 kilometre on their map and then draw an area of 1 square kilometre.
- Teacher will need to bring up their own prepared image (same as the students are using from Google Earth and project it).
- Use the Google Earth ruler function to measure 1 kilometre in length and 1 square kilometre in area to demonstrate the exact measurements.
- Students compare the exact result to their estimate.

## Step 4

- Repeat the above process for an area of 1 hectare (10 000 m<sup>2</sup>) by having students estimate on their maps an area of 100 metres by 100 metres and then compare their result to the teachers correct diagram.
  - You may need to have a second image to issue to students of an appropriate area for measuring in hectares.
  - Make comparison to the different units/sizes and appropriateness of each.
- 

## Step 5

- Students estimate the area of the school in square metres.
  - Teacher needs to issue each student with a Google Earth outline of the school showing all borders and a square that represents 1 square unit with same scale as the school outline.
  - Students approximate the area of the school in square metres by overlaying squares.
  - Extension: students approximate the area of the school using area formulas already established and by dividing school into appropriate shapes (squares, rectangles and triangles).
  - Provide students with the exact size of the school in square metres, square kilometres and hectares to check their answers.
- 

## Step 6

- Print a map of your local area with the scale shown and provide each student with a copy. Students approximate, using a ruler and given scale, various areas such as their house, local parks/ovals, shopping centres etc.
  - Technology Option: All the steps can be completed using ICT with no printed images, although it might be desirable to have students manually use scales.
  - Teachers can project an image of their school or an alternate relevant image from Google Earth, Google Maps or Six Maps with a set scale of 1 km.
  - Use the ruler function to measure out 1 square kilometre to show students an exact square kilometre. Have students compare result to estimate.
  - Repeat the process for a hectare (10 000 m<sup>2</sup>). Make comparison to the different units/sizes.
  - Ask students to use Google Earth, Maps or Six Maps in pairs or groups of three to practise the skills of measuring areas using local features - their house, local parks/ovals, shopping centres etc.
- 

## Questioning

- Were your estimation results similar to the actual areas of the object/places?
  - What methods did you use for your estimation?
  - Were your methods accurate and successful?
  - Were there any patterns or changes in your estimations?
  - What would you measure in  $mm^2$ ,  $cm^2$ ,  $m^2$ ,  $km^2$ ,  $ha$ ?
- 

## Discussion

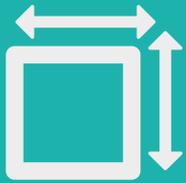
- Was your estimation close to the school size? What method/s did you use for your estimation? How can the skills of estimation be useful in today's society?

## SIX Maps Instructions

- Students go to <https://maps.six.nsw.gov.au/> and look up a place of significance to them (or as designated by the teacher).
- Use the distance tool to measure 1 kilometre. Students take a screenshot and paste it in a document.
- Use the area tool to measure an area of 1 square kilometre. Students take a screenshot and paste it in a document.
- Use the zoom tool to zoom out and create an area of 1 hectare (10 000 m<sup>2</sup>). Students take a screenshot and paste it in a document.
- Students can now use Six Maps to calculate the area of the school in hectares.

NOTE: If you don't have access to technology in the classroom.

- Look up a place on Six Maps before the lesson and print off one for each student.
- Students can then use the scale given in the bottom left hand corner to complete the activities above.



# LESSON 3

## Developing area formulae

### Overview of lesson

Students derive the formula for the area of a triangle, parallelogram, trapezium, rhombus and kite.

### Background knowledge

Selects and uses the appropriate unit to calculate areas, including areas of squares, rectangles and triangles.

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### Language considerations

Students should be able to communicate using the following language: area, square centimetre, square metre, dimensions, length, width, breadth, base (of triangle) and perpendicular height.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-13MG uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area.

---

### Syllabus content

**Students:**

- establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
- find areas of trapeziums, rhombuses and kites.

## Resources needed

- Grid paper
- Paper cut into a parallelogram
- Paper cut into a trapezium
- Paper cut into a rhombus or kite

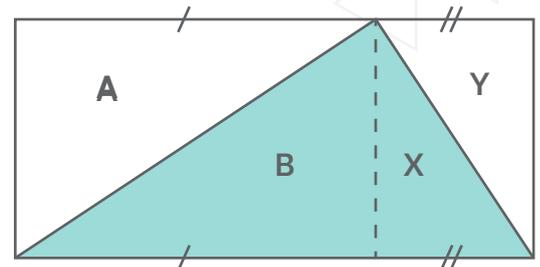
## Teacher instructions

### Step 1

- Deriving and understanding the formula and calculating the area of a triangle.

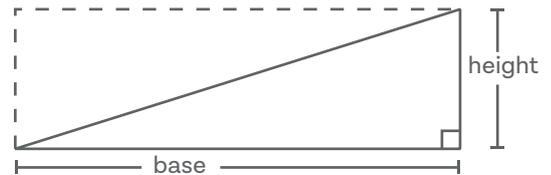
#### Method 1

- Give each student a rectangle piece of paper and have them draw a vertical line (perpendicular to the top and bottom of the rectangle).
- Draw diagonals from the top of the perpendicular line to the bottom corners of the rectangle and cut along the two drawn diagonals.
- Students attempt to overlay one triangle over the other to demonstrate that there are two equal triangles in the rectangle.



#### Method 2

- Draw one diagonal of the triangle to make two right angled triangles, then attempt to overlay one triangle over the other to demonstrate that both triangles are equal.



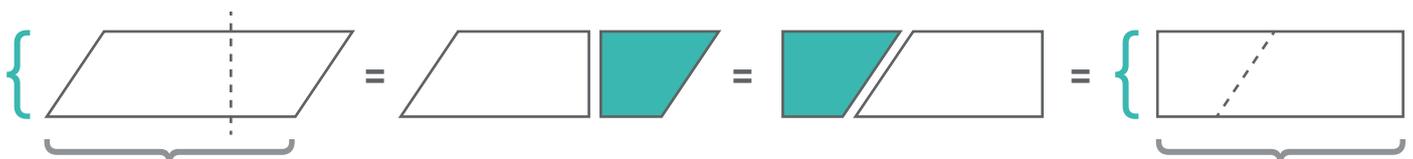
### Step 2

- Deriving and understanding the formula for calculating the area of a parallelogram.

#### Method 1

- Give each student a parallelogram (prepared prior to the lesson) and ask them to draw a vertical line that produces a right-angled triangle at one end of the parallelogram.
- Cut out the triangle carefully.
- Move the triangle to the other end of the parallelogram so that a rectangle is formed.

Note: it is worth exploring the vertical cut being made anywhere along the parallelogram (see diagram below).



- Discuss with students the formula for a parallelogram, where  $b$  is the length of the base and  $h$  is the perpendicular height (relate to a rectangle).

### Method 2

- Area of a triangle can be revisited with this dissection. Slice the triangle parallel to its base at exactly half the height. The two pieces fit together to make a parallelogram with the same base but half the height.

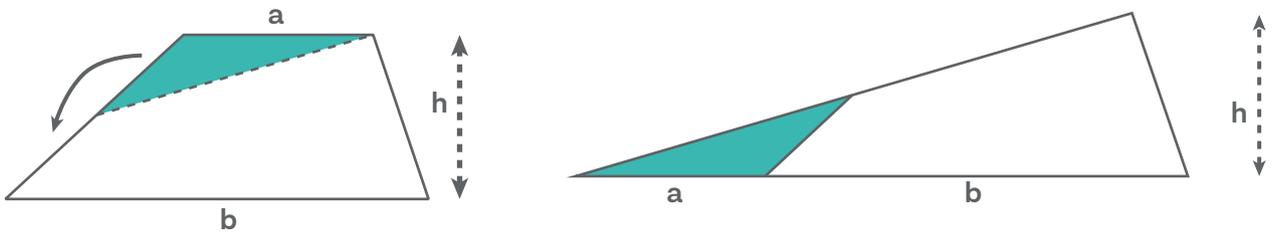


### Step 3

- Deriving and understanding the formula for calculating the area of a trapezium.

#### Method 1

- Give each student a trapezium and have them draw a line from the centre of one of the sloped edges to meet the opposite top corner of the trapezium.
- Students carefully cut along the line.
- Reorient the shape by rotating the triangle to form a larger triangle.



#### Method 2

- Area of a trapezium can be established by again using area of parallelogram in two different ways:
  - Slice the trapezium at exactly half the height and fit the two parts together to make a parallelogram whose base is the sum of the two parallel sides but whose height is half the height of the trapezium.



- Double the trapezium to form a parallelogram and then halve the area of the parallelogram.



#### Method 3

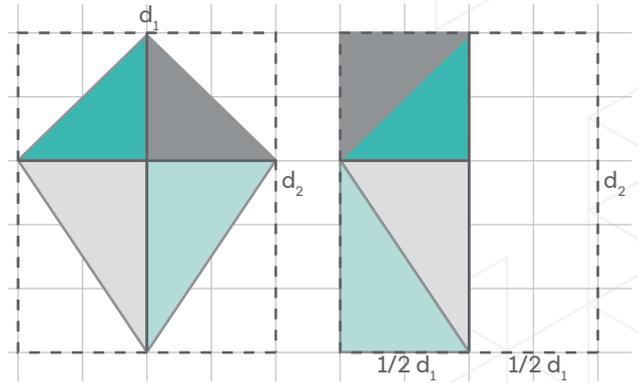
- Give each student a trapezium and have them draw vertical lines halfway along the sloped edge to meet the bottom of the trapezium. Making small triangles.
- Students then carefully cut out the small triangles.
- Students reorient the triangles to make a rectangle.



Note: the length of the rectangle is now an average of the two parallel sides (check by inspection/measurement).

## Step 4

- Deriving and understanding the formula for calculating the area of a rhombus/kite.
- Give each student a rhombus or a kite cut out of paper (prepared prior to the lesson).
- Students draw both diagonals on the rhombus or kite.
- Students cut the shape in half by cutting along the long diagonal.
- Students then take one of these halves and cut along the short diagonal, effectively creating two small triangles.
- Students reorient the triangles to form a rectangle where the width is half the length of the short diagonal and the length is equal to the long diagonal.
- Discuss with the students the formula, where  $x$  and  $y$  are the lengths of the diagonals.



## Alternatively

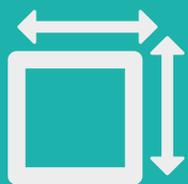
- These activities could either be done in stations where students derive the formula for themselves or the teachers guides students through each activity providing examples for each so that students can consolidate the formula.
- Geogebra can also be used to demonstrate the dissections that the class have now completed- see links below
  - <https://www.geogebra.org/m/Hb94J3S6> (trapezium)
  - <https://www.geogebra.org/m/stfH3MkX> (parallelogram)
  - <https://www.geogebra.org/m/BBGqvHs6> (triangle)
  - <https://www.geogebra.org/m/ggpaFhA4> (rhombus/kite)

## Questioning

- What have these activities shown you?
- Do you understand how the formula have been obtained from each shape?
- How can this help you in the future?

## Discussion

- Discuss the importance of having a formula and using a specified formula to calculate the area of different shapes.
- In the real world where and when would you use the formula to find the area of an object.
- PDF Resource for area cut-out demonstrations <http://betterfilecabinet.com/f10/CutoutAreaDemos.pdf>



# LESSON 4

## Area of a circle

### Overview of lesson

Students derive the area of a circle.

### Background knowledge

- Students have an understanding of/and are able to identify a circle.
- Students know the formula for circumference of a circle.
- Students have an understanding of how to find the area of a rectangle.
- Students have an understanding of, or are familiar with, terminology used in fractions.

### Language considerations

- Students should be able to communicate using the following language: area, 'vertex', circle, base, height, radius, diameter, circumference and sector.

### Syllabus outcomes

MA4-13MG uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area.

### Syllabus content

Students:

- investigate the relationship between features of circles, such as the area and the radius; use formulas to solve problems involving area.

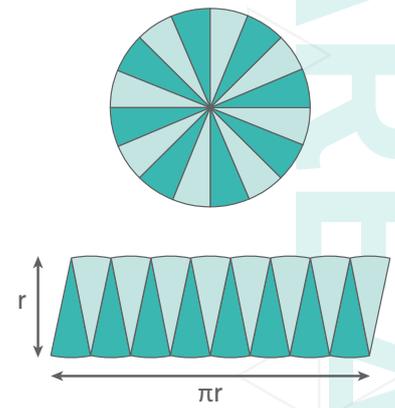
### Resources needed

- Grid paper
- Scissors
- Circle Paper
- Calculators
- Protractors and compasses

## Teacher instructions

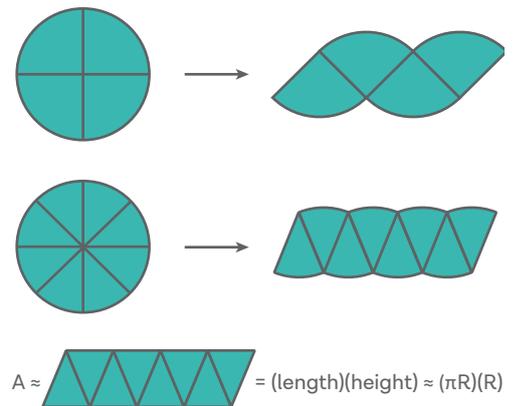
### Step 1

- Give students a circle and ask them to divide it into eighths (by folding).
- Have the students cut along the fold lines and arrange as indicated.



### Step 2

- Continue this process by giving students another larger circle and ask them to divide it into twelfths and then sixteenths (depending on the size of the circle) by folding and have them cut along the fold lines and arrange as indicated - cut a circle into a large number of sectors and arrange the sectors alternately point-up and point-down to form a rectangle with height  $r$  and base length  $\pi r$ .
- It should become clear to the students that the more sectors the circle is divided into the closer and closer the rectangle is to being absolutely straight edged rectangle).



Question: what shape was formed through the activity?  
(i.e. rectangle/parallelogram).

### Step 3

- Since the shape formed is approximately a rectangle, have students try and work out the dimensions of the rectangle in relation to the circle (e.g. height is radius and length is half circumference).
- Explain how the formula is derived based on

$$\begin{aligned} \text{Area} &= \text{length} \times \text{breadth} \\ &= \pi r^2 \end{aligned}$$

Note: Geogebra has a nice demonstration of this process <https://www.geogebra.org/m/awBAYg6P>

### Step 4

#### Discovering the area of a sector

- Give students a circle and ask them to fold it in half once (semi-circle)

Question: What is the formula for the area? (use  $180^\circ$  out of  $360 = \frac{1}{2}$  )

- Get students to repeat the process - fold the circle in half again (quadrant)

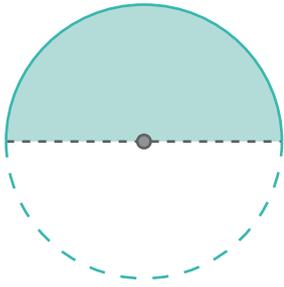
Question: What is the formula for the area? (use  $90^\circ$  out of  $360 = \frac{1}{4}$  )

- Get students to repeat the process again - fold the circle in half again (eighth)

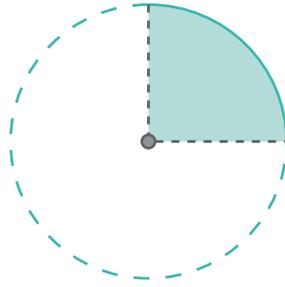
Question: What is the formula for the area? (use  $45^\circ$  out of  $360 = \frac{1}{8}$  )

## Step 5

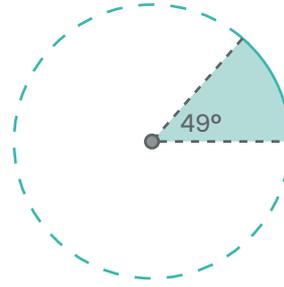
- Having now established a pattern, what is the formula for the area of any sector? (Guide students if required to derive the formula)



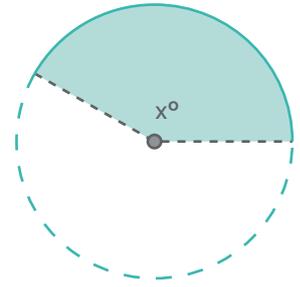
$$\text{Area} = \frac{180}{360} \pi r^2 = \frac{1}{2} \pi r^2$$



$$\text{Area} = \frac{90}{360} \pi r^2 = \frac{1}{4} \pi r^2$$



$$\text{Area} = \frac{49}{360} \pi r^2$$



$$\text{Area} = \frac{x}{360} \pi r^2$$

Think of a circle as a percentage of the circles total area, so all we do is create a percentage based on the central angle to the total angle value,  $360^\circ$ . This calculates a circle sector.

$$\text{AREA CIRCLE SECTOR} = \frac{x^\circ}{360} \pi r^2$$

## Extension activity

- Students use their circle to then construct their own sector, measure the angle at the centre using a protractor and then use the sector formula to find the area. Working in pairs students create sectors for partners to measure angle and calculate area.
- When working out the answer it is important to distinguish between an approximate and exact answer - for their answers students should be encouraged to write an approximate answer using a calculators approximation for  $\pi$  and the exact answer in terms of  $\pi$ .

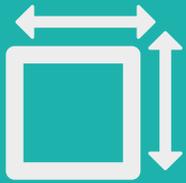
## Discussion

- Discuss the importance of having a formula and using a specified formula to calculate the area of circles.

## Alternate Demonstration of Area Formula

- Consider the following demonstration for an alternate method of demonstrating the area of a circle <https://www.geogebra.org/m/AJSmWRzp>





# LESSON 5

## Composite areas

### Overview of lesson

Students investigate the use of signage to promote business. Students are to look at the signs provided (teachers to provide) to calculate the area and perimeter of each sign. Students then create their own sign to prescribed dimensions.

### Background knowledge

- Students can identify and calculate the areas of the rectangle, square, triangle, trapezium, rhombus and kite.
- Students can find the sum of two or more areas.
- Students can identify two or more different shapes that can be joined together.

### Language considerations

- Students should be able to communicate using the following language: composite, shaded, area and total in context in order to find composite areas.

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-13MG uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area.

### Syllabus content

Students:

- find the areas of simple composite figures that may be dissected into rectangles, squares, parallelograms, trapeziums, rhombuses, kites and triangles.

### Resources needed

- Grid paper
- Ruler
- Pencils
- Prepared signage electronically or printed

## Teacher instructions

### Step 1

- Show students a simple shop sign.
  - Students calculate the perimeter and area of the sign (in this task the dimensions must be given, as the sign may not be to scale).
  - Establish the correct area and perimeter for the sign to make sure the students have calculated them correctly.
- 

### Step 2

- Give each student a blank piece of paper.
  - Students design a sign for an appropriate business of their choice using composite shapes, with given parameters.
  - Parameters could include
    - fixed area - between 1.5 and 2 m<sup>2</sup>
    - two composite shapes
    - three composite shapes
    - set shapes given
- 

### Step 3

- Students are to calculate the area of their sign.
  - Students are to calculate the perimeter of their sign.
  - All dimensions, area and perimeter should be clearly labelled on their sign design.
- 

### Technology/Alternate task

- Students can use technology to design and label their sign.
  - Students can print design to display to the class.
- 

### Extension activity

- Students research the cost of material to make a sign and have it installed.
  - Students give a description of the process used to form their design and why their design was chosen.
  - Students provide a response as to the effectiveness of their sign. (For example, is the sign eye catching, is the size adequate or too big, can it be read easily, what shapes were chosen and why were they configured in such a way).
- 

### Questioning

- Why is signage important?
  - Why is it important for business owners to be able to calculate the perimeter and area of a sign they wish to purchase and install?
  - What implications are there for the business owner if the perimeter and area is calculated incorrectly?
- 

### Discussion

- Discuss the accuracy of the measurements of each sign designed.
-



# Teaching Measurement

## VOLUME





# LESSON 1

## Drawing solids

### Overview of lesson

Students investigate, draw and construct solids from different views

### Background knowledge

Students should understand that a solid has three dimensions (length, width and depth).

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### Language considerations

Students should be able to communicate using the following language: solid, face, orientation, front view, side view and top view.

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### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-14MG uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume.

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### Syllabus content

Students:

- draw (in two dimensions) prisms, and solids formed from combinations of prisms, from different views, including top, side, front and back.
- 

### Resources needed

- Centicubes
- 1 cm grid paper
- Views sheet

## Teacher instructions

### Step 1

- Prior to the lesson, the teacher will need to construct a solid using 10 centicubes.
  - Divide the students into groups of 3 and give each group some centicubes (more than 10).
  - The teacher conceals the solid they have constructed at the front of the room.
  - Group members take turns to come up and look at the solid (for 3 seconds) and take what they have seen back to the group to try and construct the same solid as the teacher.
  - Discuss with the class what strategies they used when looking at the solid.
  - Repeat this activity using a different solid constructed by the teacher or by one member of the group with the other two members trying to build it.
- 

### Step 2

- Give each group a copy of the **View Placemat**. Teacher to construct a solid (using centicubes or other blocks, in line with the ability of the students) and show to students.
  - Students construct the same solid using their centicubes and place the solid on the view worksheet.
  - Each student then draws the solid from the views: front, left, back, right, top (encourage students to walk around the solid to visualise the solid from each view).
- 

### Step 3

- Students use 10 centicubes to construct their own solid and then sketch it from the 5 views.
- 

### Step 4

- Students construct a solid that has the views shown on the **View Worksheet**.
  - Discuss whether there is only one solid that has the following views or are the other possibilities?
- 

## Alternatives

- Students could investigate the different views of some common solids e.g. cube, rectangular prism, triangular prism, cylinder, sphere.
- 

## Extension

- Students could draw their constructed solids on isometric dot paper (alternatively students could use an online application to aid e.g. <https://illuminations.nctm.org/Activity.aspx?id=4182>).

**View Placemat**

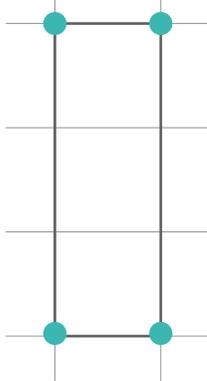
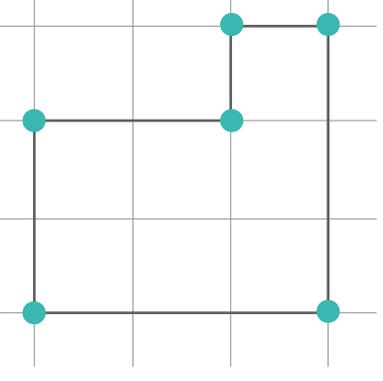
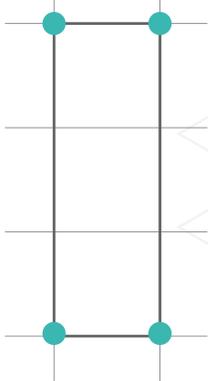
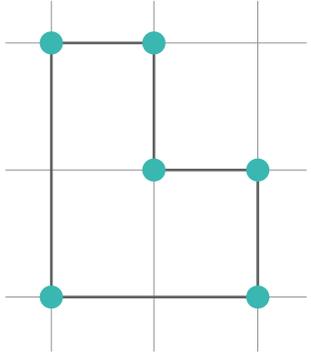
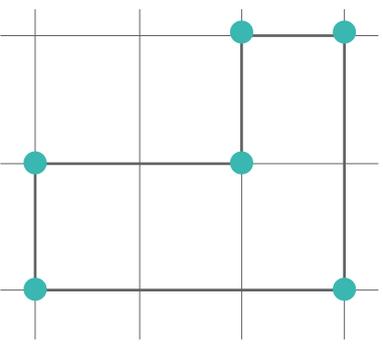
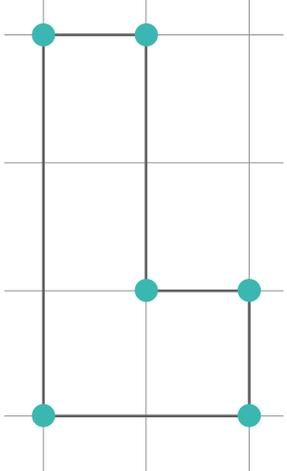
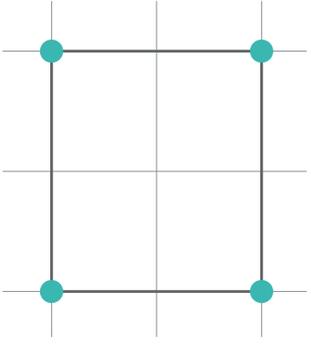
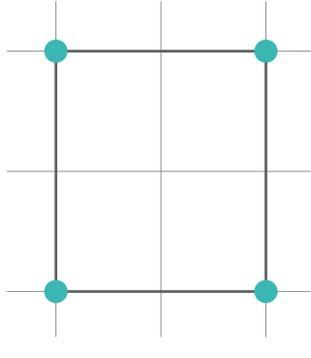
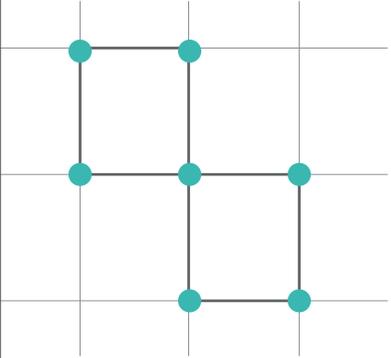
**BACK**

**LEFT**

**RIGHT**

**FRONT**

# View Worksheet

Front	Right	Top
		
		
		



# LESSON 2

## Cross-sections

### Overview of lesson

Students use everyday objects to investigate the cross-section of prisms, identify solids in their immediate environment that have either a uniform or non-uniform cross-section leading to deriving the formula for finding the volume of a prism.

### Background knowledge

Students should know how to identify solids and be able to visualise them from different views.

---

### Language considerations

Students should be able to communicate using the following language: solid, prism, cross-section, uniform, non-uniform, base, volume.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-14MG uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume.

---

### Syllabus content

#### Students:

- identify and draw the cross-sections of different prisms.
- recognise that the cross-sections of prisms are uniform.
- visualise, construct and draw various prisms from a given cross-sectional diagram.
- determine if a particular solid has a uniform cross-section.
- distinguish between solids with uniform and non-uniform cross-sections.
- develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume.

- develop the formula for the volume of prisms by considering the number and volume of 'layers' of identical shape:
  - volume of prism = base area  $\times$  height leading to  $V = Ah$
- recognise the area of the 'base' of a prism as being identical to the area of its uniform cross-section.
- find the volumes of prisms, given their perpendicular heights and the areas of their uniform cross-sections.
- find the volumes of prisms with uniform cross-sections that are rectangular or triangular.
- solve a variety of practical problems involving the volumes and capacities of right prisms.

## Resources needed

- A range of prisms and cylinders
- Prism with slices stacked horizontally e.g. loaf of bread or a slab cake
- Prism with slices stacked vertically e.g. cheese slices or a ream of paper

## Teacher instructions

### Step 1

- Show students a range of solids and have them identify the name of the solid justifying their choice (check for language used to describe 'base' of a prism and reinforce the use of the term uniform cross-section).

### Step 2

- In groups of 3-4 students brainstorm real-life examples for each prism from step 1, e.g. cylinder - food can.
- Collate each groups brainstorm to compile an exhausted list on the board.

### Step 3

- Teacher to show students a sliced loaf of bread, cake or block of Toblerone (stacked horizontally) - ask students to comment on the shape of each slice/piece.
- Show students another type of solid e.g. cheese slices or a ream of paper (stacked vertically) - ask students to comment on the shape of each slice/piece.
- Show students a solid with a non-uniform cross section (e.g. cone, a child's stackable toy etc.) and ask students to comment on the shape of the cross-section.
- Students identify solids within the classroom that have a uniform and non-uniform cross-sections (5 of each).
- Question: what happens if you can't slice the shape? How do you know if it has a uniform cross-section or not?

### Step 4

- Distribute slices of bread to pairs and have students name the shape of the cross-section of a loaf of bread.
- Have students calculate the area of slice that represents the cross-section of the loaf of bread.
- Reinforce that the cross section is the same as the base of the solid even if it is not represented as being at the bottom in the way that it is drawn.
- Discuss how to find the volume of the entire loaf of bread by stacking the slices on top of each other i.e. needing to calculate the height of each slice and adding them together, leading to the formula  $V = Ah$ .

## Step 5

- Students complete the **Volume Formula Sheet**.
- 

## Step 6

- Students use the relevant formula to calculate the volume of the solids identified in step 1.
- 

## Alternatives

- Students construct prisms using clay or kinetic sand and cut the prisms to investigate its cross-section.
- 

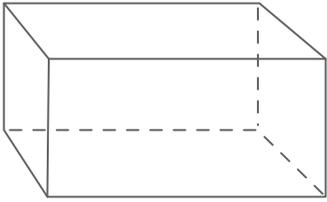
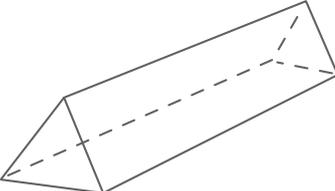
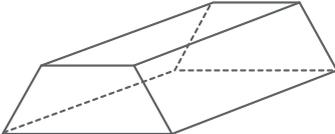
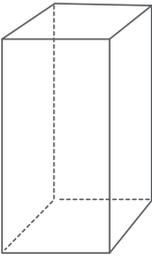
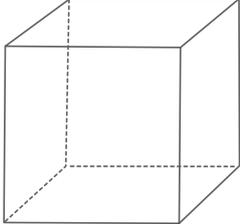
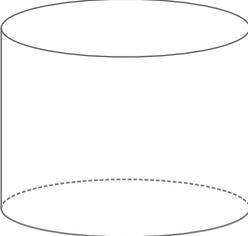
## Extension

- Use the solids students constructed in the previous lesson using centicubes, and sort according to whether they have a uniform, or non-uniform cross section.
  - For those with a uniform cross section, have students work out and draw the cross section.
  - Students can then calculate the volume for these solids using  $V = Ah$  and check by counting number of cubes used to build the solid.
- 

## Questioning

- If I cut along the height of a triangular prism and distributed the pieces would it be fair?

## Volume Formula Sheet

Prism	Name	Shape of cross section (base)	Area of base	Volume formula
	Rectangular prism	Rectangle	$A = \text{length} \times \text{width}$	$V = (\text{length} \times \text{width}) \times \text{height}$
	Triangular prism			
	Trapezoidal prism			
	Square prism			
	Cube			
	Cylinder			



# LESSON 3

## Volume and capacity

### Overview of lesson

Students construct solids to assist with the conversion of units of volume and investigate through practical methods how to convert volume to capacity. Students will have the opportunity to apply their understanding to calculating the capacity of an object within the school.

### Background knowledge

Students should know how to calculate the volume of prisms and cylinders.

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### Language considerations

Students should be able to communicate using the following language: cubic metre, cubic centimetre, cubic millimetres, volume, capacity, millilitre, litre and kilolitre.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-14MG uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume.

---

### Syllabus content

#### Students:

- choose appropriate units of measurement for volume and convert from one unit to another.
- recognise that 1000 litres is equal to one kilolitre and use the abbreviation for kilolitres (kL).
- recognise that 1000 kilolitres is equal to one megalitre and use the abbreviation for megalitres (ML).
- choose an appropriate unit to measure the volumes or capacities of different objects, e.g. swimming pools, household containers, dams.
- use the capacities of familiar containers to assist with the estimation of larger capacities.
- convert between metric units of volume and capacity, using  $1 \text{ cm}^3 = 1000 \text{ mm}^3$ ,  $1 \text{ L} = 1000 \text{ mL} = 1000 \text{ cm}^3$ ,  $1 \text{ m}^3 = 1000 \text{ L} = 1 \text{ kL}$ ,  $1000 \text{ kL} = 1 \text{ ML}$ .

## Resources needed

- Straws or pipe cleaners (for cubic metre - approximately 48 pieces per cubic metre)
  - Sticky tape
  - Centicubes
  - Jars that can be filled (depending on the ability of the class you might like to have the capacity marked on them e.g. soft drink can)
  - Measuring cups or measuring jugs
- 

## Teacher instructions

### Step 1

- Students are going to construct a cubic metre.
  - Some strategies for supporting students to construct the cubic metre include:
    - start with a square metre which they can use as a template for constructing their cube.
    - students could make 12 x 1 m lengths with the straws/pipe cleaners, cardboard strips, rolled newspaper and then put them together to make the cube.
    - students to work in 6 groups to each group constructing a square metre and then put the 6 square metres together to make the cube.
    - alternative instructions detailed at the end of the lesson.
- 

### Step 2

#### Discuss with students:

- how many cubic centimetres would fit inside a cubic metre? ( $100 \times 100 \times 100 = 1\,000\,000$ ).
  - how many cubic millimetres would fit inside a cubic centimetre? ( $10 \times 10 \times 10 = 1\,000$ ).
  - how many cubic millimetres would fit inside a cubic metre? ( $1\,000 \times 1\,000 \times 1\,000 = 1\,000\,000\,000$ ).
- 

### Step 3

- Give students solids that can be filled with water (differentiate solids based on the ability of the students).
- Have students estimate the capacity of the solid and then calculate the volume of the solid (in cubic centimetres).
- Fill the solid with water and tip the water into a measuring cup/jug and compare volume (in  $\text{cm}^3$ ) to capacity (in mL). (Students should find that the volume and capacity are equivalent which would lead them to the conclusion that  $1 \text{ cm}^3 = 1 \text{ mL}$ ).
- Students record their results in the table on the **Volume and Capacity Recording Sheet**.

Note: To assist students with estimating it may be useful to have a litre of water on display at the front of the classroom.

---

### Step 4

- If we know that  $1 \text{ mL} = 1 \text{ cm}^3$ , how many millilitres can we fit into our cubic metre? How can we express this in L, kL or ML?

## Step 5

- Students calculate the number of litres taken to fill a water tank in the school or a swimming pool.
  - Note: If possible, take the students to the object so that they can measure the dimensions of the solid themselves in order to calculate volume.
- 

## Extension

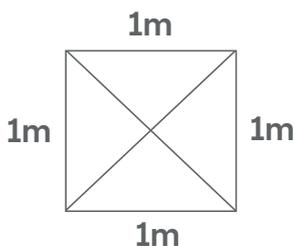
- Use the cubic metres to estimate the volume of the classroom.
  - Calculate how much water could be harvested from the school's rooftops.
    - Using Google Earth or similar capture an image of the school buildings.
    - Find a roof that is flat and calculate the area of the roof.
    - Establish your areas annual rainfall and then calculate the volume of water you could harvest. (table provided at <https://www.watercorporation.com.au/~media/files/teachers/save-water/greenfingers-rainwater-harvesting-carey-baptist-college.pdf>).
    - Convert to L, kL and ML.
- 

## How to build a cubic metre

- Materials needed: metre rulers, newspaper, tape scissors.

### Instructions:

- Step 1: Working in small groups, create two squares that are exactly one metre by one metre using tape and sheets of newspaper tightly rolled into cylinders.
- Step 2: Create an X shaped brace for each square, using more newspaper cylinders, to ensure that they remain square.



- Step 3: Join with three other groups and join/bind your six square faces to form a cubic metre.

## Volume and Capacity Recording Sheet

Object (name & sketch)	Estimate of capacity	Volume	Capacity



# LESSON 4

## Designing a drink can

### Overview of lesson

Students work with the volume of a cylinder formula to find dimensions that produce a given capacity. Students will apply their knowledge of the volume of a cylinder formula to the design of a new drink can.

### Background knowledge

Students will need to know how to calculate the volume of a cylinder. It would be helpful if students had some understanding of how to rearrange formulae or solve equations.

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### Language considerations

Students should be able to communicate using the following language: cylinder, height, radius, diameter, dimensions, volume and capacity.

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### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-14MG uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume.

---

### Syllabus content

#### Students:

- calculate the volumes of cylinders and solve related problems.
- develop and use the formula to find the volumes of a cylinder.
- recognise and explain the similarities between the volume formulas for cylinders and prisms.
- solve a variety of practical problems involving the volumes and capacities of right prisms and cylinders, e.g. find the capacity of a cylindrical drink can or a water tank.

## Resources needed

- Cardboard
  - Sticky tape/glue
  - Drink cans (for demonstration purposes)
- 

## Teacher instructions

### Step 1

- In groups of 3 students explore the dimensions and capacity of a drink can. Students should record the dimensions, calculate the volume and record the capacity.
  - Discuss with students any reasons for error in measurement (comparing the calculated volume and the stated capacity).
  - Discuss why a drink can has its particular dimensions? (e.g. fits in doors of refrigerators, vending machines, easy to hold, easy to pour).
- 

### Step 2

- In groups of 3, students are to design a new drink can that has a capacity of 400 mL.
  - Students are first to determine a set of dimensions (diameter/radius and height) that produce a capacity of 400 mL. Students are to record the strategy they used to determine these dimensions.
  - Students then use cardboard to construct a can using the dimensions they have determined.
  - Students are to record a justification for their choice of dimensions.

Note: You may need to issue students with a sheet containing a net of a cylinder to help them start thinking about their can's dimensions (an example is given at the end of the lesson).

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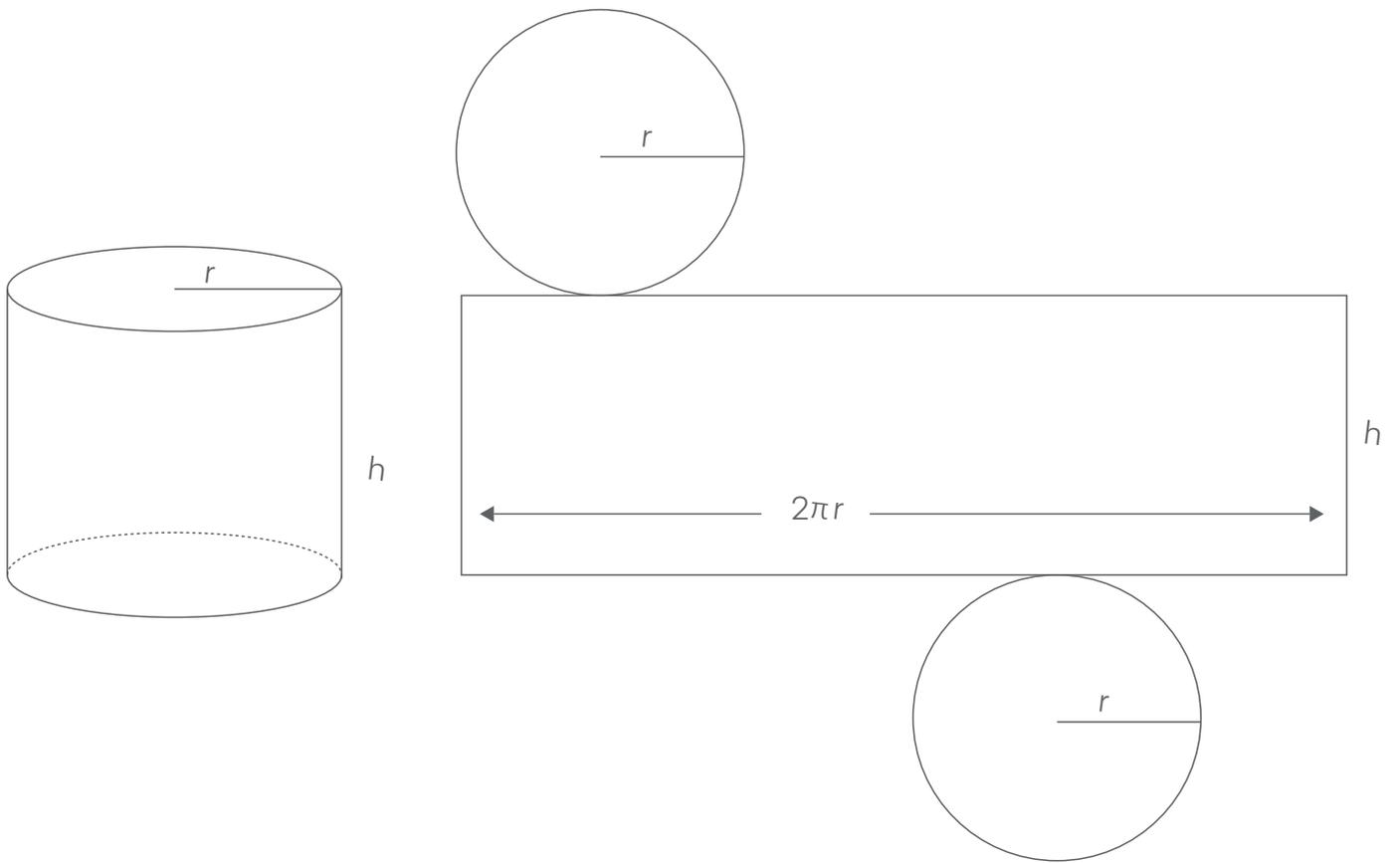
### Step 3

- In the same group, students are to name their drink and design a label for their can.
- 

## Extension

- Students calculate the area of the label both with and without an overlap.
- From Lesson 3 and the Water Harvesting Activity:
  - students calculate the dimensions of various water storage tanks (cylindrical, rectangular prism etc.) that the school would need to buy to store the water from the roof.
  - students explore what the saved water could be used for throughout the school.
  - students could explore the cost of various water tanks and write a proposal to the school's Executive outlining financial and environmental benefits of harvesting rainwater.

## Net for Step 2







# Teaching Measurement TIME





# LESSON 1

## Measuring in units of time

### Overview of lesson

Students will complete one lap of a designated route within the school. Students measure the time taken to complete the activity and arrange these times in ascending order.

### Background knowledge

Students should be able to:

- use a stopwatch.
  - read time in hours, minutes, seconds and milliseconds.
- 

### Language considerations

Students should be able to communicate using the following language: hour, minute, second, millisecond.

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### Syllabus outcomes

MA3-2WM selects and applies appropriate problem-solving strategies, including the use of digital technologies, in undertaking investigations.

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

---

### Syllabus content

Students:

- use a stopwatch to ensure and compare the duration of events.
- order a series of events according to the time taken to complete each one.

## Resources needed

- Stopwatches (alternatively students to use their device -phone/wristwatch).
  - Determine a route within the school that the students can walk/jog/jump around.
    - Pre-measuring the length of this route would help with the extension activity at the end of the task when determining the rate of the activity, or, commence the activity by having students measure the length of the route.
  - Negotiate with other school personnel about the use of an outside space for the activity.
- 

## Teacher instructions

### Step 1

#### Whilst in the classroom:

- introduce the task by explaining that students will be measuring the time taken to complete one lap of the designated route.
  - discuss the different methods of completing this lap, i.e. jog, walk, skip, walk backwards etc.
  - discuss which method students think will take the shortest and longest amount of time and explain why?
  - discuss use of measuring equipment (stopwatch) to obtain accurate measurements when completing the task.
  - discuss the units of measurement which will be appropriate (hours, minutes, seconds, milliseconds).
  - assign groups (3-4 students per group) and activities/movements.
  - distribute 2-3 stopwatches per group.
  - escort students to the start point.
- 

### Step 2

#### In groups students:

- estimate how long they think it will take to complete their movement around the route, for a particular movement type.
  - complete the route with their assigned movement, whilst other members of the group time them.
  - complete the route several times, with each member of the group timing and completing the movement lap.
  - record their results in the table provided.
- 

### Step 3

#### On return to class room:

- have a group discussion, on the time taken to complete each movement.
  - students arrange movements in order from shortest to longest. Were the results expected?
  - Whole group discussion on findings compared with estimates.
- 

## Alternate task

- If unable to use designated a route within the school, the task can be modified to the time taken to complete a particular activity in the classroom (hopping on one leg, specific number of star jumps etc.)

## Extension activities

- Vary the degree of accuracy of the measurement (seconds, milliseconds).
  - Students calculate the rate for their activity (e.g. m/s, km/h, etc.)
  - Estimate how long it would take them to complete the cross country track using each of the different types of movement.
- 

## Technology

- Compare reaction times using an online measuring tool e.g. <https://www.humanbenchmark.com/tests/reactiontime>.
- 

## Questioning

- How are you going to ensure that each movement is measured in the same way for the different groups or group members?
- 

## Discussion

- When two people in the group timed, did you both get the same time? Why? Why not?
- Were there some measurements which had very big differences?

# HOW LONG DOES IT TAKE?

Movement	Estimation	Time 1	Time 2
<b>run</b>			
<b>walk</b>			
<b>skip</b>			
<b>jump</b>			
<b>walk backwards</b>			



# LESSON 2

## Converting between 12-hour and 24-hour time

### Overview of lesson

Students place analog or digital times on a 24-hour number line.  
Converting between analog and 24-hour time

### Background knowledge

Students should be able to:

- recognise and interpret 24 hour time.
  - Typically, 24-hour time is recorded without the use of the colon (:), e.g. 3:45 pm is written as 1545 or 1545h and read as 'fifteen forty-five hours'. Discuss whether you have seen it recorded differently i.e. 15.45 or 15:45 etc. and where.
- read analog time.
- read digital time.
- recognise the difference between am and pm.

### Language considerations

Students should be able to communicate using the following language: digital, analog, 12-hour time, 24-hour time, hour, minute, second, and am and pm notation.

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-3WM recognises and explains mathematical relationships using reasoning.

MA3-13MG uses 24-hour time and am and pm notation in real-life situations, and constructs timelines

### Syllabus content

Students:

- compare 12-hour and 24-hour time and convert between them.
- tell the time accurately using 24-hour time.
- convert between 24-hour time and time given using am or pm notation.

## Resources needed

- String or rope that can be placed across the front of the room to be used as a number line (alternatively drawing a number line on the board will also work, it would just be much shorter). Number line should extend from 0000 hours to 2400 hours
  - Cards with times marked in 12-hour analog or digital time, including of course am and pm and 24-hour times. These could be on post-it notes for ease of sticking on number line on board). Enough cards for each student to have at least one
- 

## Teacher instructions

### Step 1

- Place the number line at the front of the room so that all students can see it.
  - Give each student at least one card with a time recorded in 12-hour time using either digital or analog format.
  - Explain that each student will come forward (one at a time) and place their time on the 24-hour number line, determining the appropriate position on the number line.
- 

### Step 2

- Each student places their allocated time on the 24-hour number line.
  - Students explain how or why they chose the particular position.
  - Class discussion about the accuracy of the position of the time on the number line.
  - Students then write selected times their workbook, (including both the 12-hour and 24-hour time representations).
- 

## Questioning

- Why did you place your time in that position?
  - Is that time before or after midday? Before or after 1200?
  - What is your time in 24-hour notation?
- 

## Discussion

- Why is 24-hour time used?
- Why is 12-hour time used?
- Who uses 24-hour time? Why?



# LESSON 3

## Planning a trip

### Overview of lesson

- Students plan a journey from Sydney to Dreamworld on the Gold Coast using timetables provided. They are to use three modes of transport (i.e. plane, train and bus) and arrive at Dreamworld by 10 am
- Students create a timeline of the journey. Alternatively, students can plan their own journey relevant to the school and a location. The journey must include at least two modes of transport

### Background knowledge

Students should be able to:

- recognise and interpret 24 hour time.
- recognise the difference between am and pm.
- read and interpret a timetable.
- calculate the difference between times in both 12-hour and 24-hour notation.
- use a calculator to calculate time differences.
- construct a timeline on a number line with accuracy.

---

### Language considerations

Students should be able to communicate using the following language: 12-hour time, 24-hour time, hour, minute, second, am and pm notation, daylight saving, local time, time zone.

---

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM: applies appropriate mathematical techniques to solve problems.

MA3-13MG: uses 24-hour time and am and pm notation in real-life situations, and constructs timelines.

MA4-15MG: performs calculations of time that involve mixed units, and interprets time zones.

# Syllabus content

## Students:

- compare 12-hour and 24-hour time and convert between them.
  - tell the time accurately using 24-hour time.
  - convert between 24-hour time and time given using am or pm notation.
  - compare the local times in various time zones in Australia being aware of daylight saving.
  - solve problems involving duration, including using 12-hour and 24-hour time within a single time zone.
  - add and subtract time mentally using bridging strategies (e.g. from 2:45 to 3:00 is 15 minutes and from 3:00 to 5:00 is 2 hours, so the time from 2:45 until 5:00 is 15 minutes + 2 hours = 2 hours 15 minutes).
  - add and subtract time with a calculator, including by using 'degrees, minutes, seconds' button.
  - solve a variety of problems involving duration, including where times are expressed in 12-hour and 24-hour notation, that require the use of mixed units (years, months, days, hours, and/or minutes).
- 

## Resources needed

- Copy of **Planning a Journey** worksheet, one for each student
  - Ruler for the construction of number line for timeline.
- 

## Teacher instructions

### Step 1

#### Explanation to students, they will be:

- planning a trip for students, travelling by plane, train and bus.
  - using timetables and other time constraints to determine which plane, train and bus they will need to catch to be at the destination by 10 am.
  - calculating travel time, and determining start time of the journey.
  - completing a timeline of their journey, showing all activities.
- 

### Step 2

- Distribute each student with the worksheet **Planning a Journey** worksheet Read the following information about the task to the students.
    - **What:** weekend to the Gold Coast to visit Dreamworld
    - **Flights:** Sydney to Brisbane
    - **Train:** Brisbane airport to Coomera railway station
    - **Motel:** 10min walk from Coomera railway station
    - **Bus:** motel to Dreamworld
    - **Dreamworld opening time:** 10am
  - **Additional information:**
    - you must arrive at Sydney airport an hour before the plane is due to depart.
-

- you must allow half an hour at Brisbane airport to collect your luggage
  - you can book into your accommodation and leave your luggage before going to Dreamworld (allow half an hour for this)
  - Students organise transport arrangements in the table on the worksheet.
- 

### Step 3

- Students to draw a number line/timeline of their journey from Sydney to Dreamworld.
  - Timeline must start from the time you arrive at the airport to check in, and finish on arrival at Dreamworld (i.e.10 am).
  - All parts of the journey need to be reflected in the timeline and labelled appropriately.
- 

### Questioning

- Aeroplane flights from Sydney to Brisbane take more than an hour. Why do these flights appear to take only 20 minutes?
  - How long are the flights from Sydney to Brisbane?
  - What scale was used on the timeline? Why did you choose this? Was there a more appropriate scale that you could have used?
- 

### Check

#### Students have:

- drawn the number line/timeline accurately.
  - used an appropriate scale.
  - included all transport options.
  - allowed for time to travel between places.
  - allowed time for checking in/out.
- 

### Discussion

- How did you calculate your times?
  - It is often easier to work backwards when organising a journey. To find out when you want to be somewhere (10 am at Dreamworld) and then work backwards in time, making sure you link the times correctly and give yourself enough time to get between the different modes of transport. Transport choices are usually decided by convenience, money and time.
- Is this the best route? What makes it the best route?
- Could we have left Sydney at a better time?
- Discuss changes in airport time arrivals for luggage check-in or just carry-on.

# Planning a Journey Worksheet

## What

- Weekend to the Gold Coast to visit Dreamworld
- 

## Flights

- Sydney to Brisbane
- 

## Train

- Brisbane airport to Coomera railway station
- 

## Motel

- 10min walk from Coomera railway station
- 

## Bus

- Motel to Dreamworld
- 

## Dreamworld opening time

- 10am
- 

## Additional information:

- you must arrive at Sydney airport an hour before the plane is due to depart.
- you must allow half an hour at Brisbane airport to collect your luggage.
- you can book into your accommodation and leave your luggage before going to Dreamworld (allow half an hour for this).

## Timetables

### Flights from Sydney Domestic to Brisbane Domestic Airports on a Saturday

Departs from Sydney Airport	Arrives Brisbane Airport
6.30	6.50
8.00	8.20
9.05	9.25
10.45	11.05
14.20	14.40

### Bus from Coomera Station to Dreamworld (Daily service, except Christmas day and Anzac day)

Departs Coomera Station	Arrives Dreamworld
8.55	9.00
9.30	9.35
10.00	10.05
10.45	10.50

### Brisbane Railway from Domestic Airport to Coomera Station Sat, Sun, Public Hols

Departs Airport Railway Station	Arrives Coomera Station
6.24	7.48
7.30	8.45
7.56	9.20
8.30	9.54
9.15	10.40

# MY TRIP PLANNER TO DREAMWORLD

<b>from</b>		<b>to</b>	
<b>departs</b>		<b>arrives</b>	
<b>from</b>		<b>to</b>	
<b>departs</b>		<b>arrives</b>	
<b>from</b>		<b>to</b>	
<b>departs</b>		<b>arrives</b>	



# LESSON 4

## Calculations involving time

### Overview of lesson

Students add and subtract hours, minutes and seconds

### Background knowledge

Students should be able to:

- use strategies such as bridging or compensation to add or subtract numbers.
- state that there are 60 minutes in one hour, and convert between minutes and hours.
- calculate number of weeks in year, months in year, hours in a day.

---

### Language considerations

Students should be able to communicate using the following language: hour, minute, second, difference, time difference, elapsed time.

---

### Syllabus outcomes

MA3-13MG: uses 24-hour time and am and pm notation in real-life situations, and constructs timelines.

MA4-2WM: applied appropriate mathematical techniques to solve problems.

MA4-15MG: performs calculations of time that involved mixed units, and interprets time zones.

---

### Syllabus content

Students:

- solve problems involving duration, including using 12-hour and 24-hour time within a single time zone.
- add and subtract time mentally using bridging strategies (e.g. from 2:45 to 3:00 is 15 minutes and from 3:00 to 5:00 is 2 hours, so the time from 2:45 until 5:00 is 15 minutes + 2 hours = 2 hours 15 minutes).
- add and subtract time with a calculator, including by using 'degrees, minutes, seconds' button.
- round answers to time calculations to the nearest minute or hours.
- interpret calculator displays for time calculations, e.g. 2.25 on a calculator display for a time calculation means 2  $\frac{1}{4}$  hours or 2 hours 15 minutes.

- solve a variety of problems involving duration, including where times are expressed in 12-hour and 24-hour notation, that require the use of mixed units (years, months, days, hours, and/or minutes).

---

## Resources needed

Up to 3 (at least 2) dice for each group (these can be easily made using blank, writable dice)

- One die to have whole numbers less than 10
- One die to have 5, 10, 15, 20, 30, 45 (representing minutes)
- One die to have 5, 10, 15, 20, 30, 45 (representing seconds)

Note: this third die is optional depending on the ability of students

---

## Teacher instructions

### Step 1

- Before activity explain to the students the concept.
  - Students will be rolling all dice and recording the time shown in hours, minutes and seconds.
  - Students then roll the dice again and record the time shown again in hours, minutes and seconds.
  - Students then calculate the sum and difference of the times, recording their answers in hours, minutes and seconds.
- Demonstration of bridging or compensation strategies as required.
- Calculators may be used depending on the level of the students.

---

### Step 2

- Students complete the activity and record answers in the table provided.
- 

### Time Total and Difference

Time 1	Time 2	Sum	Difference

### Step 3

- How many years/months/days in the class?
  - Each student to calculate their age in years, months and days (task could be varied depending on ability level i.e. nearest year, month etc.).
  - Each student records their age on the board or via a shared document.
  - Students work in groups to calculate the combined age of the students in the class.
- 

### Questions

- What is the biggest difference between the ages?
  - What is the smallest difference between the ages?
  - What will be the combined age of the class in one year's time?
- 

### Step 4

- The shortest day of the year (called the winter solstice) occurs in the southern hemisphere. In NSW this is around 21 June each year. The day is approximately 10 hours 24 minutes and 6 seconds long. The longest day of the year is 13 hours 53 minutes and 7 seconds long.

Question: What is the time difference in time between the longest and shortest day of the year?

.....

### Questioning

- Advertisements for companies often suggest that there may be “110 years of experience”. How is this possible if the company has only been open for 30 years?
  - When does the longest day of the year in the Southern Hemisphere occur? Why?
- .....

### Discussion

- How long does an average person spend asleep in their lifetime?
- .....

### Extension

- Blink Activity (details at the end of this lesson).

## Extension: On the Blink

1. Copy the following table.

Student Name	Trial 1	Trial 2	Mean blinks per minute

2. Get comfortable and relax. Have a partner watch your eyes and count how often you blink in a minute. Do this twice. Record your results in the table and find the mean.
3. Repeat the experiment with you observing your partner. Record your results.
4. Calculate how often you each blink in the following period. (Write your answers in scientific notation, correct to two significant figures.)
  - e. an hour
  - f. a waking day of 16 hours
  - g. a year
  - h. an average lifetime of 75 years
9. If a blink takes approximately 0.5 seconds, calculate how long you will each have your eyes closed in the following periods.
  - j. a minute
  - k. an hour
  - l. a waking day
  - m. a waking year
  - n. an average waking lifetime



# LESSON 5

## Time zones

### Overview of lesson

Students investigate time zones around the world

### Background knowledge

Students should be able to:

- identify that Australia is divided into three main time zones (in non-daylight saving periods)  
–see attachment at the end of the lesson.
- identify that Daylight saving occurs in Australia for some states during the Summer months.  
Daylight Saving Time (DST) is the practice of advancing clocks one hour during the warmer months of the year.

### Language considerations

Students should be able to communicate using the following language: hour, minute, second, difference, time difference, elapsed time, Universal Time (UT), time zone, daylight saving, and Australian Eastern Standard Time (AEST).

### Syllabus outcomes

MA3-13MG: uses 24-hour time and am and pm notation in real-life situations, and constructs timelines.

MA4-1WM: communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM: applies appropriate mathematical techniques to solve problems.

MA4-15MG: performs calculations of time that involved mixed units, and interprets time zones.

### Syllabus content

Students:

- compare 12-hour and 24-hour time and convert between them.
- tell the time accurately using 24-hour time.
- convert between 24-hour time and time given using am or pm notation.
- compare the local times in various time zones in Australia, including daylight saving.
- solve problems involving duration, including using 12-hour and 24-hour time within a single time zone.

- add and subtract time mentally using bridging strategies (eg. from 2:45 to 3:00 is 15 minutes and from 3:00 to 5:00 is 2 hours, so the time from 2:45 until 5:00 is 15 minutes + 2 hours = 2 hours 15 minutes).
- add and subtract time with a calculator, including by using 'degrees, minutes, seconds' button.
- solve a variety of problems involving duration, including where times are expressed in 12-hour and 24-hour notation, that require the use of mixed units (years, months, days, hours, and/or minutes).

---

## Resources needed

- Students will need to have access to the internet for this task as they will be searching online trip planners to prepare an itinerary.
- Class set of time zones around the world <https://i.pinimg.com/564x/71/a9/77/71a977243b39aed76ecad161741bbf3c--world-time-zones-time-zone-map.jpg>

---

## Prior to lesson

- Arrange for use of computers during lesson.
- Identify an event happening in a different time zone anywhere in the world that is of relevance to your students. (Adjust event and location to suit the ability of your class - for lower ability classes you might like to pick a location that is closer to home but still in a different time zone).
  - Olympics
  - World Cup
  - Eurovision
  - Movie premier
  - Rugby International
- Determine location, time difference with current location and starting time of the event.
- A short visual display (PowerPoint, YouTube clip etc.) on the event may be beneficial for students who are unfamiliar with it or its location.

---

## Teacher instructions

### Step 1

#### Explanation to students:

- introduce the lesson as an investigation of time differences in the world, incorporating different time zones in different countries.
- introduce the event that you have pre-prepared.
- show the visual display if available.

---

### Step 2

#### Questioning

- If an event starts at 4:00pm on Saturday in London, does it start at 4:00 pm on Saturday in Sydney? Why? Why not?
  - Without the use of a calculator or other electronic device, have students estimate whether it will be after 4:00 pm on Saturday afternoon in Sydney or before. Why? Why not?

### Step 3

- Using the map of time zones around the world, students calculate the time it would be in 5 other places at the start of this event.
- 

### Step 4

Itinerary planned for celebrating birthday several times in one year.

#### Pre-questioning

- Is it possible to have your birthday twice in one year? How? Why?
  - Is it possible for this to happen by flying from Sydney to New Zealand? Why? Why not?
  - Which way would you have to travel in order to be able to celebrate your birthday more than once in a year?
  - Are there other things to consider? Travel time?
- 

### Activity

- Students plan an itinerary that would allow them to have their birthday at least twice in one year.
  - Use of an online trip planner with departure and arrival times may help with this. e.g. <https://www.planapple.com>
- 

### Questions

1. Did everyone go to the same country? The same venue?
  2. What was the furthest distance travelled?
  3. What was the shortest?
- 

### Step 5

#### From the previous activity:

- students to come up with what they would do in each location to celebrate their birthday and present each birthday as a postcard.
  - this can be completed electronically and then emailed to teacher or uploaded to google drive or google classroom.
- 

### Questioning

- Using the map of time zones around the world in the additional resources.
- If it is Friday, 5 pm, Universal Time what day and time is it in Rio de Janeiro? (Rio is on the eastern coast of South America)
- If it is Monday, 11 am in Rio de Janeiro, what day and time is Universal Time?
- If it is Thursday 11:30 pm in Rio de Janeiro, what day and time is it in Sydney?
- How many hours ahead is Sydney to Rio de Janeiro?
- NSW has daylight saving during its summer months. How many hours ahead is Sydney to Rio de Janeiro during these months?
- Name a town, city, area or country that is in zone B.
- Is this place ahead or behind Universal Time?

## Discussion

- Using <https://www.timeanddate.com/time/map/> a really interesting discussion is to look at why the world time zones are not just straight vertical zones but zig-zag around borders etc.
- 

## Useful Online resources

- <https://www.timeanddate.com/time/map/>
- [http://www.convertit.com/Go/ConvertIt/World\\_Time/Time\\_Zones\\_Map\\_Large.ASP](http://www.convertit.com/Go/ConvertIt/World_Time/Time_Zones_Map_Large.ASP)

# AUSTRALIAN TIME ZONES

- With a land mass close to 7.7 million square kilometres, Australia is the world's sixth largest country and is divided into three separate time zones.
- 

## Australian Eastern Standard Time (AEST)

- Covers the eastern states of Queensland, New South Wales (with the exception of the town of Broken Hill), Victoria, Tasmania and the Australian Capital Territory.
  - AEST is equal to Coordinated Universal Time plus 10 hours (UTC +10).
- 

## Australian Central Standard Time (ACST)

- Covers the state of South Australia, the town of Broken Hill in western New South Wales and the Northern Territory.
  - ACST is equal to Coordinated Universal Time plus 9 ½ hours (UTC +9 ½).
  - Central Standard Time is 0.5 hour behind Eastern Standard Time.
- 

## Australian Western Standard Time (AWST)

- Covers Western Australia.
- AWST is equal to Coordinated Universal Time plus 8 hours (UTC +8).
- Western Standard Time is 2 hours behind Eastern Standard Time.

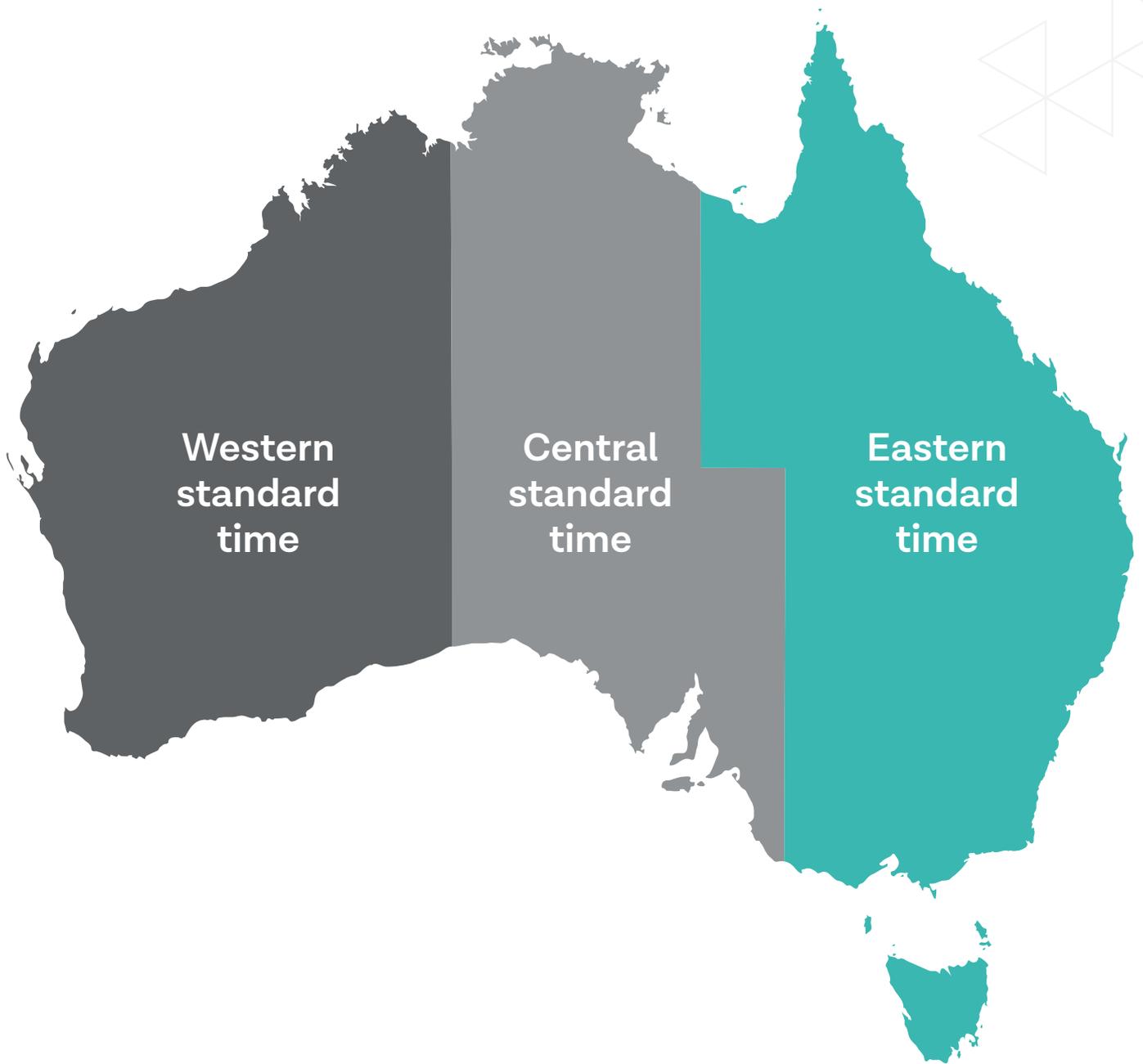
NOTE: When it is 12.00 noon in Sydney (Eastern Standard Time) it is 11.30 am in Adelaide (Central Standard Time) and 10.00 am in Perth (Western Standard Time).

---

## Daylight Saving Time

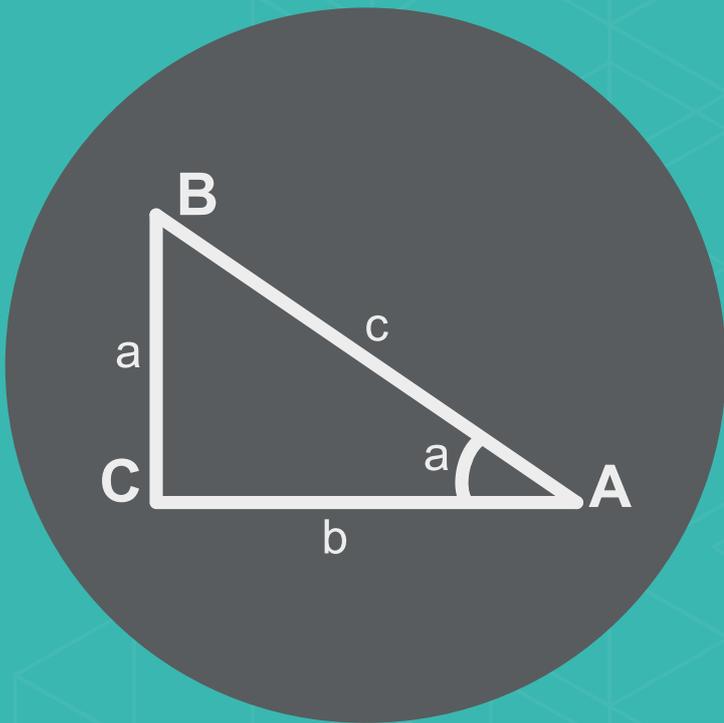
- Daylight Saving Time is the practice of advancing clocks one hour during the warmer months of the year. In Australia, Daylight Saving Time is observed in New South Wales, Victoria, South Australia, Tasmania, and the Australian Capital Territory.
- Daylight Saving Time begins at 2am (AEST) on the first Sunday in October and ends at 2am (AEST) (which is 3am Australian Eastern Daylight Time) on the first Sunday in April.
- Where Daylight Saving Time is observed:
  - NSW, ACT, Vic and Tas move from AEST to Australian Eastern Daylight Time (AEDT), and clocks are advanced to UTC +11.
  - SA and the NSW town of Broken Hill move from ACST to Australian Central Daylight Time (ACDT) and clocks are advanced to UTC +10 ½.
  - Daylight Saving Time is not observed in Queensland, the Northern Territory or Western Australia.

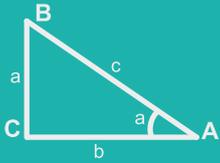
NOTE: Normally Australia has 3 time zones but in summer, because of daylight saving, there are 5 different time zones.





# Teaching Measurement PYTHAGORAS' THEOREM





# LESSON 1

## Irrational numbers

### Overview of lesson

Students should gain an understanding of irrational numbers, through the investigation of numbers which give exact and approximate answers.

### Background knowledge

Students should be able to:

- round a number to a given number of decimal places.
  - recognise and work with square numbers.
- 

### Language considerations

- Students should be able to communicate using the following language: square number, rational numbers, irrational numbers, square root, surd and integer.
  - Students need to understand the difference between an 'exact' answer and an 'approximate' answer.
- 

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-5NA operates with fractions, decimals and percentages.

---

### Syllabus content

Students:

- investigate the concept of irrational numbers.
- use technology to explore decimal approximations of surds.
- recognise that surds can be represented by decimals that are neither terminating nor have a repeating pattern.

## Resources needed

- Exact or approximate worksheet
- Scissors and glue
- Number line from 0 to 100 with 10 equally spaced intervals marked on it

## Teacher instructions

### Step 1

#### Students:

- draw two columns in their workbooks labelled 'exact answer' and 'approximate answer'.
- cut out **Exact or Approximate** questions into strips.
- place each question under the appropriate heading.
- come up with another item that could go under each heading.

## Questions

- What other examples did you come up with?
- How can you determine if a question is going to have an exact answer or an approximate answer?

### Step 2

- Using the blank number line students plot the first 10 square numbers on the number line.
- Students then write under each square number the product that gives the square number (e.g. under 4 write  $2^2$  etc.).

### Step 3

- Students pick a number on the number line that lies between two square numbers and calculate the square root of that number and record the answer in their workbook.
- Discuss students answers and look for similarities and patterns.
- e.g.  $\sqrt{5}$ ,  $\sqrt{6}$ ,  $\sqrt{7}$  and  $\sqrt{8}$  will all start with a 2 (i.e. 2. .... and the closer we get to 9 the closer the root is to 3).

### Step 4

#### Students:

- estimate the answer to different roots e.g.  $\sqrt{17}$ ,  $\sqrt{89}$  etc. by thinking about the ideas discussed in Steps 2 and 3.
- check answers using a calculator.

## Questioning

- How can you record answers to these root questions?
- When we round off a number, how do you let other people know that you have rounded off?
- Introduce the 'approximately equal to' notation (either  $\doteq$  or  $\approx$ ) and explain when it is used.

## Extension

- Using  $\sqrt{17}$  as an example students calculate its value the using a calculator answering to 5, 4, 3, 2 and 1 decimal places and recording these answers.
  - Students square each of their answers and make observations.

to 1 d.p.= 4.1	$4.1^2 = 16.81$
to 2 d.p. = 4.12	$4.12^2 = 16.9744$
to 3 d.p. = 4.123	$4.123^2 = 16.999129$
to 4 d.p. = 4.1231	$4.1231^2 = 16.99995361$
to 5 d.p. = 4.12311	$4.12311^2 = 17.0000360721$

- Draw attention to the fact that the only way to write  $\sqrt{17}$  exactly is to keep it as a square root, which is called a surd.
- 

## Questioning

- Is it possible to have a number, which is not a square whole number, which has a terminating root?
  - Using  $\sqrt{12.25}$  as an example, have students explain why  $\sqrt{12.25} = 3.5$  and  $\sqrt{6.25} = 2.5$
  - Get students to find four decimals between 0 and 100 that have a terminating square root e.g.  $\sqrt{23.04} = 4.8$
  - Is it possible for fractions to be square numbers? E.g.  $\sqrt{\left(\frac{4}{9}\right)} = \frac{2}{3}$
- 

## Discussion

- If we want an exact answer, why do we leave it as a surd?

## Exact or Approximate Worksheet

Instructions:

Draw two columns in your workbook and label them Exact Answer and Approximate Answer

Consider the answers to the following questions and place each question in the column that matches its answer

$10 \div 5$

Number of smarties each child receives if 64 smarties are shared between 8 children

\$2 shared between 3 children

The measurement of the length of my foot

The number 12 written to 2 decimal places

$16 \div 5$

The distance between school and home

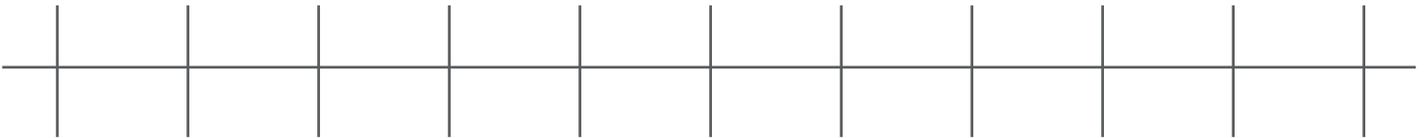
The number 9

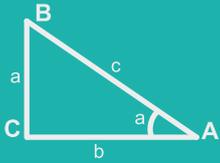
Number of drinks sold in the canteen today

$2 \div 3$  written to 2 decimal places

---

## Number line for step 2





# LESSON 2

## Discovering Pythagoras' theorem

### Overview of lesson

By dissecting squares and rearranging the parts students will gain an understanding of Pythagoras' theorem a statement of a relationship among the areas of squares rather than just being able to recite the formula

### Background knowledge

Students should be able to:

- identify a right-angled triangle.
- state the properties of a right-angled triangle.

### Language considerations

- Students should be able to communicate using the following language: shape, two-dimensional shape (2D shape), triangle, right-angled triangle, features, properties, side, opposite, length, vertex (vertices), angle, and right angle.
- A 'feature' of a shape or object is a generally observable attribute of a shape or object. A 'property' of a shape or object is an attribute that requires mathematical knowledge to be identified. The difference between a 'feature' and a 'property' is an important concept that needs to be understood for geometry proofs further down the track and is worth looking at here.

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-16MG applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems.

# Syllabus content

## Students:

- investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles.
- identify the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle.
- establish the relationship between the lengths of the sides of a right-angled triangle in practical ways, including with the use of digital technologies.
- describe the relationship between the sides of a right-angled triangle.
- use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle.
- identify a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third.
- use the converse of Pythagoras' theorem to establish whether a triangle has a right angle.

---

## Resources needed

- A prepared 3, 4, 5 triangle
- Pythagoras' Theorem dissection activity worksheet (optional)
- Scissors and glue

---

## Teacher instructions

### Step 1

- Students think about how many different types of triangles exist?
- Draw the table below or provide students with the table showing different ways of classifying triangles.

	<b>Obtuse</b>	<b>Acute</b>	<b>Right</b>
<b>Equilateral</b>			
<b>Isosceles</b>			
<b>Scalene</b>			

---

### Step 2

- Students complete the table, by indicating if the angle type exists in each triangle type, using a tick (if they exist) and a cross (if they don't exist) (answers provided below).

	Obtuse	Acute	Right
Equilateral	X	✓	X
Isosceles	✓	✓	✓
Scalene	✓	✓	✓

### Step 3

- Students use the **Right-Angled Triangles Worksheet** and measure the lengths of each side. Ask students to identify the position of the longest side in each right-angled triangle, leading to the introduction of the term hypotenuse.

### Step 4

- Depending on the ability of the class, give the students the 3 cm, 4 cm, 5cm right-angled triangle or ask them to construct a right-angled triangle with sides 3 cm, 4 cm and 5 cm.
- Students draw a square off each of the sides of the triangle with grid markings.
- Students cut and place the 3x3 square on top of the 5x5 square and see if the 4x4 square can fit in the remaining space (students can cut up the 4x4 square as necessary).

Alternatively: use the **Pythagoras' Theorem Dissection Worksheet**.

Note:

- Geogebra also has a nice demonstration <https://www.geogebra.org/m/yy5bKdW9>.
- There are many dissection proofs and it is worth showing students a few different ones.
- Henry Perigal (1801 – 1898) had the second dissection carved onto his gravestone <https://plus.maths.org/content/dissecting-table>.

### Step 5

- Introduce Pythagoras' theorem, both in words and then as a formula.
  - the square of the hypotenuse is equal to the sum of the squares of the two shorter sides which can be translated algebraically into  $c^2 = a^2 + b^2$  or  $r^2 = p^2 + q^2$ , using any letters, although a, b and c are most commonly used.

### Step 6

- Students construct a right-angled triangle (this is ideally done on grid paper, particularly for lower ability classes).
- Students swap their right-angled triangle with the person next to them and have each student apply Pythagoras' theorem ( $c^2=a^2+b^2$ ) to see if it is true.

### Step 7

- Question: how could we use Pythagoras' theorem to find the shorter side?
- Have students test a rearrangement of Pythagoras' theorem ( $a^2=c^2-b^2$ ) on the triangles they have constructed.

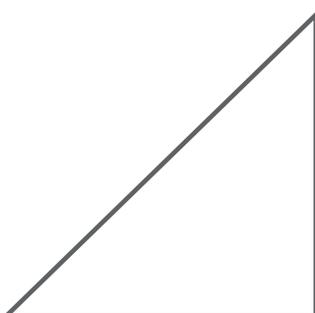
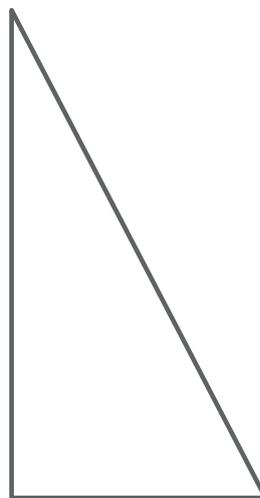
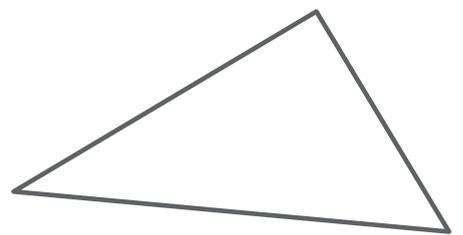
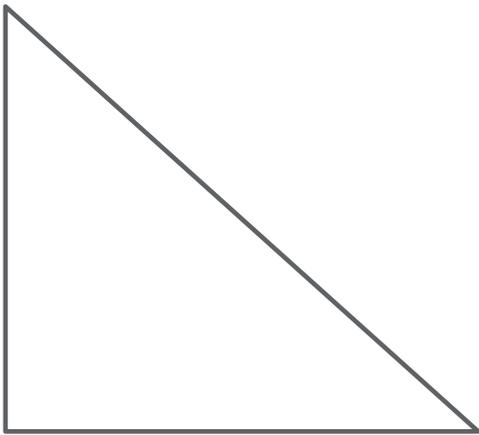
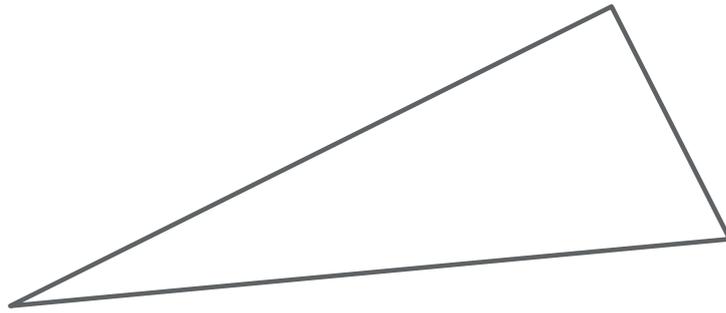
## Questioning

- Would Pythagoras' Theorem still work if we doubled the length of each of the sides in the triangle we just constructed?
  - Would Pythagoras' Theorem still work if we halved the length of each of the sides in the triangle we just constructed?
  - Would Pythagoras' Theorem still work if we tripled the length of each of the sides in the triangle we just constructed?
  - Use these combinations to introduce the concept of a Pythagorean triad. What other Pythagorean triads are there (have students think of two square numbers that add to another square number)? Does this work for any right-angled triangle?
  - At this point show students this YouTube video from 'Who wants to be a Millionaire' where a young man lost \$15 000 because he didn't know Pythagoras Theorem. <https://youtu.be/BbX44YSsQ2I>
- 

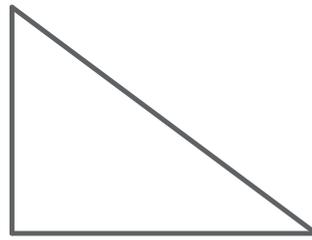
## Discussion

- Class discussion about the observations from the activity. Include in the discussion a consideration of the area of the each of the squares and the relationship between the areas.

# Right-Angled Triangles Worksheet

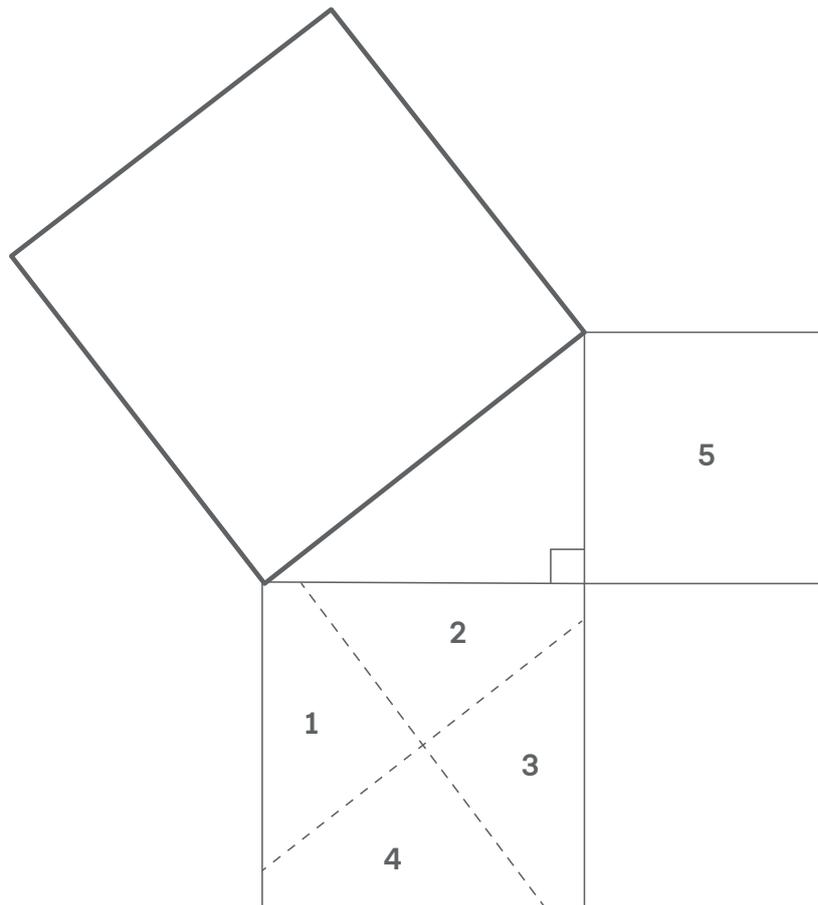


## 3 cm, 4 cm, 5 cm triangle



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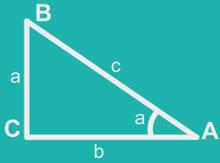
## Pythagoras' Theorem Dissection Worksheet



### Instructions

1. Cut off the small and medium squares. (Numbered 1-4 and 5)
2. Cut the medium square along the dotted lines.
3. Try to arrange the pieces (1-5) inside the larger, darker square.

What can you conclude from this exercise?



# LESSON 3

## Pythagoras theorem

### Overview of lesson

Using the converse of Pythagoras' theorem to construct right-angled triangles

### Background knowledge

Students should be able to:

- identify the hypotenuse of a right-angled triangle.
- recognise and use Pythagoras' theorem to find the unknown side of a right-angled triangle.

### Language considerations

Students should be able to communicate using the following language: right angle hypotenuse, vertex, vertices, triad, converse, Pythagoras' theorem, exact and approximate.

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-16MG applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems.

### Syllabus content

Students:

- investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles.
- establish the relationship between the lengths of the sides of a right-angled triangle in practical ways.
- describe the relationship between the sides of a right-angled triangle.
- use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle.
- identify a Pythagorean triad as a set of three numbers such that the sum of the squared of the first two equal the square of the third.
- use the converse of Pythagoras' theorem to establish whether a triangle has a right angle.

## Resources needed

- 20 m lengths of string/rope (enough for each group to have one)
- Measuring tape
- Athletic field event markers (as shown below)

NOTE: These are best as they have a solid base to use as the vertices of the triangle, and make a sharp angle. They can then be positioned into the ground where needed and will hold their position.

Negotiate with other school personnel about the use of space outside the classroom for the activity.

## Teacher instructions

### Step 1

**Whilst in the classroom:**

- give students a set square or have them use their own.
- have students measure the sides of the set square and test the validity of Pythagoras' theorem for the set square.
- introduce the concept of a 'converse' (switching the hypothesis and conclusion of a conditional statement).
- students write down the converse of Pythagoras' theorem.

### Questions

- Why is it called a set square (ensures a right angle at two intersecting lines)?
- What is it used for?
- Did Pythagoras' theorem work? Why? Why not?

### Step 2

**Whilst in classroom:**

- introduce the task, using a length of string/rope (20 m long), you will work in groups to construct a right-angled triangle on the school oval/grassed area.
- you will be using the athletic field event markers to identify the vertices of the triangles, then place the string/rope around to form the sides of the triangle.
- measure the sides of the triangle and use the converse of Pythagoras' theorem to check if it is right angled.
  - if not right angled students will need to make adjustments to their vertices until the triangle becomes right angled.

Note: Students do not have to use the entire length of string/rope for their triangle.

Equipment students will need to take with them.

- Calculator
- Pen
- Book/worksheet

## Step 3

### Outside in groups of 3-4:

- assign groups and allocate equipment (1 x rope, 3 x markers, 1 x tape measure).
  - students construct right-angled triangles, take measurements and then check for a right angle using the converse of Pythagoras theorem.
  - students record their results and then refine and redo as appropriate.
- 

## Extension activity

- Is our classroom square? Break the class into at least four groups. Using the string used outdoors, determine whether the classroom is square using diagrams and calculations for justification.
- 

## Questioning

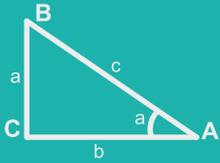
- Why is it important to measure accurately?
  - Was there one side that you adjusted more than the others? Why?
- 

## Discussion

- Where would Pythagoras' theorem be used?
- Why would we want to construct a right-angled triangle?
- Who would use a right-angled triangle? For what? (E.g. A builder to ensure that rooms are square, determine screen size of television, the shortest distance between two points.) <https://youtu.be/ehgJu-hERul>

## Recording Sheet

Triangle Pythagoras Construction	Side 1 (a)	Side 2 (b)	Side 3 (c - hyp)	$a^2+b^2$	$c^2$	Right angled (Y/N)
1						
2						
3						
4						
5						



# LESSON 4

## Pythagoras' theorem in composite figures

### Overview of lesson

Applying Pythagoras Theorem to a variety of practical problems including composite shapes and 3-dimensional shapes

### Background knowledge

Students should be able to accurately apply Pythagoras theorem multiple times.

### Language considerations

Students should be able to communicate using the following language: horizontal, vertical, Pythagoras' theorem, hypotenuse and right angle.

### Syllabus outcomes

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.

MA4-2WM applies appropriate mathematical techniques to solve problems.

MA4-16MG applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems.

### Syllabus content

#### Students:

- investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles.
- use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle.
- write answers to a specified or sensible level of accuracy, using an 'approximately equals' sign, i.e.  $\doteq$  or  $\approx$
- solve a variety of practical problems involving Pythagoras' theorem, approximating the answer as a decimal.
- apply Pythagoras' theorem to solve problems involving the perimeters and areas of plane shapes.

### Resources needed

- Three shapes worksheet (printed on card, enough for students to work in pairs).

## Teacher instructions

### Step 1

- Students use the three shapes on the **Three Shapes Worksheet** to construct a composite shape which will become a garden bed to be constructed in the school.
- 

### Step 2

- Give dimensions for the horizontal and vertical sides of each shape.
  - Discuss strategies with students for calculating the perimeter (we don't know the length of the slant edge of a trapezium or the hypotenuse of the right-angled triangle).
- 

### Step 3

- Discuss what we need to know about the composite shape to find out how much edging material is required? (i.e. perimeter).
  - Students to calculate the perimeter of their composite shape.
- 

### Step 4

- Discuss what we need to know about the composite shape to find how much paving is required? (i.e. area).
  - Students to calculate the area of their composite shape.
- 

### Step 5

- Discuss what we need to know about the composite shape to find how much soil was required? (i.e. volume).
  - Students to calculate the volume of their composite shape given a depth of 30 cm.
- 

### Step 6

- Students use the internet or a suitable catalogue to determine the cost of fencing the garden bed.
  - Students to decide on what proportion of the garden bed will be paved and what proportion will be filled with soil and determine the cost of each.
  - Students could also research what plants are suitable and how many they would need to complete the garden bed.
- 

## Extension - Pythagoras in 3 dimensions

### Resources

You will need:

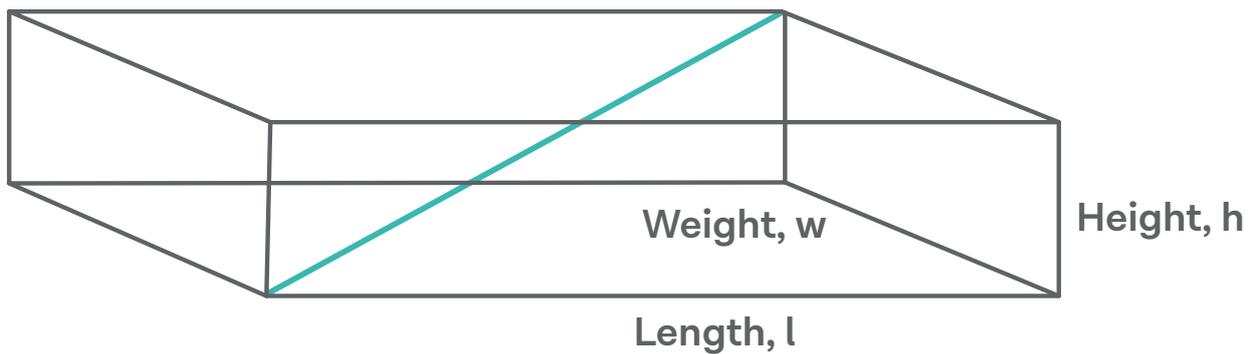
- a variety of rectangular boxes e.g. shoe boxes, small cartons etc. (Teacher will need to have calculated the longest internal diagonal of each box prior to lesson)
- rulers and small measuring tape

## Teacher instructions

### Step 1

#### Students:

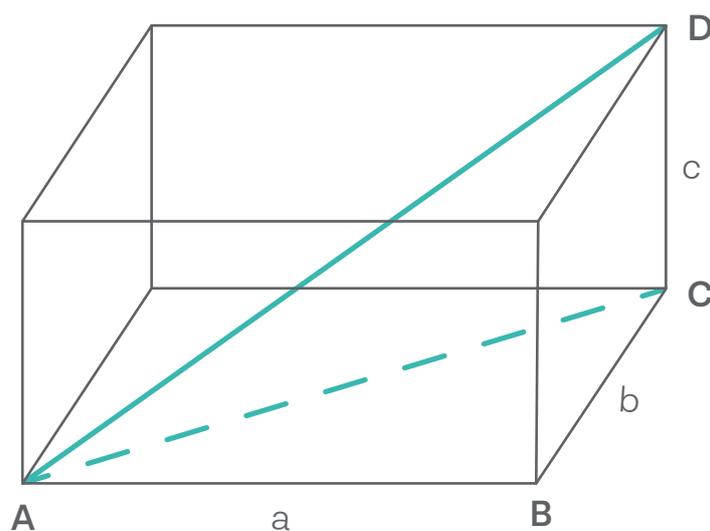
- work in small groups with their given box.
- use a ruler or tape to find and record the dimensions of the box, then determine the longest item that would fit in the box.
  - Discuss as a class why some of their answers do not match the teacher's answer and why some do? Establish where students measured and then look at a typical box and guide discussion to arrive at the fact that the longest item would lie along the internal diagonal as shown.



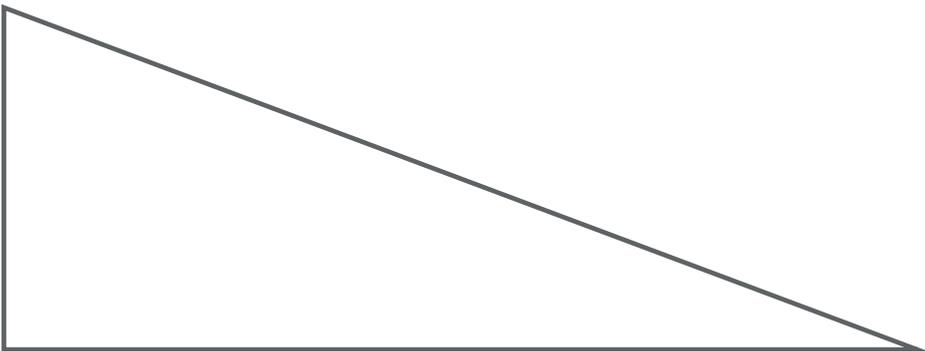
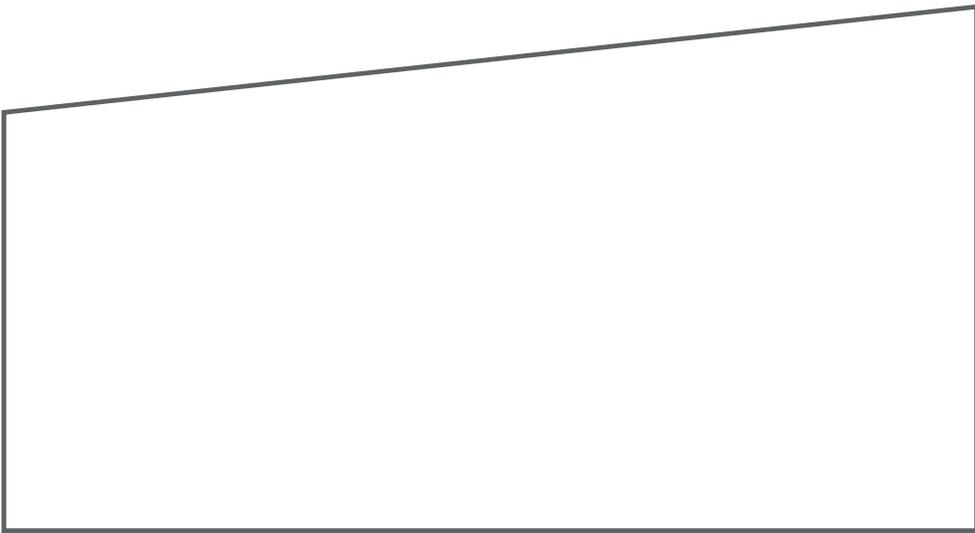
### Step 2

#### Students:

- use the **Rectangular Prism Worksheet** and draw an internal diagonal in each of the rectangular prisms.
- draw and create right-angled triangles in their prisms.
- use Pythagoras' theorem to determine the exact length of the internal diagonal i.e. the length AC in triangle ABC and then find AD in triangle ADC.



## Three Shapes Worksheet



# Rectangular Prism Worksheet

