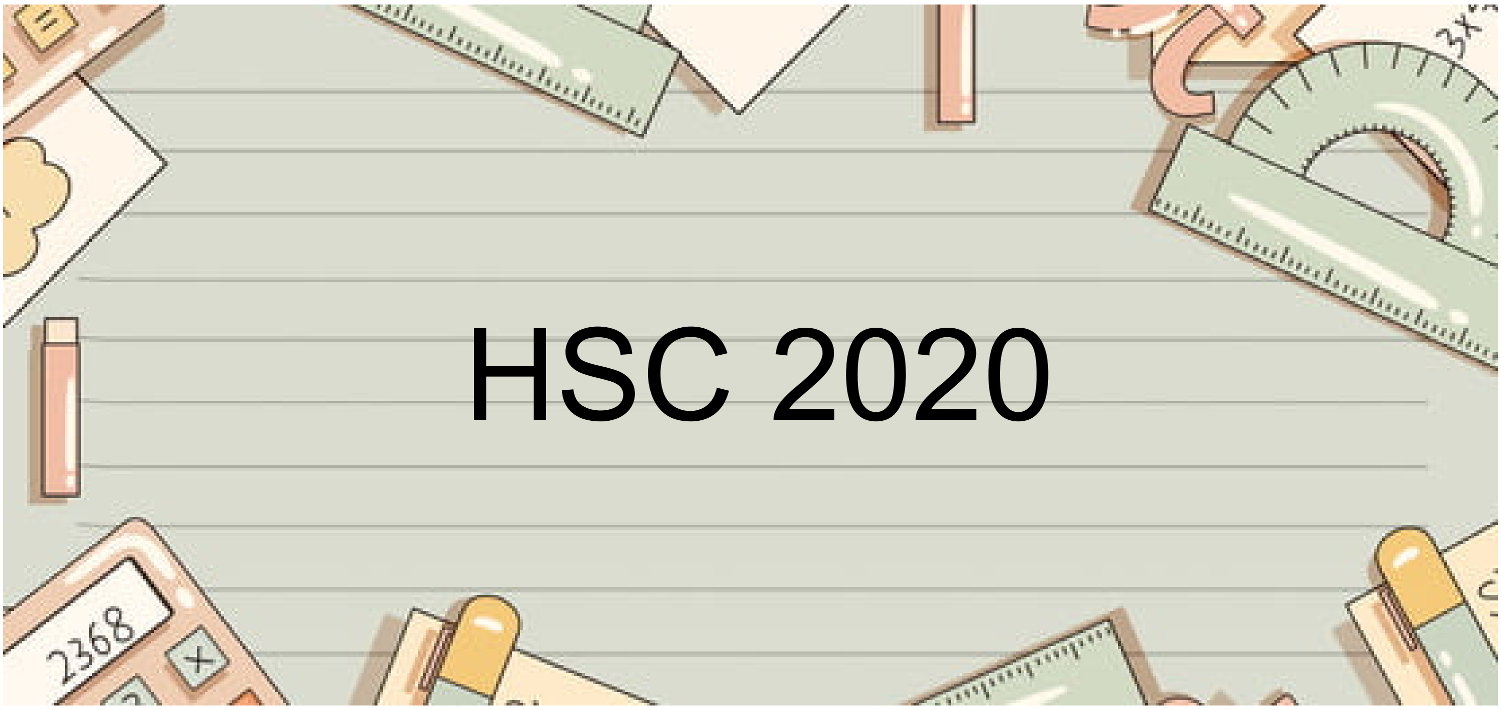
2020 HSC Study Day Series

AURORA COLLEGE

HSC Mathematics

Advanced



NSW Department of Education

www.aurora.nsw.edu.au

2020 HSC Study Day Series

AURORA COLLEGE

Details

Date: Thursday 6th August 2020

Time: 8:50am – 3:10pm

Location: Adobe Connect room https://connect.schools.nsw.edu.au/aurora-hsc-study1/

Materials: Available to download via this Dropbox link

Recordings: The sessions will be recorded and accessible for registered participants after the event via the same Dropbox link above. These recordings will be accessible until the HSC exam.

Program

Time	Session
8:50 – 9:00 am	**Welcome**
9:00 – 10:00 am	**Calculus** *Stuart Palmer, Mathematics Consultant*
10:05 – 10:45 am	**Financial Mathematics** *Robert Gorton, Wadalba Community School*
10:45 – 11:15 am	Morning tea break
11:15 – 12:15 pm	**Functions** *Stuart Palmer, Mathematics Consultant*
12:20 – 1:20 pm	**Statistical Analysis** *Robert Gorton, Wadalba Community School*
1:20 – 2:00 pm	Lunch break
2:00 – 3:00 pm	**Trigonometric functions** *Stuart Palmer, Mathematics Consultant*
3:00 – 3:10 pm	**Conclusion**

Image on front cover attribution: Sourced from https://pngtree.com/free-backgrounds-photos/maths

2020 HSC Study Day Series

Setting up Adobe Connect

Teachers will need:

- A good, stable Dept of Ed internet connection using an ethernet cable (wifi not recommended)
- Data projector
- Speakers

The sessions will be held via Adobe Connect. Please ensure there is only one connection per school. The presentation can be displayed on a data projector through any computer with an ethernet cable and speakers. The information below will help with setting up if you are not familiar with Adobe Connect.

- You will need to perform all necessary setup in advance of your online session so that you have time to resolve any connection or access issues. The Adobe room will be opened 30 mins prior to commencing to allow time for set up.
- Test your computer prior to accessing your online room by going to the Meeting Connection Diagnostic. Ensure you install any add-ins, if prompted to do so by the connection test.
- The following guide may also be useful Quick Start Guide for Participants.

Entering the Adobe room

Teachers log in once for their class. Students are NOT to log in individually. To enter your online room, click on the Adobe Connect link provided above. Enter by typing in your Department of Education ID (eg: *jane.citizen@detnsw)* in the *Username* field then your DoE password in the *Password* field. The first thing you should do when you enter the room is complete the audio setup wizard. ('Meeting' drop down menu-> Audio Setup Wizard)

For technical help:

If you are having any issues with technology, please contact the Aurora College IT Support Team on 1300 610 733 or support@aurora.nsw.edu.au

Rights and responsibilities

Duty of care for students throughout the day remains with the registered schools and their respective teachers. Please ensure adequate supervision is provided during the day. Respectful and active participation in the event is strongly encouraged through the 'chat' pod.

Evaluation

Constructive feedback is essential, links to online surveys will also be distributed during and shortly after the event. There are two surveys and they both close on 21st September:

- Teachers https://www.surveymonkey.com/r/HSCSTUDYDAYSTEACHER2020
- Students https://www.surveymonkey.com/r/HSCSTUDYDAYSSTUDENT2020

We look forward to your participation.

HSC Advanced Mathematics Sessions with Stuart Palmer
Topic: Calculus

CALCULUS IS A HUGE TOPIC!
The SIX main components of the Advanced calculus topics:

1. You need to be very capable at **differentiating and integrating all sorts of functions**. These 'skill drill' sheets are designed to increase your fluency, speed and accuracy prior to examinations. There are eight of them in total, each for a different set of functions, with answers, here.
   After that you should be able to do these HSC questions, some of which include equations of tangents and normals:

   Exponential functions with base a: 2020sample-4,
   Logarithmic functions with base a: 2020sample-10,
   Exponential functions with base e: 2020sample-16, 2018-11g, 2017-3, 2016-12d(i), 2015-11e, 2014-15c, 2013-11d, 2011-2d, 2010-1e, 2010-2a, 2009-2a(ii), 2008-2a(ii), 2008-3b(i), 2007-2a(i), 2006-5b(i), 2005-5c, 2004-3a(i), 2003-3a(i), 2002-2a, 2001-3c(i)
   Logarithmic functions with base e: 2017-11d, 2016-8, 2015-11f, 2012-12a(i), 2011-1e, 2010-2c, 2005-2d, 2003-2a, 2002-2b(ii), 2001-3c(i)
   Trigonometric functions: 2020sample-23, 2019-11b, 2018-5, 2018-11f, 2018-12b, 2017-11c, 2016-11f, 2015-6, 2014-13a(i), 2013-4, 2013-11c, 2012-11d, 2012-12a(ii), 2011-4a, 2010-2a, 2009-2a(i), 2008-2a(iii), 2007-2c, 2007-2a(ii), 2006-2a, 2006-2c, 2005-2b(i), 2004-3a(ii), 2004-5b(i), 2003-1b, 2003-3a(ii)
   From Extension 1 (Same concept, harder questions)**:** 2016-12c, 2014-11f, 2012-11d, 2011-1b, 2009-1e, 2007-7a, 2002-1b, 2001-1b
   Tangents and normals: 2019-14d, 2019-16c(i), 2018-15c(ii), 2017-12a, 2015-12c(i), 2012-11c, 2011-2c, 2010-7b, 2009-1d, 2009-6c(ii), 2004-8b(ii), 2003-8d, 2001-2a

2. **Using calculus to find key points and to draw curves**, which is covered in this booklet.
3. Solving **optimisation problems**, which is covered in this booklet.
4. Solving **problems involving rates of change**, including velocity and acceleration.

   It is very difficult to draw the line between Advanced and Extension 1. These are possibly within the scope of Advanced but are possibly more likely to appear in Extension 1.:
   - **Rates of change involving differentiation of a function:** 2005-6b, 2002-7b
   - **Displacement, velocity, acceleration involving differentiation of a function:** 2018-12d, 2016-16a, 2014-13c, 2013-14a, 2012-15b, 2011-7b, 2007-5b, 2006-8a,2004-5b, 2003-7b, 2002-8b, 2001-7c
   - **From Extension 1** (Same concept, harder questions)**:** 2017-12d, 2016-7

   Exponential growth and decay: It is very difficult to draw the line between Advanced and Extension 1. These are possibly within the scope of Advanced but are possibly more likely to appear in Extension 1:
   2019-12c, 2018-13c, 2017-14c, 2016-13c, 2016-16b, 2015-15a, 2014-13b, 2013-16b, 2012-14c, 2011-10a, 2010-8a, 2009-6b, 2008-5c, 2007-8a, 2006-6b, 2005-6a, 2004-7b, 2003-6c, 2002-8a, 2001-8a

5. **Integration**, including:
   a. Anti-derivatives (primitives) and indefinite integrals
   b. Definite integrals
   c. Calculating areas of regions:
      i. bounded by a curve and the x-axis or a curve and the y-axis
      ii. bounded by two curves
   d. Estimating the area of regions using the trapezoidal rule
   e. Solving **problems involving rates of change**, including velocity and acceleration.
   f. Finding the amount by which a quantity has changed over time.

Indefinite integrals: 2019-9, 2007-2b(i), 2005-1c, 2002-2d(i)
Integrals involving logarithms: 2015-11e, 2013-11g, 2012-12b, 2010-2d(ii), 2008-2c(i), 2006-2b(ii), 2005-2c(i), 2004-3b(ii), 2003-3d(i), 2002-1d, 2001-1d
Integrals involving exponentials: 2014-4, 2013-11e, 2006-2b(i)
Find a function: 2020sample-33, 2019-14b(i), 2017-9, 2017-13d, 2015-15c, 2014-11f, 2013-16a, 2011-4c, 2008-5a, 2008-9, 2002-6b(i)
Displacement, velocity and acceleration:
It is very difficult to draw the line between Advanced and Extension 1. These are possibly within the scope of Advanced but are possibly more likely to appear in Extension 1:
Rates of change involving integration of a function: 2017-15c, 2015-14a, 2006-9b
Displacement, velocity, acceleration involving integration of a function: 2020sample-33, 2016-16a, 2013-14a, 2012-15b, 2011-9b, 2010-7a, 2009-7a, 2007-5b, 2005-9a, 2003-7b, 2002-9c, 2001-9c
Trapezoidal rule:
- **From Advanced / 2 Unit:** 2020sample-29b, 2015-5, 2013-15a(i), 2010-3d, 2001-5d
- **From General / Standard:** A printable collection of trapezoidal rule questions is here.

Calculating integrals using area formulas: 2020sample-18, 2016-9, 2007-10a(iii)
The relationship between integration and area: 2018-7, 2018-10, 2013-14d, 2012-10, 2008-4c(iv), 2007-10a(iii), 2005-7b(ii)
Calculating total change using a definite integral or area under curve: 2020sample-30, 2019-8,2015-9, 2011-5c, 2011-9b, 2010-2e, 2008-6b(iv) needs to be done with the trapezoidal rule, 2007-10a(i) needs to be done with the trapezoidal rule, 2005-9a(iii), 2003-7b(iv)
Definite integrals: 2009-2b(iii)
- **Reverse chain:** 2019-11e, 2016-11d,
- **Trigonometric:** 2017-14b(i), 2015-11g, 2014-11e, 2012-11g, 2008-2c(ii), 2005-2c(ii), 2003-3d(ii),
- **Logarithmic:** 2012-9, 2011-4b, 2001-3a
- **Exponential:** 2004-3b(i), 2002-2d(ii)

Area in relation to x-axis: 2020sample-38c, 2019-12d, 2018-11e, 2018-15d, 2015-10, 2011-6c, 2010-5c, 2006-7b
Area in relation to y-axis: 2016-13d, 2008-10a
Area between two curves: 2020sample-35, 2018-15c, 2017-14d, 2015-7, 2015-16a, 2014-12d, 2013-13b, 2012-13b, 2010-4b, 2009-10def, 2007-7b, 2006-5b, 2005-8b, 2004-8b, 2003-4c, 2002-4d, 2001-4c
Differentiate a given function, hence integrate an unusual but related function: 2019-13c, 2016-12d, 2014-13a, 2011-4d, 2009-10ef, 2008-3b, 2006-5b

6. Using a given graph with an unknown equation to draw another graph:

Using a graph of a function f(x) with no equation to draw a graph of f ’(x):
2014-14e, 2011-9c, 2005-7b(ii), 2009-8a
Using a graph of a function f ’(x) with no equation to draw a graph of f(x):
2020sample-33, 2010-9b, 2007-10a(iv)

HSC Advanced Mathematics Sessions with Stuart Palmer
Topic: Calculus

Using calculus to find key points and sketch curves

This is a big-ticket item! Basically five or more marks every year are devoted to this concept:

- The following questions did not require a sketch to be drawn: 2014-14a, 2013-12a, 2004-9c,
- The following questions use differentiation and stationary points to assist with curve-sketching: 2020sample-14, 2018-13a, 2017-13b, 2016-13a, 2015-13c, 2012-14a, 2011-7a, 2010-6a, 2009-10,2008-8a, 2007-6b, 2006-5a, 2005-4b, 2004-4b,2003-5a, 2002-6b(ii)(iii), 2001-6c
- This one involves a little bit of 'integrate and find the constant': 2019-14b
- **From Extension 1** (Same concept, harder questions**):** 2012-13d(i), 2011-4a, 2010-3b(i), 2007-6b(i), 2006-7a, 2005-7b
- **From Extension 2** (Same concept, harder questions**):** 2017-12a, 2016-13a, 2009-5c

2020 Sample HSC Examination

Question 14 (6 marks)

A function is given by $f(x) = 18x^2 - x^4$.

(a) Find the stationary points and determine their nature. **4**

(b) Sketch the curve, labelling the stationary points and axis intercepts. **2**

2008 HSC Question 8a

Let $f(x) = x^4 - 8x^2$.

(i) Find the coordinates of the points where the graph of $y = f(x)$ crosses the axes. **2**

(ii) Show that $f(x)$ is an even function. **1**

(iii) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **4**

(iv) Sketch the graph of $y = f(x)$. **1**

2003 HSC Question 5a

Consider the function $f(x) = x^4 - 4x^3$.

(i) Show that $f'(x) = 4x^2(x-3)$. **1**

(ii) Find the coordinates of the stationary points of the curve $y = f(x)$, and determine their nature. **3**

(iii) Sketch the graph of the curve $y = f(x)$, showing the stationary points. **1**

(iv) Find the values of x for which the graph of $y = f(x)$ is concave down. **2**

Optimisation problems

This is a section of Year 12 Advanced Topic C3.2 (Applications of the derivative).
The HSC examiners like to use this concept to make judgements about Band 5 and 6 achievement.
This is one of the most challenging aspects of the Advanced Syllabus, because:

- There are many steps to be completed in the solution and many places where things can go wrong.
- High-level algebraic skills are required.
- Any minor error in the early stages can make it difficult or impossible to finish the solution.
- Every question is unique and may involve concepts from a variety of other topics.
- The textbooks can't come close to covering every conceivable problem.
- The questions in the HSC Examinations in previous years have usually been towards the end of the paper, when students are getting tired and more likely to make errors.

All the HSC questions from 1980 to present have been collected in this document. Solutions are here.
I have attempted to place them into four categories from easiest (which are not simple) to most difficult.
Solutions will be attached to this document over time.

There has been one or two of these every year since 1980. You will notice that most of them are worth 5 marks or more. Sometimes this is also assessed in Extension 1.

SHOW EVERY STEP OF WORKING and DON'T CUT CORNERS

If you think the value of x is 60 and you think $x + y = 180$. It may seem obvious that y is 120, but show the steps, because the 60 might be incorrect. The markers will want to see how you got from 60 to 120. That might score you a mark, even though the 60 was incorrect.

DON'T 'FUDGE'

If the answer was given in the question and you did not get that answer, don't fudge the figures to make it look like you got the given answer. The markers do not like fudging at all. It may cost you marks because you are introducing more errors into your proof. You would be better off:

- Making a note and coming back later if you have time, or
- Trying to find your mistake, but this is often difficult, it may be fruitless and it can consume many minutes which could have been more profitably spent on other questions.

DON'T GIVE UP

Sometimes the last part of the question is the easiest part. If you need the answer from part b to do another part, just guess an answer for part (b) which seems plausible and then use it in the other parts. You can get full marks if your method beyond part b is correct. Show every step of working.

How best to work through optimisation problems (A step-by-step guide for students and teachers)

Step 1: Read **ALL** the information carefully and **ALL** the parts of the question, highlighting the most useful snippets, such as:

- Is there a quantity or measurement (either known or unknown) or some relationship which is constant. For example:
    - The perimeter is 20cm.
    - There is enough material for 20 metres of fencing.

  - The volume (V) is constant.
  - There might be two triangles in the diagram which are similar.
- What is it that you need to minimise or maximise? It could be, for example:
  - Cost or time.
  - Perimeter, area or surface area.
  - Volume or capacity.
  - Displacement, velocity or acceleration.

Step 2: Is a diagram given?
- If so, is all the given information on the diagram? Can you deduce then add extra information?
- If not, it might be wise to draw one.
- Add notes about what it is that you are trying to optimise.
- If the questions contain the answers (which they often do) make note of that on the diagram too.

Step 3: Start with the constant (the thing you CAN'T change). Try to form an equation from that then simplify and rearrange it. For example if $6x + 8y = 56$ then ask yourself:
- Can you divide through by a number to make the numbers smaller?
- What does it look like if x is the subject? What does it look like if y is the subject? Sometimes one of these is much simpler than the other.

Step 4: Now look at the thing to be optimised. Form an equation. Typically the RHS contains two variables, so it is not ready to be differentiated. Usually, one may be eliminated using an equation from the previous step.

Step 5: Go hunting for stationary points. This is how to do it:
- Find the derivative.
- Let the derivative be equal to 0, just like you did when hunting for stationary points for graphs.
- Solve the equation and, if time permits, check them by putting them back into the equation.
- Write down ALL the solutions. Some of them may not make sense in the context and may be labelled 'Not valid in this context'. For example, time can't be negative. This might be obvious, but do it.
- Check that the solutions make sense on your diagram and/or from the question.

Step 6: Determine the nature of the stationary point(s), by either:
- Substituting the solution(s) into the **second derivative** to determine whether it is:
  - positive (ie concave up) which means minimum
  - negative (ie concave down) which means maximum

  OR, if the second derivative is looking nasty,
- Using the first derivative test in a table with three values in the top row and three gradients in the bottom row. If possible, **put numbers in the bottom row**, not just symbols or 'pos' and 'neg'.

Step 7: Go back and re-read the question:
- Does your answer make sense? Is it possible in the context?
- Have you done enough? You may need to do some more calculations. You may have found x, but you may also be required to find y, or the maximum area, say.
- If the question contained the answer (which it often does), is that what you got?

We are now going to use the HSC questions on the following pages to model the advice given above.

Then you can use the steps to work through all the past HSC questions, here, from 'base camp' through to 'thrill-seekers'. Enjoy!

1988 HSC Question 9a

This one comes from the group: Level 0: Base camp.

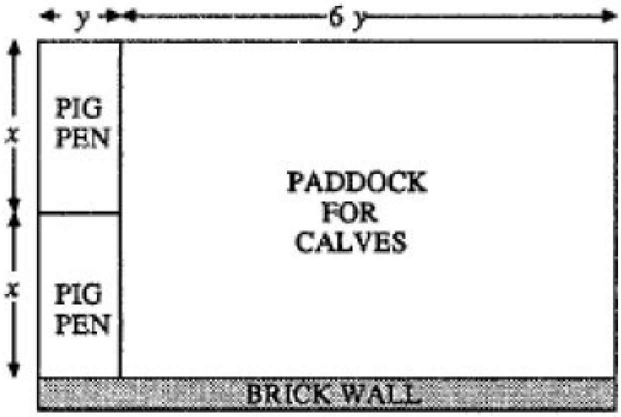


Figure not to scale. All measurements in metres.

Farmer Brown wishes to construct three rectangular enclosures, as shown above, in which to put pigs and calves. The paddock for the calves is to be six times as long and twice as wide as a pig pen. One pig pen and the calves' paddock have an existing brick wall as a boundary fence as shown. All other fences are to be constructed from 56 metres of wire mesh.

(i) Let x metres be the width of a pig pen and y metres be its length. Show that

$$y = 7 - \frac{3}{4}x \ .$$

(ii) Hence show that the total area A square metres contained in the three enclosures is given by

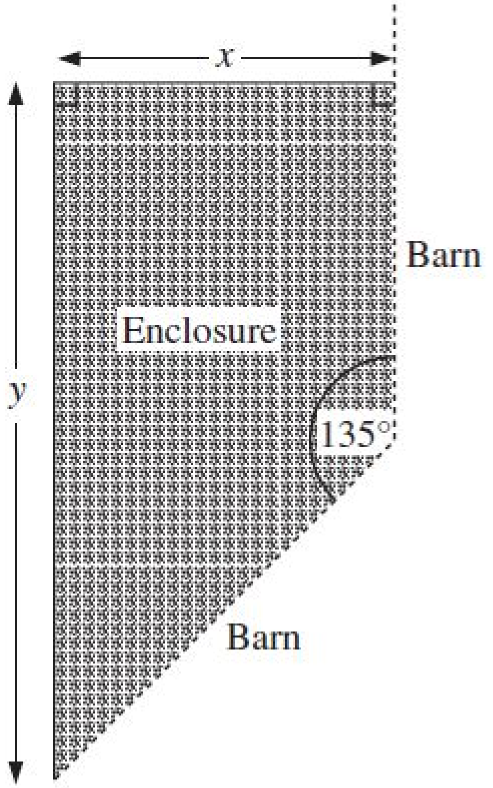
$$A = 14x\left(7 - \frac{3}{4}x\right) \ .$$

(iii) Show that A is a maximum when half the wire fencing has been placed parallel to the brick wall.

2000 HSC Question 8b

This one comes from the group: Level 2: More challenging.

An enclosure is to be built adjoining a barn, as in the diagram. The walls of the barn meet at 135°, and 117 metres of fencing is available for the enclosure, so that $x + y = 117$ where x and y are as shown in the diagram. **5**



(i) Show that the shaded area of the enclosure in square metres is given by

$$A = 117x - \frac{3}{2}x^2 .$$

(ii) Show that the largest area of the enclosure occurs when $y = 2x$.

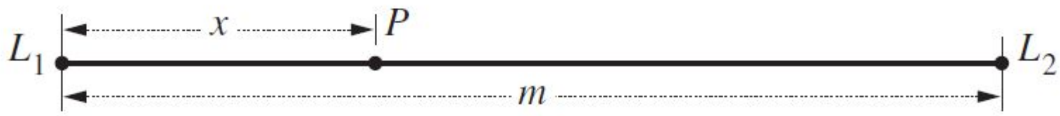
2007 HSC Question 10b

This one comes from the group: Level 3: For thrill-seekers.

The noise level, N, at a distance d metres from a single sound source of loudness L is given by the formula

$$N = \frac{L}{d^2}.$$

Two sound sources, of loudness L_1 and L_2 are placed m metres apart.



The point P lies on the line between the sound sources and is x metres from the sound source with loudness L_1.

(i) Write down a formula for the sum of the noise levels at P in terms of x. **1**

(ii) There is a point on the line between the sound sources where the sum of the noise levels is a minimum. **4**

Find an expression for x in terms of m, L_1 and L_2 if P is chosen to be this point.

1998 HSC Question 10b

This one also comes from the group: Level 3: For thrill-seekers.

A fish farmer began business on 1 January 1998 with a stock of 100 000 fish. He had a contract to supply 15 400 fish at a price of \$10 per fish to a retailer in December each year. In the period between January and the harvest in December each year, the number of fish increases by 10%. **7**

(i) Find the number of fish just after the second harvest in December 1999.

(ii) Show that F_n, the number of fish just after the nth harvest, is given by

$$F_n = 154\,000 - 54\,000(1 \cdot 1)^n.$$

(iii) When will the farmer have sold all his fish, and what will his total income be?

(iv) Each December the retailer offers to buy the farmer's business by paying \$15 per fish for his entire stock. When should the farmer sell to maximise his total income?

HSC Study Day 2020

MATHEMATICS

Financial Mathematics

Robert Gorton Robert.gorton1@det.nsw.edu.au
Wadalba Community School

Overview

- Syllabus outcomes
- Summaries
- Past HSC questions
- Useful Resources

Syllabus Outcomes (per topic)

MA-M1 Modelling Financial Situations

A student:

- models and solves problems and makes informed decisions about financial situations using mathematical reasoning and techniques MA12-2
- applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems MA12-4
- chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
- constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Syllabus Outcomes –

M1.1: Modelling investments and loans

Students:

- solve compound interest problems involving financial decisions, including a home loan, a savings account, a car loan or superannuation **AAM**
  - identify an annuity (present or future value) as an investment account with regular, equal contributions and interest compounding at the end of each period, or a single-sum investment from which regular, equal withdrawals are made
  - use technology to model an annuity as a recurrence relation and investigate (numerically or graphically) the effect of varying the interest rate or the amount and frequency of each contribution or a withdrawal on the duration and/or future or present value of the annuity
  - use a table of interest factors to perform annuity calculations, eg calculating the present or future value of an annuity, the contribution amount required to achieve a given future value or the single sum that would produce the same future value as a given annuity

Syllabus Outcomes –

M1.2: Arithmetic sequences and series

Students:

- know the difference between a sequence and a series
- recognise and use the recursive definition of an arithmetic sequence: $T_n = T_{n-1} + d$, $T_1 = a$ **AAM**
- establish and use the formula for the n^{th} term (where n is a positive integer) of an arithmetic sequence: $T_n = a + (n-1)\,d$, where a is the first term and d is the common difference, and recognise its linear nature **AAM**
- establish and use the formulae for the sum of the first n terms of an arithmetic sequence: $S_n = \frac{n}{2}(a + l)$ where l is the last term in the sequence and $S_n = \frac{n}{2}\{2a + (n-1)\,d\}$ **AAM**
- identify and use arithmetic sequences and arithmetic series in contexts involving discrete linear growth or decay such as simple interest (ACMMM070) **AAM**

Syllabus Outcomes –

M1.3: Geometric sequences and series

Students:

- recognise and use the recursive definition of a geometric sequence: $T_n = rT_{n-1}$, $T_1 = a$ (ACMMM072) **AAM**
- establish and use the formula for the n^{th} term of a geometric sequence: $T_n = ar^{n-1}$, where a is the first term, r is the common ratio and n is a positive integer, and recognise its exponential nature (ACMMM073) **AAM**
- establish and use the formula for the sum of the first n terms of a geometric sequence: $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$ (ACMMM075) **AAM**
- derive and use the formula for the limiting sum of a geometric series with $|r| < 1$: $S = \frac{a}{1-r}$ **AAM**
  - understand the limiting behaviour as $n \to \infty$ and its application to a geometric series as a limiting sum
  - use the notation $\lim_{n\to\infty} r^n = 0$ for $|r| < 1$

Syllabus Outcomes –

M1.4: Financial applications of sequences and series

Students:

- use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMMM076) **AAM**
  - calculate the effective annual rate of interest and use results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
  - solve problems involving compound interest loans or investments, eg determining the future value of an investment or loan, the number of compounding periods for an investment to exceed a given value and/or the interest rate needed for an investment to exceed a given value (ACMGM096)
  - recognise a reducing balance loan as a compound interest loan with periodic repayments, and solve problems including the amount owing on a reducing balance loan after each payment is made
- solve problems involving financial decisions, including a home loan, a savings account, a car loan or superannuation **AAM**
  - calculate the future value or present value of an annuity by developing an expression for the sum of the calculated compounded values of each contribution and using the formula for the sum of the first n terms of a geometric sequence
  - verify entries in tables of future values or annuities by using geometric series

Summaries – Sequences and Series

- Identify the difference between a sequence and a series
- Distinguish between arithmetic and geometric sequences and series
- Find the *n*th term of arithmetic and geometric series
- Find the sum to *n* terms of arithmetic and geometric series
- Understand and apply the limiting sum formula for infinite geometric series

Nelson, Maths In Focus Advanced Year 12

Summaries – Investments, Annuities and Loans

- Identify arithmetic and geometric growth and decay
- Solve practical problems of growth and decay
- Solve problems involving compound interest investments using repeated calculations, table and formulas
- Solve problems involving annuities using repeated calculations, tables and geometric series
- Solve problems involving reducing balance loans using repeated calculations, tables and geometric series.

Nelson, Maths In Focus Advanced Year 12

Knowing the Exam

The paper will consist of two sections.

Section I (10 marks)

- There will be objective-response questions to the value of 10 marks.

Section II (90 marks)

- Questions may contain parts.
- There will be 37 to 42 items.
- At least two items will be worth 4 or 5 marks.
- The Mathematics Advanced examination will include items that are common with the Mathematics Standard 2 HSC examination. Common items will be worth 20 to 25 marks and will be distributed throughout Sections I and II.

The examination will be based on the Mathematics Advanced Year 12 course and will focus on the course objectives and Year 12 outcomes. The Mathematics Advanced Year 11 course will be assumed knowledge for this examination and may be examined.

Knowing the Exam – Collated by Robert Yen

MATHEMATICS STANDARD 2	MATHEMATICS ADVANCED
YEAR 11 LINEAR FUNCTIONS Linear functions and modelling Direct (linear) variation **YEAR 12 NON-LINEAR FUNCTIONS** Quadratic functions and modelling Reciprocal function (hyperbola) Inverse (linear) variation	**YEAR 11 FUNCTIONS** Linear functions and modelling Direct (linear) variation Quadratic functions and modelling Reciprocal function (hyperbola) Inverse (linear) variation
YEAR 12 FUNCTIONS Solving simultaneous equations graphically Break-even analysis Exponential functions and modelling	**YEAR 11 FUNCTIONS** Solving simultaneous equations graphically Break-even analysis Exponential functions and modelling
YEAR 12 FINANCIAL MATHS Compound interest $FV = PV(1 + r)^n$ Declining-balance depreciation Reducing balance loans Credit cards	**YEAR 12 FINANCIAL MATHS** Compound interest $A = P(1 + r)^n$
YEAR 12 ANNUITIES Annuities using technology, recurrence relations. tables of interest factors	**YEAR 12 ANNUITIES** Annuities using technology, recurrence relations, tables of interest factors

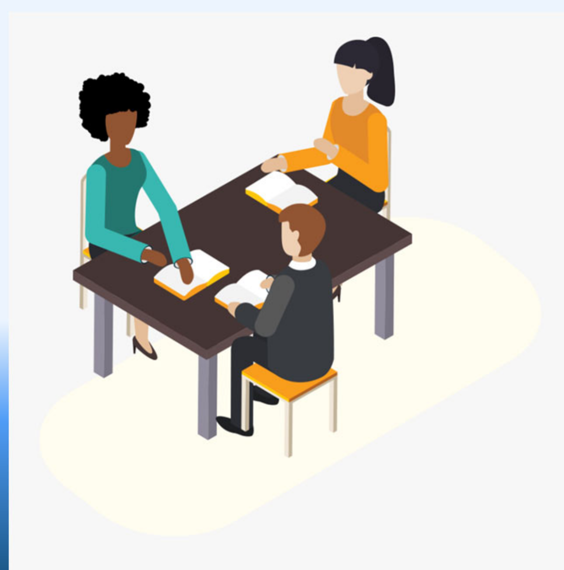
Knowing the Exam – Collated by Robert Yen

YEAR 12 TRIGONOMETRY Pythagoras' theorem, trigonometry Compass and true bearings The sine and cosine rules $A = \frac{1}{2} ab \sin C$	**YEAR 11 TRIGONOMETRY** Pythagoras' theorem, trigonometry Compass and true bearings The sine and cosine rules $A = \frac{1}{2} ab \sin C$
YEAR 11 TRAPEZOIDAL RULE? Trapezoidal rule $A \approx \frac{h}{2}(d_f + d_l)$	**YEAR 12 TRAPEZOIDAL RULE?** Trapezoidal rule $A \approx \frac{b-a}{2n}\left\{ f(a) + f(b) + 2\left[f(x_1) + \ldots + f(x_{n-1}) \right] \right\}$

Knowing the Exam – Collated by Robert Yen

YEAR 11 PROBABILITY Theoretical probability Relative frequency Arrays and tree diagrams Probability simulations	**YEAR 11 PROBABILITY** Theoretical probability Relative frequency Arrays and tree diagrams Probability simulations
YEAR 11 STATISTICS Types of statistical data Statistical graphs Mean, median, mode Range, IQR, standard deviation Deciles, percentiles Outliers, shapes, modality	**YEAR 12 STATISTICS** Types of statistical data Statistical graphs Mean, median, mode Range, IQR, standard deviation Deciles, percentiles (in normal distribution) Outliers, shapes, modality
YEAR 12 STATISTICS Scatterplots Line of best fit Least-squares regression line Pearson's correlation coefficient Normal distribution z-scores $z = \frac{x - \bar{x}}{s}$	**YEAR 12 STATISTICS** Scatterplots Line of best fit Least-squares regression line Pearson's correlation coefficient Normal distribution z-scores $z = \frac{x - \mu}{\sigma}$

Your Turn!



Past HSC Questions

2017 Standard Multiple Choice Question 17

What amount must be invested now at 4% per annum, compounded quarterly, so that in five years it will have grown to $60 000?

(A) $8919
(B) $11 156
(C) $49 173
(D) $49 316

2016 Standard Multiple Choice Question 8

The table shows the future value of an investment of $1000, compounding yearly, at varying interest rates for different periods of time.

Future values of an investment of $1000

Number of years	*Interest rate per annum*				
	1%	2%	3%	4%	5%
1	1010.00	1020.00	1030.00	1040.00	1050.00
2	1020.10	1040.40	1060.90	1081.60	1102.50
3	1030.30	1061.21	1092.73	1124.86	1157.63
4	1040.60	1082.43	1125.51	1169.86	1215.51
5	1051.01	1104.08	1159.27	1216.65	1276.28

Based on the information provided, what is the future value of an investment of $2500 over 3 years at 4% pa?

2016 Standard Multiple Choice Question 8

(A) $1124.86
(B) $2812.15
(C) $3624.86
(D) $5312.15

2019 Standard Question 42

The table shows the future values of an annuity of $1 for different interest rates for 4, 5 and 6 years. The contributions are made at the end of each year.

Future value of an annuity of $1

Years	*Interest rate per annum*			
	1%	*2%*	*3%*	*4%*
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

An annuity account is opened and contributions of $2000 are made at the end of each year for 7 years.

For the first 6 years, the interest rate is 4% per annum, compounding annually.
For the 7th year, the interest rate increases to 5% per annum, compounding annually.

Calculate the amount in the account immediately after the 7th contribution is made.

2019 Standard Question 42

2019 Standard Question 42

Criteria	Marks
• Provides correct answer or correct numerical expression	3
• Provides correct value just before the 7th investment is made, or equivalent merit	2
• Identifies correct value from the table or equivalent merit	1

2017 Mathematics Question 12c

In an arithmetic series, the fifth term is 200 and the sum of the first four terms is 1200. **3**

Find the value of the tenth term.

Criteria	Marks
• Provides correct solution	3
• Finds the three correct equations and attempts to solve, or equivalent merit	2
• Finds a valid equation linking a and d, or equivalent merit	1

2017 Mathematics Question 16b

A geometric series has first term a and limiting sum 2.

Find all possible values for a.

Criteria	Marks
• Provides correct solution	3
• Obtains $-1 < 1 - \frac{a}{2} < 1$, or equivalent merit	2
• Recognises $\frac{a}{1-r} = 2$, or equivalent merit	1

2018 Mathematics Question 11d

In an arithmetic series, the third term is 8 and the twentieth term is 59.

(i) Find the common difference. **1**

(ii) Find the 50th term. **2**

Criteria	Marks
• Provides correct answer	1

Criteria	Marks
• Provides correct solution	2
• Finds first term, or equivalent merit	1

2019 Mathematics Question 11d

What is the limiting sum of the following geometric series? **2**

$$2000 - 1200 + 720 - 432 \ldots$$

Criteria	Marks
• Provides correct solution	2
• Identifies the common ratio, or equivalent merit	1

2019 Mathematics Question 16a

A person wins \$1 000 000 in a competition and decides to invest this money in an account that earns interest at 6% per annum compounded quarterly. The person decides to withdraw \$80 000 from this account at the end of every fourth quarter. Let A_n be the amount remaining in the account after the nth withdrawal.

(i) Show that the amount remaining in the account after the withdrawal at the end of the eighth quarter is **2**

$$A_2 = 1\,000\,000 \times 1.015^8 - 80\,000\left(1 + 1.015^4\right).$$

(ii) For how many years can the full amount of \$80 000 be withdrawn? **3**

2019 Mathematics Question 16a

Question 16 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains the expression for A_1, or equivalent merit	1

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	3
• Correctly sums the series and equates A_n to zero, or equivalent merit	2
• Obtains an expression for A_n, or equivalent merit	1

2019 Mathematics Question 16a

2019 Mathematics Question 16a

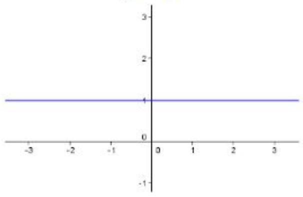
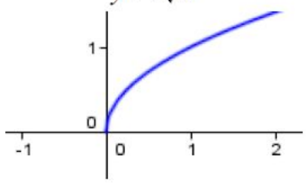
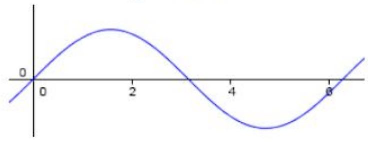
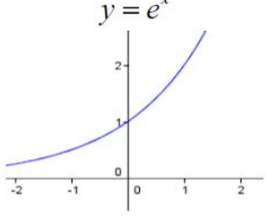
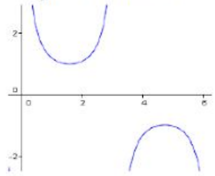
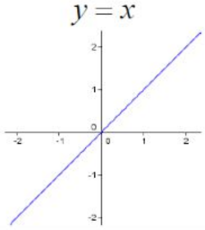
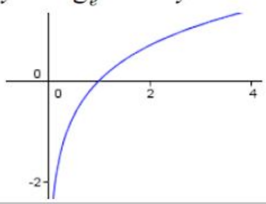
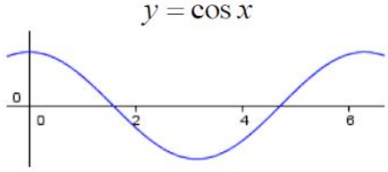
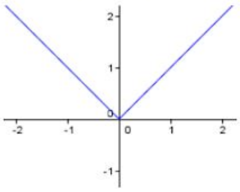
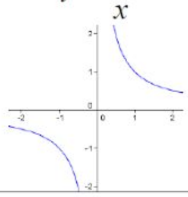
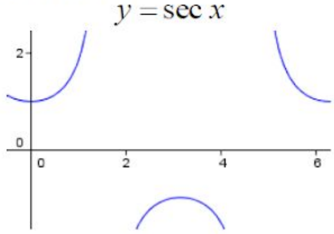
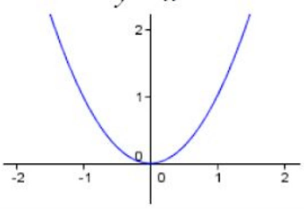
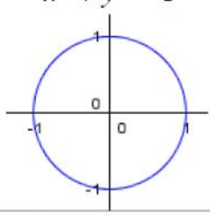
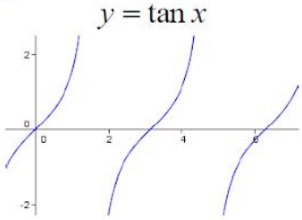
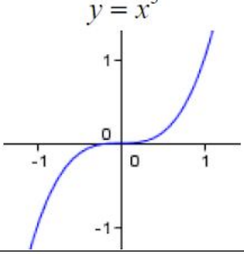
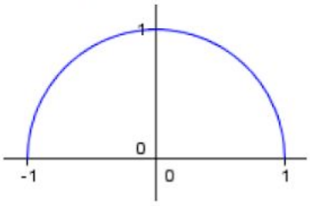
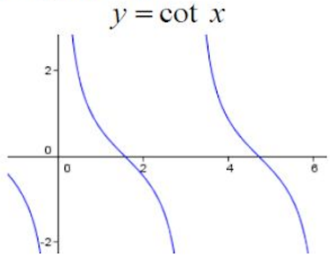
Useful Resources

- https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/resources/hsc-exam-papers Past HSC Papers with solutions, marking criteria and Markers notes.
- https://www.hscninja.com/course/ Past papers by topics
- https://www.mathswhiz.com.au/pages/hsc-past-papers-mathematics-advanced-general-2-unit-trials - Past papers from a variety of schools with worked solutions

HSC Advanced Mathematics Sessions with Stuart Palmer

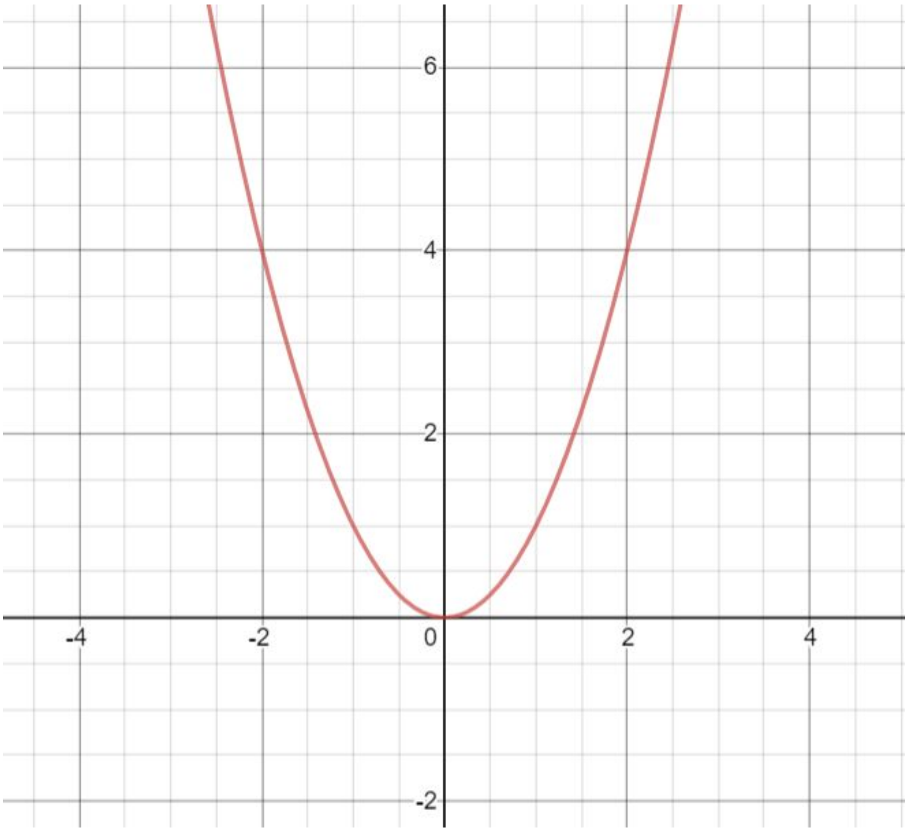
Topic: Functions

First and foremost, over the years you have studied a variety of graphs which you should now be able to recognise or graph on sight. You need to carry these words, equations and shapes inside your head.

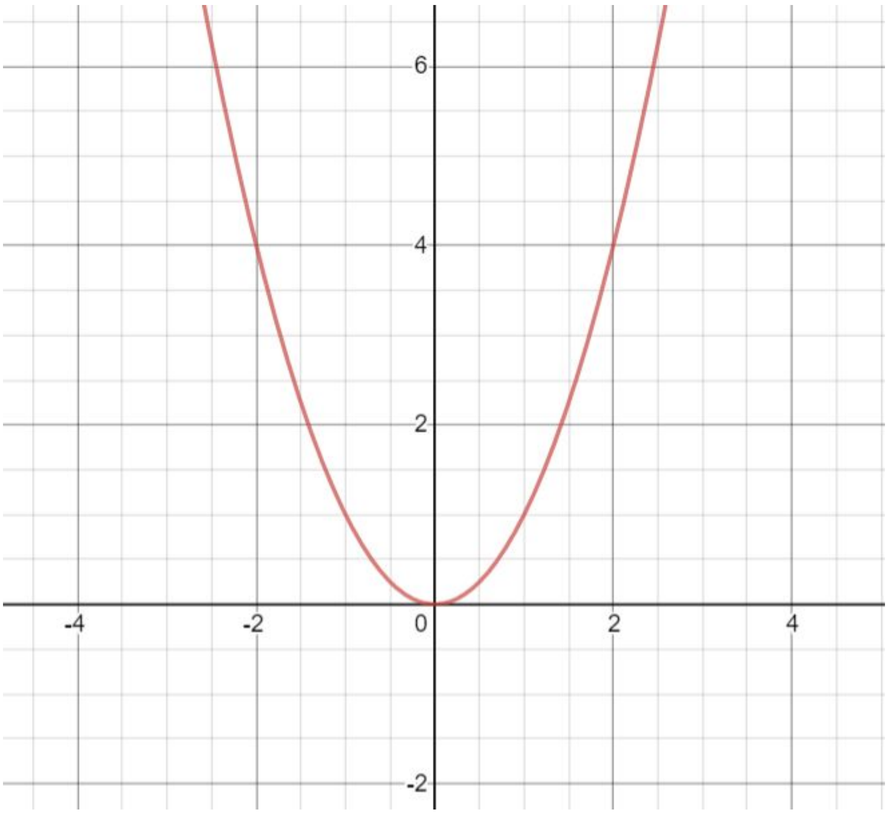
Number plane graphs that students could be required to draw by hand or recognise from memory in the HSC Examination		
horizontal lines $y = 1$ 	**square root** $y = \sqrt{x}$ 	**sine curve** $y = \sin x$ 
vertical lines $x = 1$ 	**exponential** $y = e^x$ 	**cosec curve** $y = \operatorname{cosec} x$ 
oblique lines $y = x$ 	**logarithm** $y = \log_e x$ or $y = \ln x$ 	**cosine curve** $y = \cos x$ 
absolute value $y = \lvert x \rvert$ 	**hyperbola** $y = \frac{1}{x}$ 	**secant curve** $y = \sec x$ 
parabola $y = x^2$ 	**circle** $x^2 + y^2 = 1$ 	**tangent curve** $y = \tan x$ 
cubic $y = x^3$ 	**semicircle** $y = \sqrt{1 - x^2}$ 	**cotangent curve** $y = \cot x$ 

You also need to know how to transform graphs based on changes to their equations:

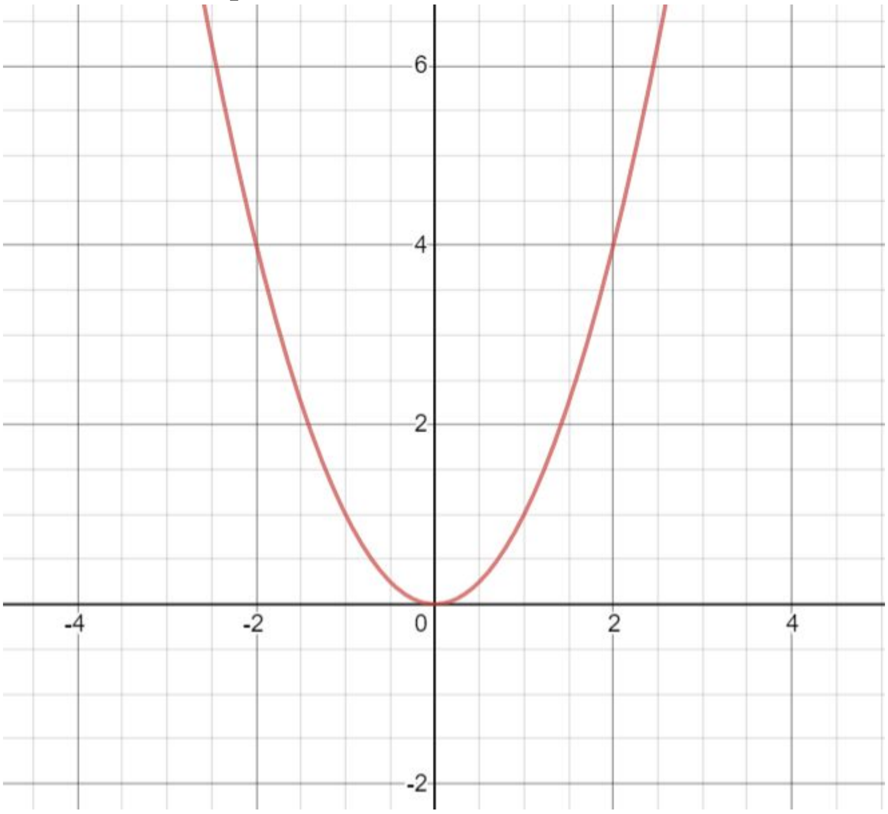
Use $y = x^2$ to sketch $y = x^2 - 2$ and $y = x^2 + 2$. Label them.



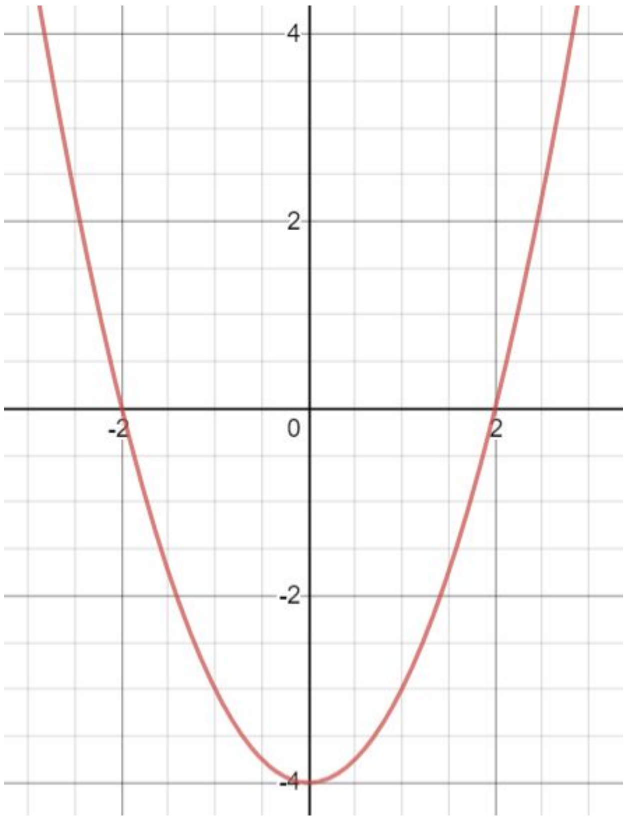
Use $y = x^2$ to sketch $y = (x-2)^2$ and $y = (x+2)^2$. Label them.



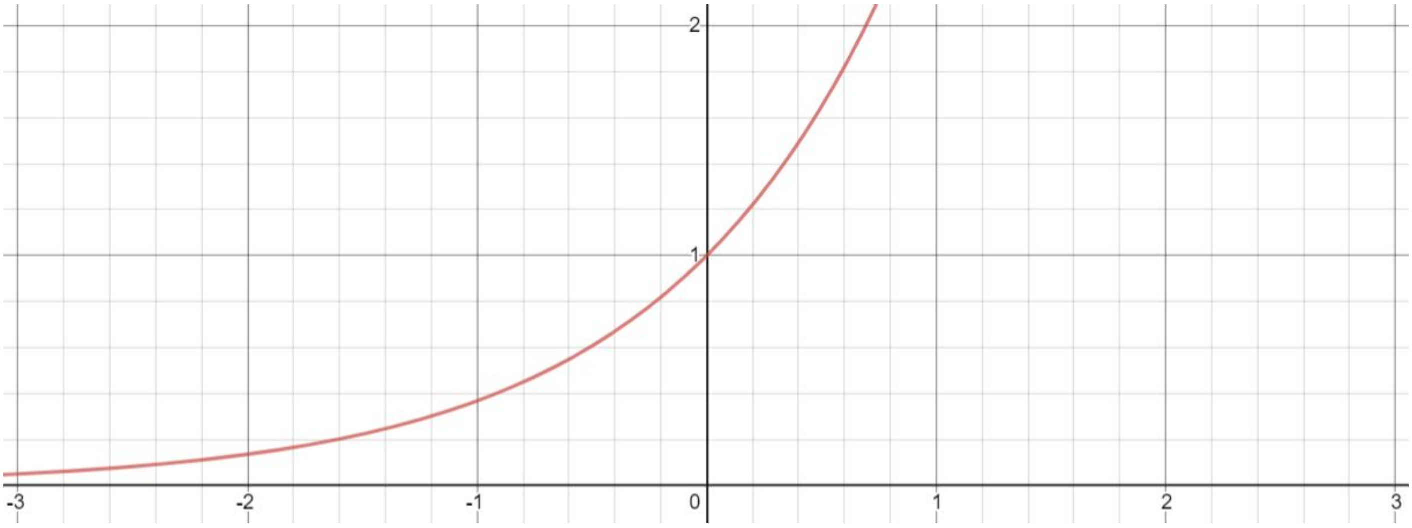
Use $y = x^2$ to sketch $y = 2x^2$ and $y = \frac{x^2}{2}$. Label them.



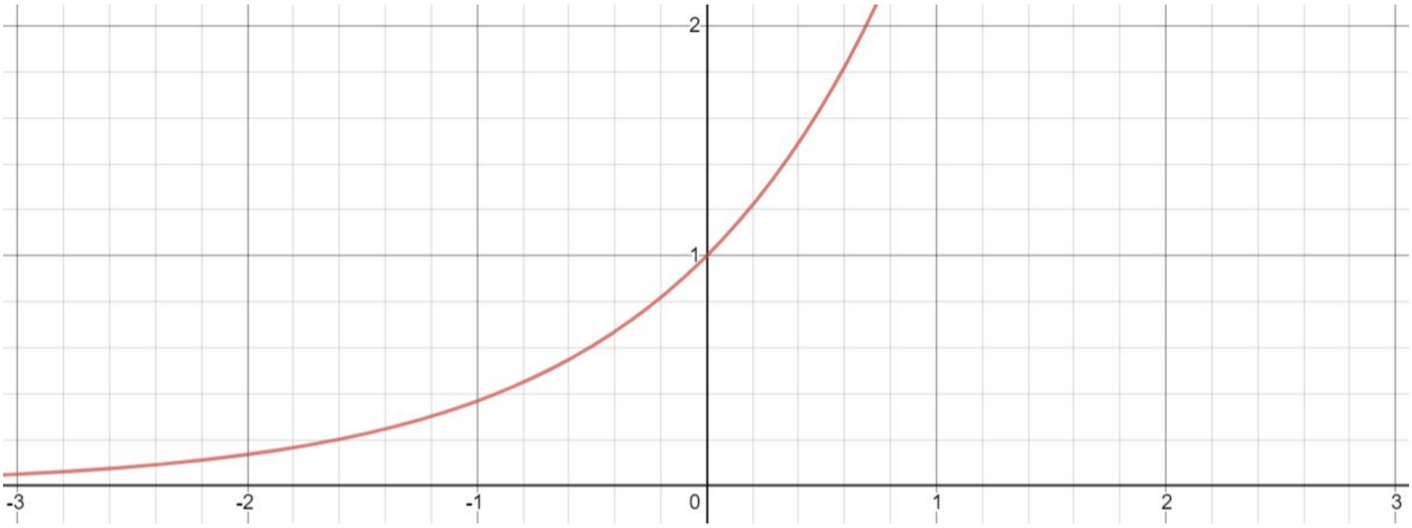
Use $y = x^2 - 4$ to draw $y = 4 - x^2$



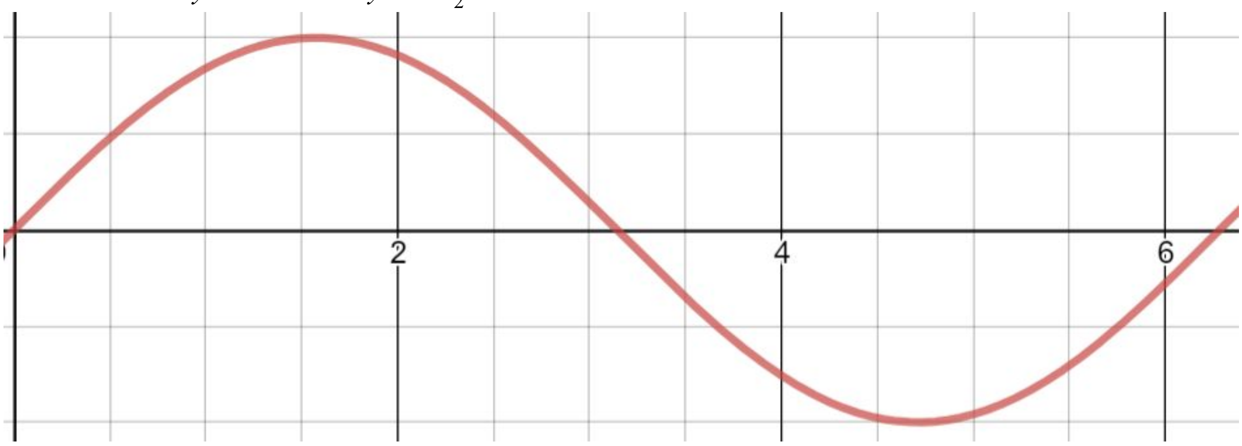
Use $y = e^x$ to draw $y = e^{-x}$



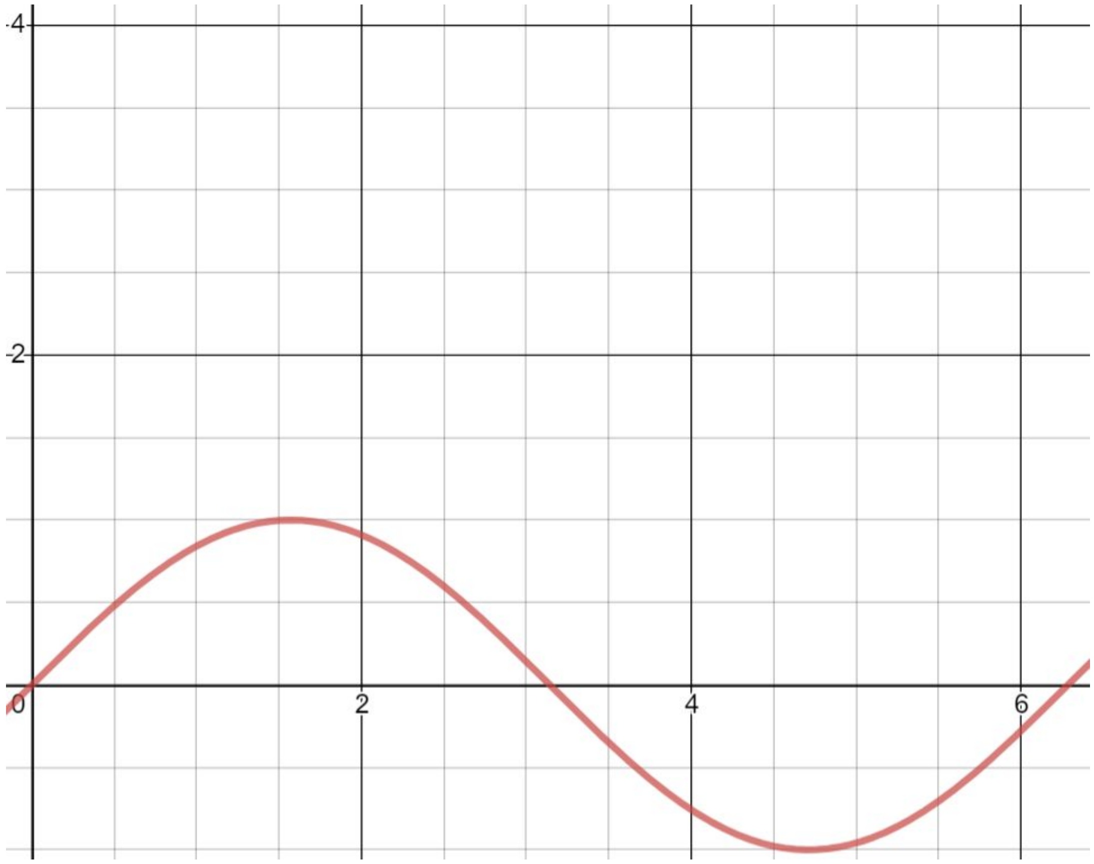
Use $y = e^x$ to draw $y = e^{2x}$ and $y = e^{0.1x}$. Label them.



Use $y = sinx$ to draw $y = sin2x$ and $y = sin\frac{1}{2}x$. Label them.

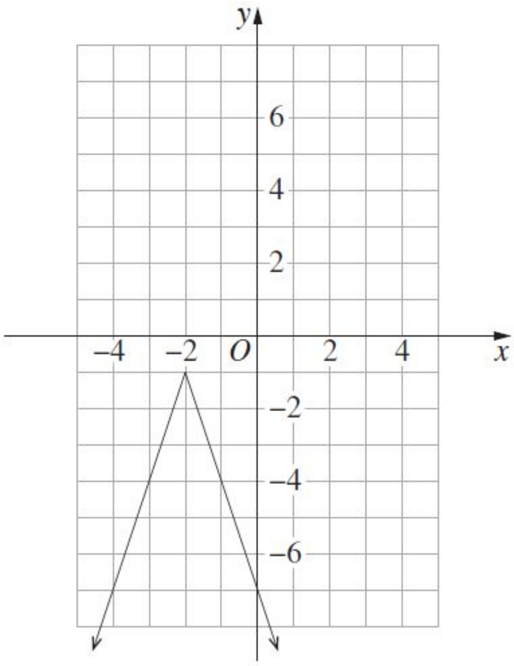


Use $y = sinx$ to draw $y = 2sin2x + 2$



The function $f(x) = |x|$ is transformed and the equation of the new function is of the form $y = kf(x + b) + c$, where k, b and c are constants. **2**

The graph of the new function is shown.



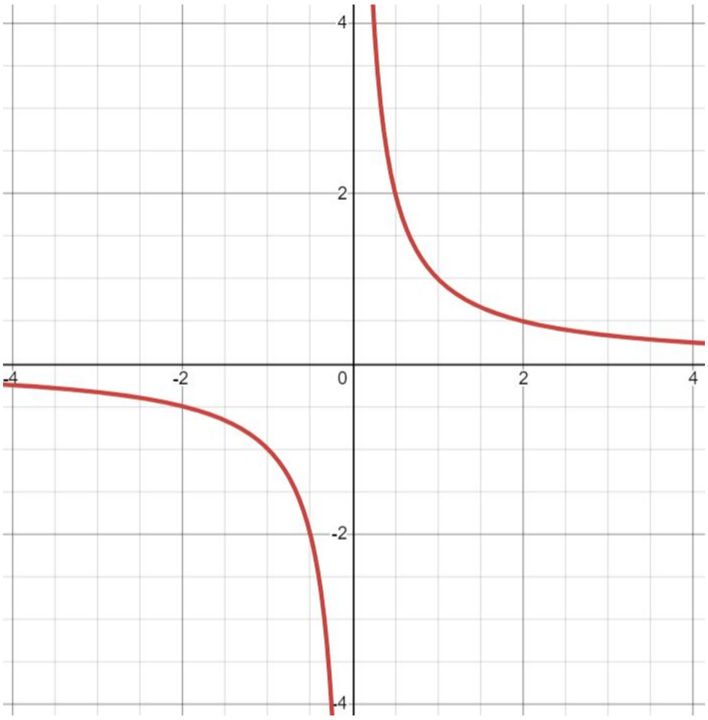
What are the values of k, b and c?

Dealing with horizontal and vertical asymptotes

Consider the equation xy = 1.
It says: 'The product of two unknown numbers is one.'
The two numbers can be, for example

- 1 and 1, or
- 0.5 and 2, or
- -0.5 and -2, etc

All of these points together form the basic garden variety hyperbola:



But there is one thing for sure, **neither number can be zero**:

- x can't be zero, so **the y-axis is a vertical asymptote**, because every point on the y-axis has x = 0:
  - (0,-2), (0,-1), (0,0), (0,1), (0,2), (0,3) etc.
- y can't be zero, so **the x-axis is a horizontal asymptote**, because every point on the x-axis has y = 0:
  - (-2,0), (-1,0), (0,0), (1,0), (2,0) etc.

The same can be said for xy = 2, which is usually written as $y = \frac{2}{x}$
Likewise for xy = 3, 4, 5, 6 or any number other than 0.
Open Desmos, graph xy = 1, then change the 1 to a 2, 3, 4, 5, etc

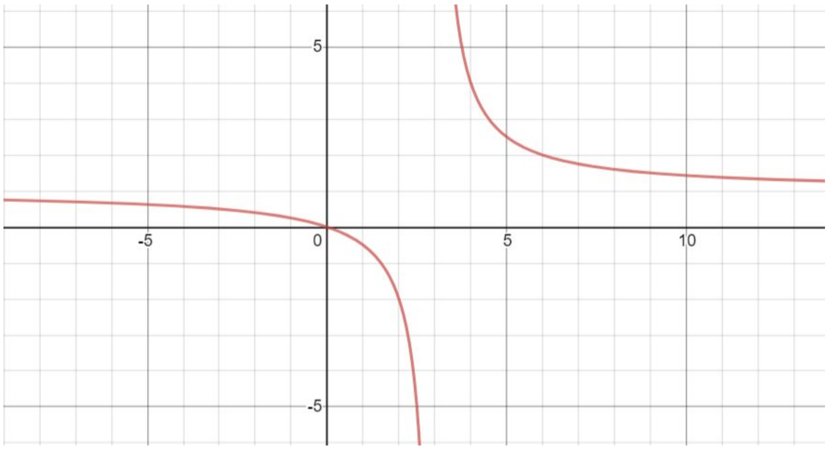
When the equation is $\frac{1}{x-2}$, the hyperbola is translated 2 units to the right.
The x-axis is still a horizontal asymptote. The vertical line x = 2 is a vertical asymptote.
Note: When the graph is translated 2 units to the right the asymptotes go with it.

What about vertical translations?
Consider $y = \frac{1}{x} + 2$. This translates the basic hyperbola up by 2.
Note the following. There are some other ways to write that equation:

$y = \frac{1}{x} + 2$ is the same as $y = \frac{1}{x} + \frac{2x}{x}$ which is $y = \frac{1+2x}{x}$ which is $y = \frac{2x+1}{x}$

What about $y = \frac{x}{x-3}$?

You can do this: $y = \frac{x}{x-3}$ is the same as $y = \frac{x-3+3}{x-3}$ then split into $y = \frac{x-3}{x-3} + \frac{3}{x-3}$ which is $y = 1 + \frac{3}{x-3}$.
This we can draw: Basic hyperbola, dilated vertically by 3, translated right by 3, then shifted up by 1.
Vertical asymptote is x = 3. Horizontal asymptote is y = 1.



What about $y = \frac{2x}{x-3}$?
That is $y = 2(\frac{x}{x-3})$. The previous graph is in the brackets. So this is the previous graph dilated vertically by 2, so the horizontal asymptote is now y = 2, not y = 1.

What about $y = \frac{2x+1}{x-3}$? Note that the 2x in the numerator is double the x in the denominator.
That is $y = \frac{2x-6+7}{x-3}$ which is $y = \frac{2x-6}{x-3} + \frac{7}{x-3}$ which is $y = 2 + \frac{7}{x-3}$ which you know how to draw.

Examining the behaviour of a graph in the vicinity of an asymptote

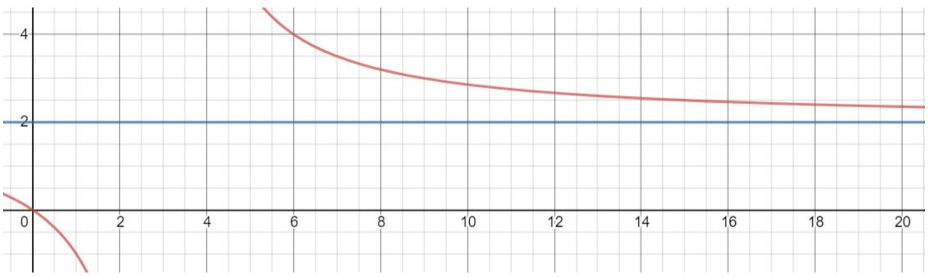
You can do this with vertical asymptotes:
Let's stick with $y = \frac{2x}{x-3}$. We know that x = 3 is a vertical asymptote. This table can help. It tells us what is happening near the asymptote:

x	2.9	3	3.1
y	-58	**undefined**	62

Another way to detect a horizontal asymptote:
Let's stick with $y = \frac{2x}{x-3}$. This table can help. It steers you towards a horizontal asymptote:

x	10	20	50	100
y	2.86	2.35	2.3	2.06

Conclusion: As x is approaching infinity, y is approaching 2, from above.



You can do the same thing with negative values of x (ie -10, -20, -50, -100)

HSC Study Day 2020

MATHEMATICS

Statistical Analysis

Robert Gorton
Robert.gorton1@det.nsw.edu.au
Wadalba Community School

Overview

- Syllabus outcomes
- Summaries
- Past HSC questions
- Useful Resources

Syllabus Outcomes (per topic)

MA-S1 Probability and Discrete Probability Distributions

- › uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions MA11-7
- › uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8
- › provides reasoning to support conclusions which are appropriate to the context MA11-9

Syllabus Outcomes (per topic)

S1.1: Probability and Venn diagrams

Students:

- understand and use the concepts and language associated with theoretical probability, relative frequency and the probability scale
- solve problems involving simulations or trials of experiments in a variety of contexts **AAM**
  - identify factors that could complicate the simulation of real-world events (ACMEM153)
  - use relative frequencies obtained from data as point estimates of probabilities (ACMMM055)
- use arrays and tree diagrams to determine the outcomes and probabilities for multi-stage experiments (ACMEM156) **AAM**
- use Venn diagrams, set language and notation for events, including $\bar{A}$ (or A' or A^c) for the complement of an event A, $A \cap B$ for 'A and B', the intersection of events A and B, and $A \cup B$ for 'A or B', the union of events A and B, and recognise mutually exclusive events (ACMMM050) **AAM**
  - use everyday occurrences to illustrate set descriptions and representations of events and set operations (ACMMM051)
- establish and use the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054) **AAM**
- understand the notion of conditional probability and recognise and use language that indicates conditionality (ACMMM056)

Syllabus Outcomes (per topic)

S1.1: Probability and Venn diagrams

- use the notation $P(A|B)$ and the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$ for conditional probability (ACMMM057) **AAM**
- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)
- use the multiplication law $P(A \cap B) = P(A)P(B)$ for independent events A and B and recognise the symmetry of independence in simple probability situations (ACMMM059)

Syllabus Outcomes (per topic)

S1.2: Discrete probability distributions

- define and categorise random variables
  - know that a random variable describes some aspect in a population from which samples can be drawn
  - know the difference between a discrete random variable and a continuous random variable
- use discrete random variables and associated probabilities to solve practical problems (ACMMM142) **AAM**
  - use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable (ACMMM137)
  - recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes (ACMMM138)
  - examine simple examples of non-uniform discrete random variables, and recognise that for any random variable, X, the sum of the probabilities is 1 (ACMMM139)
  - recognise the mean or expected value, $E(X) = \mu$, of a discrete random variable X as a measure of centre, and evaluate it in simple cases (ACMMM140)
  - recognise the variance, $\mathrm{Var}(X)$, and standard deviation (σ) of a discrete random variable as measures of spread, and evaluate them in simple cases (ACMMM141)
  - use $\mathrm{Var}(X) = E((X-\mu)^2) = E(X^2) - \mu^2$ for a random variable and $\mathrm{Var}(x) = \sigma^2$ for a dataset
- understand that a sample mean, $\bar{x}$, is an estimate of the associated population mean μ, and that the sample standard deviation, s, is an estimate of the associated population standard deviation, σ, and that these estimates get better as the sample size increases and when we have independent observations

Syllabus Outcomes (per topic)

MA-S2 Descriptive Statistics and Bivariate Data Analysis

A student:

› solves problems using appropriate statistical processes MA12-8
› chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
› constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Syllabus Outcomes (per topic)

S2.1: Data (grouped and ungrouped) and summary statistics

- classify data relating to a single random variable
- organise, interpret and display data into appropriate tabular and/or graphical representations including Pareto charts, cumulative frequency distribution tables or graphs, parallel box-plots and two-way tables **AAM**
  - compare the suitability of different methods of data presentation in real-world contexts (ACMEM048)
- summarise and interpret grouped and ungrouped data through appropriate graphs and summary statistics **AAM**
- calculate measures of central tendency and spread and investigate their suitability in real-world contexts and use to compare large datasets
  - investigate real-world examples from the media illustrating appropriate and inappropriate uses or misuses of measures of central tendency and spread (ACMEM056) **AAM**
- identify outliers and investigate and describe the effect of outliers on summary statistics
  - use different approaches for identifying outliers, for example consideration of the distance from the mean or median, or the use of below $Q_1 - 1.5 \times IQR$ and above $Q_3 + 1.5 \times IQR$ as criteria, recognising and justifying when each approach is appropriate
  - investigate and recognise the effect of outliers on the mean, median and standard deviation
- describe, compare and interpret the distributions of graphical displays and/or numerical datasets and report findings in a systematic and concise manner **AAM**

Syllabus Outcomes (per topic)

S2.2: Bivariate data analysis

- construct a bivariate scatterplot to identify patterns in the data that suggest the presence of an association (ACMGM052)
- use bivariate scatterplots (constructing them where needed), to describe the patterns, features and associations of bivariate datasets, justifying any conclusions **AAM**
  - describe bivariate datasets in terms of form (linear/non-linear) and in the case of linear, also the direction (positive/negative) and strength of association (strong/moderate/weak)
  - identify the dependent and independent variables within bivariate datasets where appropriate
  - describe and interpret a variety of bivariate datasets involving two numerical variables using real-world examples in the media or those freely available from government or business datasets
- calculate and interpret Pearson's correlation coefficient (r) using technology to quantify the strength of a linear association of a sample (ACMGM054)
- model a linear relationship by fitting an appropriate line of best fit to a scatterplot and using it to describe and quantify associations **AAM**
  - fit a line of best fit to the data by eye and using technology (ACMEM141, ACMEM142)
  - fit a least-squares regression line to the data using technology (ACMGM057)
  - interpret the intercept and gradient of the fitted line (ACMGM059)

Syllabus Outcomes (per topic)

S2.2: Bivariate data analysis

- use the appropriate line of best fit, both found by eye and by applying the equation of the fitted line, to make predictions by either interpolation or extrapolation **AAM**
  - distinguish between interpolation and extrapolation, recognising the limitations of using the fitted line to make predictions, and interpolate from plotted data to make predictions where appropriate
- solve problems that involve identifying, analysing and describing associations between two numeric variables **AAM**
- construct, interpret and analyse scatterplots for bivariate numerical data in practical contexts **AAM**
  - demonstrate an awareness of issues of privacy and bias, ethics, and responsiveness to diverse groups and cultures when collecting and using data

Syllabus Outcomes (per topic)

MA-S3 Random Variables

A student:

› solves problems using appropriate statistical processes MA12-8
› chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
› constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10

Syllabus Outcomes (per topic)

S3.1: Continuous random variables

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)
- understand and use the concepts of a probability density function of a continuous random variable **AAM**
  - know the two properties of a probability density function: $f(x) \geq 0$ for all real x and $\int_{-\infty}^{\infty} f(x)dx = 1$
  - define the probability as the area under the graph of the probability density function using the notation $P(X \leq r) = \int_a^r f(x)dx$, where $f(x)$ is the probability density function defined on $[a, b]$
  - examine simple types of continuous random variables and use them in appropriate contexts
  - explore properties of a continuous random variable that is uniformly distributed
  - find the mode from a given probability density function
- obtain and analyse a cumulative distribution function with respect to a given probability density function
  - understand the meaning of a cumulative distribution function with respect to a given probability density function
  - use a cumulative distribution function to calculate the median and other percentiles

Syllabus Outcomes (per topic)

S3.2: The normal distribution

- identify the numerical and graphical properties of data that is normally distributed
- calculate probabilities and quantiles associated with a given normal distribution using technology and otherwise, and use these to solve practical problems (ACMMM170) **AAM**
  - identify contexts that are suitable for modelling by normal random variables, eg the height of a group of students (ACMMM168)
  - recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ, and the use of the standard normal distribution (ACMMM169)
  - visually represent probabilities by shading areas under the normal curve, eg identifying the value above which the top 10% of data lies
- understand and calculate the z-score (standardised score) corresponding to a particular value in a dataset **AAM**
  - use the formula $z = \frac{x-\mu}{\sigma}$, where μ is the mean and σ is the standard deviation
  - describe the z-score as the number of standard deviations a value lies above or below the mean

Syllabus Outcomes (per topic)

S3.2: The normal distribution

- use z-scores to compare scores from different datasets, for example comparing students' subject examination scores **AAM**
- use collected data to illustrate the empirical rules for normally distributed random variables
  - apply the empirical rule to a variety of problems
  - sketch the graphs of $f(x) = e^{-x^2}$ and the probability density function for the normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ using technology
  - verify, using the Trapezoidal rule, the results concerning the areas under the normal curve
- use z-scores to identify probabilities of events less or more extreme than a given event **AAM**
  - use statistical tables to determine probabilities
  - use technology to determine probabilities
- use z-scores to make judgements related to outcomes of a given event or sets of data **AAM**

Summaries – Probability and Discrete Probability Distributions

- The probability of a particular outcome occurring is calculated using the formula:

$$P(\text{outcome}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

- The probability of an outcome must be between 0 and 1, inclusive

$$0 \leq P(\text{outcome}) \leq 1$$

- If $P(\text{outcome})$=0, the outcome is impossible
- If $P(\text{outcome})$=1, the outcome is certain to occur

Summaries – Probability and Discrete Probability Distributions

Complementary events

- The complement of an outcome is the list of all other possible outcomes
- $P(\bar{A})$ means the complement of A occurring (A not occurring)
- $P(\bar{A}) = 1 - P(A)$

Summaries – Probability and Discrete Probability Distributions

Mutually exclusive events

- If two events are mutually exclusive, they cannot both occur at the same time
- For mutually exclusive events, A and B:

$$P(A \text{ or } B) = P(A) + P(B)$$

- For Non-mutually exclusive events, A and B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Summaries – Probability and Discrete Probability Distributions

Multiplication Principle

- In multi-stage events, the probability of a final outcome is the product of all of the outcomes for each stage
- Tree diagrams can be used to show sample space and calculate the probabilities of all possible outcomes using multiplication along the branches

Summaries – Probability and Discrete Probability Distributions

Independent versus dependent events

- If two events are independent, then the outcome of each event is not affected by the outcome of the other event.

  For independent events:

  $$P(\text{A and } B) = P(A) \times P(B)$$

- If the outcome of an event is affected by the outcome of another event, then these two events are dependent.

  $P(B|A)$ notation means "the probability of event B, given that event A has occurred.

  For dependent events:

  $$P(\text{A and } B) = P(A) \times P(B|A)$$

Summaries – Descriptive Statistics and Bivariate Data Analysis

- Identify types of data
- Display data in tables and graphs
- Measures of Central Tendency – Mean, Median, Mode
- Measures of Spread – Range, Quantiles, IQR, Variance, Standard Deviation
- Identify outliers
- Recognise modality and the shape of data sets
- Identifying bias
- Compare and contrast 2 sets of data

Nelson, Maths in Focus, Year 12

Summaries – Descriptive Statistics and Bivariate Data Analysis

- Interpret scatterplots of bivariate data
- Identify correlation in bivariate data
- Calculate Pearson's correlation coefficient
- Apply lines of best fit
- Least-squares regression line
- Interpolate and extrapolate from a data set

Nelson, Maths in Focus, Year 12

Summaries – **Random Variables**

- Recognise continuous random variables
- Properties of a probability density function (PDF)
- Find a cumulative distribution function (CDF)
- Find probabilities of continuous data
- Calculate measures of central tendency and spread for a continuous probability distribution
- Recognise the normal distribution and identify properties
- Calculate properties and quantiles for normal distributions
- Understand the standard normal distribution and z-scores
- Apply the normal distribution to solving practical problems

Nelson, Maths in Focus, Year 12

Knowing the Exam

The paper will consist of two sections.

Section I (10 marks)

- There will be objective-response questions to the value of 10 marks.

Section II (90 marks)

- Questions may contain parts.
- There will be 37 to 42 items.
- At least two items will be worth 4 or 5 marks.
- The Mathematics Advanced examination will include items that are common with the Mathematics Standard 2 HSC examination. Common items will be worth 20 to 25 marks and will be distributed throughout Sections I and II.

The examination will be based on the Mathematics Advanced Year 12 course and will focus on the course objectives and Year 12 outcomes. The Mathematics Advanced Year 11 course will be assumed knowledge for this examination and may be examined.

Knowing the Exam – Collated by Robert Yen

MATHEMATICS STANDARD 2	MATHEMATICS ADVANCED
YEAR 11 LINEAR FUNCTIONS Linear functions and modelling Direct (linear) variation **YEAR 12 NON-LINEAR FUNCTIONS** Quadratic functions and modelling Reciprocal function (hyperbola) Inverse (linear) variation	**YEAR 11 FUNCTIONS** Linear functions and modelling Direct (linear) variation Quadratic functions and modelling Reciprocal function (hyperbola) Inverse (linear) variation
YEAR 12 FUNCTIONS Solving simultaneous equations graphically Break-even analysis Exponential functions and modelling	**YEAR 11 FUNCTIONS** Solving simultaneous equations graphically Break-even analysis Exponential functions and modelling
YEAR 12 FINANCIAL MATHS Compound interest $FV = PV(1 + r)^n$ Declining-balance depreciation Reducing balance loans Credit cards	**YEAR 12 FINANCIAL MATHS** Compound interest $A = P(1 + r)^n$
YEAR 12 ANNUITIES Annuities using technology, recurrence relations. tables of interest factors	**YEAR 12 ANNUITIES** Annuities using technology, recurrence relations, tables of interest factors

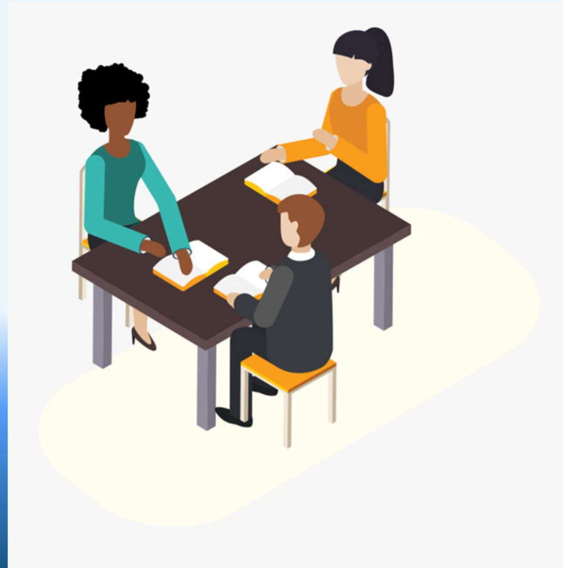
Knowing the Exam – Collated by Robert Yen

YEAR 12 TRIGONOMETRY Pythagoras' theorem, trigonometry Compass and true bearings The sine and cosine rules $A = \frac{1}{2} ab \sin C$	**YEAR 11 TRIGONOMETRY** Pythagoras' theorem, trigonometry Compass and true bearings The sine and cosine rules $A = \frac{1}{2} ab \sin C$
YEAR 11 TRAPEZOIDAL RULE? Trapezoidal rule $A \approx \frac{h}{2}(d_f + d_l)$	**YEAR 12 TRAPEZOIDAL RULE?** Trapezoidal rule $A \approx \frac{b-a}{2n}\left\{ f(a)+f(b)+ 2\left[f(x_1)+\ldots+f(x_{n-1})\right]\right\}$

Knowing the Exam – Collated by Robert Yen

YEAR 11 PROBABILITY Theoretical probability Relative frequency Arrays and tree diagrams Probability simulations	**YEAR 11 PROBABILITY** Theoretical probability Relative frequency Arrays and tree diagrams Probability simulations
YEAR 11 STATISTICS Types of statistical data Statistical graphs Mean, median, mode Range, IQR, standard deviation Deciles, percentiles Outliers, shapes, modality	**YEAR 12 STATISTICS** Types of statistical data Statistical graphs Mean, median, mode Range, IQR, standard deviation Deciles, percentiles (in normal distribution) Outliers, shapes, modality
YEAR 12 STATISTICS Scatterplots Line of best fit Least-squares regression line Pearson's correlation coefficient Normal distribution z-scores $z = \frac{x - \bar{x}}{s}$	**YEAR 12 STATISTICS** Scatterplots Line of best fit Least-squares regression line Pearson's correlation coefficient Normal distribution z-scores $z = \frac{x - \mu}{\sigma}$

Your Turn!



Past HSC Questions

2016 Standard Multiple Choice Question 23

A group of 485 people was surveyed. The people were asked whether or not they smoke. The results are recorded in the table.

	Smokers	*Non-smokers*	*Total*
Male	88	176	264
Female	68	153	221
	156	329	485

A person is selected at random from the group.

What is the approximate probability that the person selected is a smoker OR is male?

(A) 33%
(B) 18%
(C) 68%
(D) 87%

2016 Standard Multiple Choice Question 23

2017 Standard Question 29 c)

A group of Year 12 students was surveyed. The students were asked whether they live in the city or the country. They were also asked if they have ever waterskied.

The results are recorded in the table.

	Have waterskied	Have never waterskied
Live in the city	150	2500
Live in the country	70	800

2017 Standard 29 c

A newspaper article claimed that Year 12 students who live in the country are more likely to have waterskied than those who live in the city. **2**

Is this true, based on the survey results? Justify your answer with relevant calculations.

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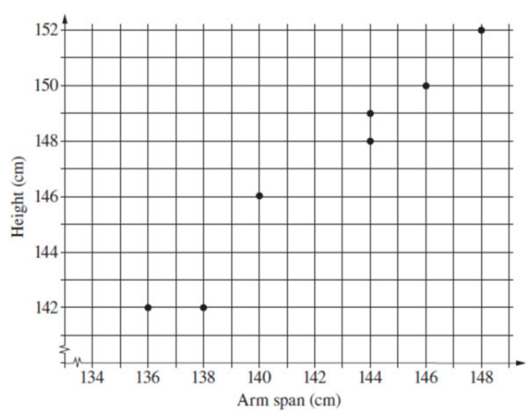
Criteria	Marks
• Provides correct solution	2
• Provides correct numerator or denominator, or equivalent merit	1

2019 Standard 23

Question 23 (3 marks)

A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatterplot of these measurements.



2019 Standard Question 23

(a) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. **1**

....

Question 23 (a)

Criteria	Marks
• Provides correct answer with or without correct rounding	1

2019 Standard Question 23

(b) Identify the direction and the strength of the linear association between height and arm span. **1**

....

Question 23 (b)

Criteria	Marks
• Provides correct direction and strength	1

2019 Standard 2 Question 23

(c) The equation of the least-squares regression line is shown. **1**

$$\text{Height} = 0.866 \times (\text{arm span}) + 23.7$$

A child has an arm span of 143 cm.

Calculate the predicted height for this child using the equation of the least-squares regression line.

....

....

Question 23 (c)

Criteria	Marks
• Provides correct answer	1

2019 Standard 2 Question 39

Two netball teams, Team *A* and Team *B*, each played 15 games in a tournament. For each team, the number of goals scored in each game was recorded. **5**

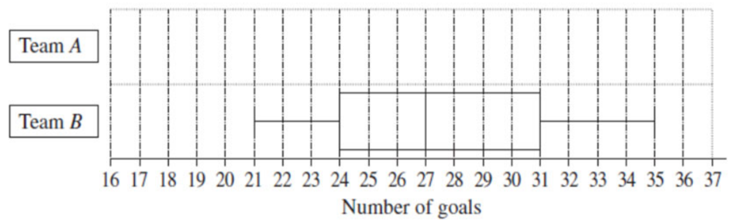
The frequency table shows the data for Team *A*.

Number of goals	*Frequency*
19	1
20	0
21	1
22	1
23	1
24	3
25	0
26	4
27	3
28	1

The data for Team *B* was analysed to create the box-plot shown.

2019 Standard 2 Question 39

The data for Team *B* was analysed to create the box-plot shown.



Compare the distributions of the number of goals scored by the two teams. Support your answer with the construction of a box-plot for the data for Team *A*.

2019 Standard 2 Question 39

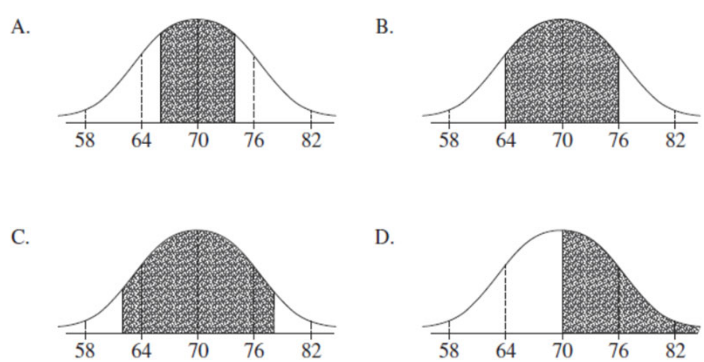
2019 Standard 2 Question 39

Criteria	Marks
• Provides correct box-plot AND comparisons of 'central tendency, spread and skewness'	5
• Provides correct box-plot AND compares two of 'central tendency, spread and skewness', or equivalent merit	4
• Provides correct box-plot AND compares one of 'central tendency, spread and skewness', or equivalent merit	3
• Provides correct box-plot OR compares two of 'central tendency, spread and skewness' or equivalent merit	2
• Provides one correct quartile for team *A* or correct comparison of the minimum or maximum scores, or equivalent merit	1

2019 Standard 2 Question 15 Multiple Choice

The scores on an examination are normally distributed with a mean of 70 and a standard deviation of 6. Michael received a score on the examination between the lower quartile and the upper quartile of the scores.

Which shaded region most accurately represents where Michael's score lies?



2019 Standard 2 Question 39

2015 Standard 2 Question 28 c)

The results of two tests are normally distributed. The mean and standard deviation for each test are displayed in the table. **2**

	Mathematics	English
$\bar{x}$	70	75
s	6.5	8

Kristoff scored 74 in Mathematics and 80 in English. He claims that he has performed better in English.

2015 Standard 2 Question 28 c)

Is Kristoff correct? Justify your answer using appropriate calculations.

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2015 Standard 2 Question 28 c)

Question 28 (b)

Criteria	Marks
• Determines that the claim is correct, justified with correct and appropriate calculations	2
• Correctly calculates one z-score or equivalent merit	1

SmarterMaths Bank Question

A probability density function $f(x)$ is given by

$$f(x) = \begin{cases} \frac{1}{12}(8x - x^3) & 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

The median m of this function satisfies the equation

A. $-m^4 + 16m^2 - 6 = 0$

B. $m^4 - 16m^2 = 0$

C. $m^4 - 16m^2 + 24 = 0.5$

D. $m^4 - 16m^2 + 24 = 0$

SmarterMaths Bank Question

The probability density function $f(x)$ of a random variable X is given by

$$f(x) = \begin{cases} \frac{x+1}{12} & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of b such that $P(X \le b) = \frac{5}{8}$. *(3 marks)*

Useful Resources

- https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/resources/hsc-exam-papers Past HSC Papers with solutions, marking criteria and Markers notes.
- https://www.hscninja.com/ Past papers by topics
- https://www.mathswhiz.com.au/pages/hsc-past-papers-mathematics-advanced-general-2-unit-trials - Past papers from a variety of schools with worked solutions
- https://hschub.nsw.edu.au/mathematics/mathematics –Videos by topics

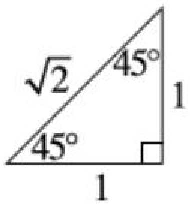
HSC Advanced Mathematics Sessions with Stuart Palmer
Topic: Trigonometry

In this topic, the information on the HSC Reference Sheet is helpful, but a few minutes of 'enhancement' could pay dividends several times during the three-hour marathon.

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



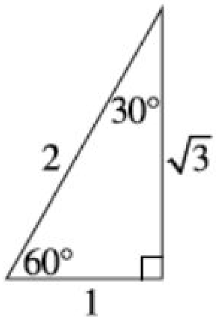
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



It's nice that NESA have given you those two triangles, but I think this is much more useful:
You could draw this table in the empty space on page 4 of the Reference Sheet.
Short-term pain, long-term gain!

radians	degrees	sinA	cosA	tanA	These graphs are good too!
	0, 360				
	30				
	45				
	60				
	90				
	180				
	270				

If you have access to that table and the graphs during examinations, the following facts become obvious.
You can just look them up rather than thinking about them.
That frees up your mind for other more important things.

- What is $\sin 45^\circ$?
- What is $\sin \frac{\pi}{6}$?
- What is $\cos \pi$?
- What is $\sec \frac{\pi}{6}$? (There's no calculator button for that!)
- What is $\cot \frac{\pi}{2}$? (Surprise!)
- What are the solutions for $\sin x = 0$?
- What are the solutions for $\cos x = \frac{1}{2}$?

What other useful results can you get from these?

Trigonometric identities $\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$ $\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$ $\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$ $\cos^2 x + \sin^2 x = 1$	

These are VERY useful:

$$y = \sin f(x) \qquad \frac{dy}{dx} = f'(x)\cos f(x)$$

$$y = \cos f(x) \qquad \frac{dy}{dx} = -f'(x)\sin f(x)$$

$$y = \tan f(x) \qquad \frac{dy}{dx} = f'(x)\sec^2 f(x)$$

So are these:

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

But I still like this time-saving memory aid device:
(because I often put a negative sign where a positive should be or vice versa)

HSC Advanced Mathematics Sessions with Stuart Palmer
Topic: Trigonometry

You need to be fluent with Pythagoras' theorem and SOH CAH TOA:

- **From General/Standard:** 2017-8, 2017-26d, 2014-26b, 2013-4, 2012-4, 2012-27d, 2011-9, 2009-4, 2009-23ai, 2008-14, 2007-8, 2005-8, 2005-25bii, 2004-5, 2003-27ci, 2002-27bii, 2001-20
- **From 2 Unit:** 2019-14c, 2002-10a(i)(ii)

You need to be very fluent with the sine rule, the cosine rule, bearings and calculating the area of a triangle:

- **Advanced:** 2020sample-12, 2020sample-21,
- **From General/Standard (Angles of elevation and depression):** 2015-9, 2011-4, 2010-24d, 2009-23a, 2008-20, 2006-3
- **From General/Standard (Bearings):** 2018- 7, 2017-30cii, 2016-25, 2014-23, 2012-20, 2011-24c, 2010-10, 2009-27b, 2008-17, 2007-26aiii, 2006-13, 2005-27c, 2003-26b, 2002-27a
- **From 2 Unit:** 2019-11a, 2019-15b, 2018-12a, 2018-14a, 2017-13a, 2016-11c, 2015-13a, 2014-13d, 2013-14bi, 2013-14c, 2012-13aii, 2011-8ai, 2006-1d, 2005-3b, 2004-3c, 2003-4a, 2002-2c, 2002-4c, 2001-3d
- **From Extension 1** (Same concept, harder questions)**:** (Note: These may need to be done later in the topic) 2015-12c, 2012-14ci, 2010-5a, 2008-6a, 2005-7ai, 2004-3d, 2003-7a, 2001-7b(i)(ii)

When do you need to have your calculator in RAD mode?

HSC Advanced Mathematics Sessions with Stuart Palmer
Topic: Trigonometry

Arc length and area of sector:

- 2019-13b, 2018-16ai, 2017-11e, 2016-7, 2014-11g, 2013-13c, 2012-11f, 2011-10b(i)(ii), 2010-6b, 2009-5c, 2008-7b, 2007-4c, 2006-4a, 2005-4a, 2004-4a, 2003-1c, 2003-9b, 2002-5b, 2001-5

The trigonometric identities are hardly ever examined in the HSC Examination:

- 2017-7, 2010-5b(i)(ii)

Trigonometric equations which are quadratic equations in disguise:

- 2016-1, 2015-12a, 2014-7, 2014-15a, 2012-6, 2011-2b, 2009-1e, 2007-4a, 2005-2a, 2004-8ai, 2003-9a, 2002-4b

Trigonometric graphs:

- 2020sample-27, 2019-7, 2017-14a, 2016-6, 2016-8, 2013-6, 2010-8c, 2006-7b(i)(ii), 2002-10a(iii), 2001-4c(i), 2000-6a, 1996-7a, 1996-10a(i)

Solving trigonometric equations:

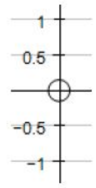
- 2019-13a, 2016-11g, 2015-12a, 2014-7, 2012-6, 2011-2b, 2009-1e, 2007-4a, 2008-6a, 2007-7b(i), 2005-2a, 2004-8a, 2003-9a, 2002-4b, 1999-10a

Solving practical problems:

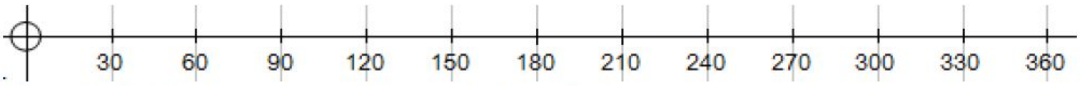
- **From 2 Unit:** 2018-15a (daylight hours), 2013-13a (wild horses), 2009-7b (tides), 2002-8b (particle on line)
- **From Extension 1:** 2016-13a (tides), 2004-7a (tides), 1997-3a (how many solutions?)

How to draw trigonometric graphs (Year 12) on lined paper in the HSC Examination

Tips:



- Draw the axes with pen so they don't run out if you need to re-draw the curve.
- For the **vertical axis** of sine curves and cosine curves, use the lines on the page as the scale and make every gap between the lines worth 0.5 (for $y = \sin x$) or 1 (for $y = 2\sin x$), so that **the amplitude will be maximum 2 or 3 lines high.**
- For the **horizontal axis**, make the section from **0 to 2π 12 cm long**. That way, every centimetre represents 30 degrees. Then, in a different colour, mark off multiples of 1.9 cm for every radian. This is very useful if you need to draw a straight line and a trigonometric curve on the same diagram.



- Plot all the high points and low points and intercepts.
- Use a calculator to check that those points really DO lie on the curve.
- Use pencil to draw the curve, making sure it is nice and smooth and does not 'wobble.
- When it is done, go over it with pen.
- If the domain is not from 0 to 2π it might be helpful to draw from 0 to 2π then 'modify'.

Question 1: Draw the graph $y = \sin x$ for $0 \le x \le 2\pi$

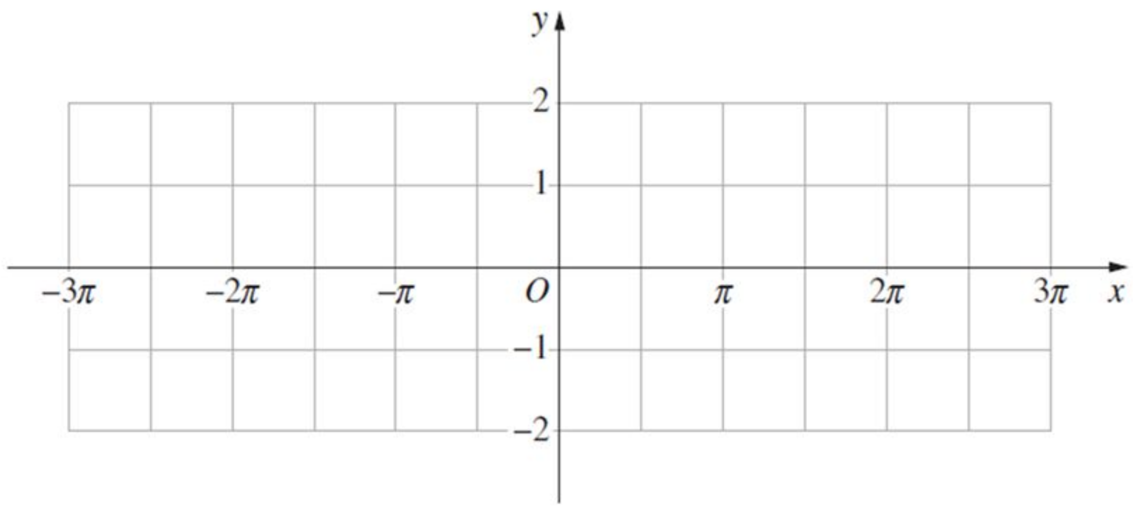
Question 2: Draw the graph $y = 3\sin 2x$ for $0 \le x \le 2\pi$ and $y = 2x - 1$

Question 3: Draw the graph $y = 2\sin(3x + \pi)$ for $0 \leq x \leq 2\pi$

NESA 2020 Sample HSC Examination

Question 26 (3 marks)

By drawing graphs on the number plane, determine how many solutions there are to the equation $\sin x = \left|\frac{x}{5}\right|$ in the domain $(-\infty, \infty)$. **3**



Question 27 (2 marks)

The function $f(x) = \cos x$ is transformed to $g(x) = 3\cos 2x$. **2**

Describe in words how both the amplitude and period change in this transformation.

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....

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How to solve trigonometric equations

Question 1:

Solve the equation $\sin\theta = \frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$

S	A
T	C

Question 2:

Solve the equation $\sin\theta = \frac{1}{2}$ for $0^\circ \le \theta \le 720^\circ$

S	A
T	C

Question 3:

Solve the equation $\sin\theta = \frac{1}{2}$ for $-180^\circ \le \theta \le 180^\circ$

S	A
T	C

Question 4:

Solve the equation $\sin\theta = -\frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$

S	A
T	C

Question 5:

Solve the equation $2\sin\theta = \frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$

S	A
T	C

Question 6:

Solve the equation $\sin 2\theta = \frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$

S	A
T	C

Question 7:

Solve the equation $\sin^2\theta = \frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$

S	A
T	C

Question 8:

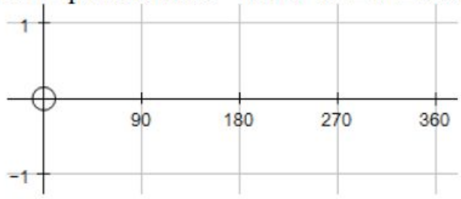
Solve the equation $2\sin 3\theta = 1$ for $0^\circ \le \theta \le 360^\circ$

S	A
T	C

Some special cases (in which it is best to use the graph)

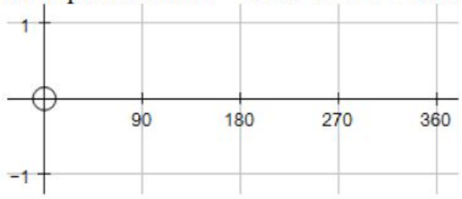
Question 9:

Solve the equation $\sin\theta = 0$ for $0^\circ \le \theta \le 360^\circ$



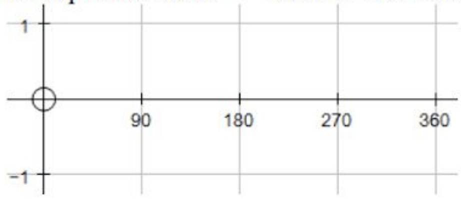
Question 10:

Solve the equation $\sin\theta = 1$ for $0^\circ \le \theta \le 360^\circ$



Question 11:

Solve the equation $\sin\theta = -1$ for $0^\circ \le \theta \le 360^\circ$

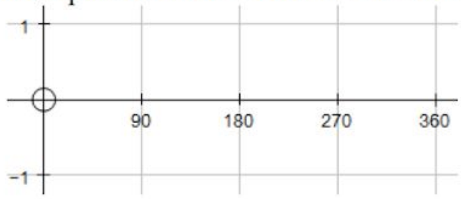


Question 12:

Solve the equation $\cos\theta = 0$ for $0^\circ \le \theta \le 360^\circ$

Question 13:

Solve the equation $\cos\theta = 1$ for $0^\circ \le \theta \le 360^\circ$

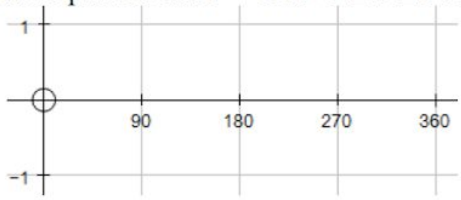


Question 14:

Solve the equation $\cos\theta = -1$ for $0^\circ \le \theta \le 360^\circ$

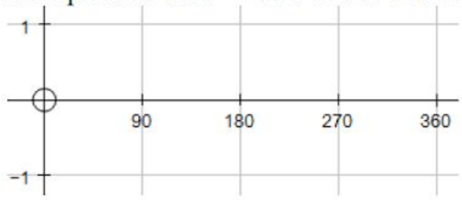
Question 15:

Solve the equation $\tan\theta = 0$ for $0^\circ \le \theta \le 360^\circ$



Question 16:

Solve the equation $\tan\theta = 1$ for $0^\circ \le \theta \le 360^\circ$



2009 HSC Question 7b

Between 5 am and 5 pm on 3 March 2009, the height, h, of the tide in a harbour was given by

$$h = 1 + 0.7\sin\frac{\pi}{6}t \quad \text{for} \quad 0 \le t \le 12,$$

where h is in metres and t is in hours, with $t = 0$ at 5 am.

(i) What is the period of the function h? **1**

(ii) What was the value of h at low tide, and at what time did low tide occur? **2**

(iii) A ship is able to enter the harbour only if the height of the tide is at least 1.35 m. **3**

Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour.

2018 HSC Question 15a

The length of daylight, $L(t)$, is defined as the number of hours from sunrise to sunset, and can be modelled by the equation

$$L(t) = 12 + 2\cos\left(\frac{2\pi t}{366}\right),$$

where t is the number of days after 21 December 2015, for $0 \le t \le 366$.

(i) Find the length of daylight on 21 December 2015. **1**

(ii) What is the shortest length of daylight? **1**

(iii) What are the two values of t for which the length of daylight is 11? **2**