# One small step IS one giant leap

Students will explore how the application of a logarithmic scale can reduce the challenges of comparing extremely large and small values.

Students will need at least one digital device per pair to interact with Desmos or Polypad during this lesson.

## Visible learning

This learning episode incorporates Path content and assumes students are confident with related Core content.

### Learning intention

* To understand and use logarithmic scales.

### Success criteria

* I can create a logarithmic scale with a base of 10.
* I can create a logarithmic scale with a base other than 10.
* I can convert a single logarithm into 2 logarithms.
* I can simplify 2 logarithms with the same base and create a single logarithm.
* I can solve problems on a log scale using the laws of logarithms.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* establishes and applies the laws of logarithms to solve problems **MA5-LOG-P-01**

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## Activity structure

Please use the associated PowerPoint *One small step IS one giant leap* to display images in this lesson.

### Launch

1. Show students the video ‘Solar System Size and Distance’ (1:20) ([bit.ly/3RSmTH3](https://bit.ly/3RSmTH3)).
2. After the video has finished, show students the image of the planets in the solar system on slide 2 of the associated PowerPoint *One small step IS one giant leap.*
3. Place students into visibly, random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and have them work at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).
4. Ask them to undertake the following:

After hearing the information in the video, in your groups list down the positive and negative aspects of this image of the solar system.

Students may recognise that the planets are too large in comparison to one another and that the distance between each of the planets is shown as being equal.

Students may recognise that although the scale is inaccurate, this image provides a clear idea of what the planets look like and that this would not be possible if they were scaled down accurately to fit on the screen.

1. Inform students that the difficulties presented in the video and the problems with the depiction of the solar system shown on the image result from trying to scale down exceedingly large values using a linear model but there is, however, a better way to approach these types of problems.

### Explore

1. In their groups from earlier, ask students to answer the following questions, providing their reasoning for their responses:

Would it feel more significant to increase from $10 to $100, or from $1010 to $1100?

What if it was an increase from $1000 to $10 000 instead?

1. Draw students back to the centre of the room and conduct a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) of the responses provided, highlighting interesting decisions and reasonings that emphasise although getting an extra $90 is always significant, it is more significant when the value increases by a magnitude of 10, rather than a difference of 90.
2. Have students draw a number line (or print out the blank number line from Appendix A ‘Linear scale’) on A3 paper and place this into appropriately sized plastic sleeves hung around the room for each group.
3. Students are to mark down the places of $1 and $10 and then each of the next nine $10 increments up to $100.
4. On the same number line, starting again by marking down the places of $1 and $10, have students plot the point that is ten times greater ($100), and then continue to multiply their subsequent answers by 10, recording each of the next 4 multiplicative increments ($1000, $10 000, $100 000 and $1 000 000).

Students will need to have the flexibility of non-permanent surfaces as they will likely need to erase their work, re-think their scales and re-design their work during this activity.

1. Have students utilise a notice and wonder strategy ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) to record their thoughts once this activity has been finalised.

**Students may notice:**

* the number line works well for linear increases/differences
* the number line was restrictive when attempting to plot multiplicative differences between values
* the number line was good at being able to show how significant the differences were when there was a multiplicative increase/difference
* because the physical length of the number line was fixed, the first set of data was difficult to differentiate on the scale once larger values from the second set of data were required to be included.

Students may wonder:

* How quickly will this become ineffective at showing the relative position of the differing values?
* Is there a better way to compare significantly different values, without losing meaning or accuracy?

1. Using a method such as the Pose-Pause-Pounce-Bounce question strategy (PDF 200KB) ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) ask students the following in a whole class discussion:

What is a common property between these 2 scales?

1. Students should be guided towards identifying that each of these scales begins with $1 and $10, and both scales increase by an operation involving the value ‘10’ as a significant component, whether it be a difference or a scale factor.
2. Have students pause their exploration of the number line and scales and provide the following list of operations in a centrally visible area. Encourage students to consider applying these to the values they are working with to determine which one, if any, is helpful in adjusting the scales:
3. addition of 10 to each value
4. subtraction of 10 to each value
5. multiplication of 10 to each value
6. division of 10 to each value
7. exponentials/indices with a base of 10 (examples include and so on)
8. logarithms with a base of 10 (examples include and so on).
9. Once students have explored how exponentials and indices may be applied, if they do not progress to considering logarithms with a base of 10 then this concept should be introduced to the whole class.
10. Having now established this connection, have students attempt to redesign their scales by manipulating the values using ‘10’ in some way, maintaining consideration of the beginning values of $1 and $10.

Allow time for students to discuss how they might be able to adjust the scales, encouraging groups to look around the room and ‘steal’ good ideas that may differ from their own.

Students may naturally think to divide by 10, but will soon find that this does not improve the scales to any great degree.

1. After allowing time for students to attempt to plot the values on a single number line, applying the relationships established by using logarithms with a base of 10, have students engage in a gallery walk to view and explore each other’s work.
2. Use the *One small step IS one giant leap* PowerPoint, slides 3–4, to provide a brief overview of the process and concept of a logarithmic scale for students to consider.

The unequal increments, when written in exponential form, can be difficult to process and often the use of scales (such as decibels for example) are used to provide a linear progression looking only at the exponent value, so the scale appears to have linear increments again, but instead represent the logarithmic differences.

### Summarise

1. Explain to students that this helps us to understand the importance and significance of logarithmic scales, as they compare values based on changes in magnitude rather than linear differences.

Logarithmic relationships are a proportionate way to compare values, as opposed to an additive approach when utilising linear scales.

1. Use the *One small step IS one giant leap* PowerPoint, slides 5–12, for explicit teaching of the concepts related to the laws and properties of logarithmic scales.

The explicit teaching technique used in the PowerPoint is ‘Your turn’. The first slide is a worked example which should be displayed for the students before using the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually explain to themselves what is happening in each step.
4. Students hold a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs, students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. By using these slides students should gain a better understanding of the following law/property of logarithms:

Students could be challenged to explore the same concepts using logarithms with a base other than 10, including the natural log, .

1. Have students create notes to their future forgetful self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) encouraging them to use algebraic terms to generalise the rule.
2. Students should be directed to ‘Logarithms 1-B’ (<https://www.openmiddle.com/logarithms-1-b/>) and attempt to find as many solutions as possible.

For students requiring more challenging applications, they can explore logarithms with no limit applied to the digits in the bases as indicated in the problem provided.

### Apply

1. Show the following video ‘To Scale: THE SOLAR SYSTEM’ (7:07) ([bit.ly/3v0aUOy](https://bit.ly/3v0aUOy)) to the students.
2. Once completed, in a class discussion randomly select students to answer the question:

Did the filmmakers re-create the solar system based on a logarithmic or linear scale? Make sure you can provide justification for your response.

1. Once students have had a chance to briefly discuss this concept, ensure that there is a consensus that the creators of the film used a linear model to scale down the solar system.
2. In randomly assigned groups of 3, have students determine the relative size of each of the planetary bodies and the sun that would be needed to be used if the oval (or a similarly large space, such as a netball or basketball court) was to be used to recreate the distances of each planet from the sun.

Google maps and Google Earth can be used to determine the measurements of external fields, ovals or courts, or the external dimensions of buildings that may house courts in your school.

Alternatively, school plans with measurement scales can be accessed and downloaded via the ‘AMS on the Web’ essential found in NSW Department of Education employee’s Profile centre when first accessing the Staff Portal.

1. Students will need to research the size of the planetary bodies and the distances between each of them with the sun which can be found using data collated by NASA ([jpl.nasa.gov/edu/pdfs/scaless\_reference.pdf](https://www.jpl.nasa.gov/edu/pdfs/scaless_reference.pdf) (45.2 KB)).
2. Students should find that the size of the planetary bodies, and even the sun itself, become unreasonably small and would be essentially impossible to use with the resources available.
3. Once this has been realised, have students consider the same task but using a logarithmic scale instead.
4. Students will likely need to consider a different logarithmic scale for the comparison of the size of the sun and planets (comparing diameters), and a different logarithmic scale for the relative distances between the sun and the planets.

Referenced in the measurements is the astronomical unit (AU), one of which is equivalent to approximately 150 million kilometres (based off the average distance of the Earth to the sun).

1. Students will need to plan out what this logarithmically scaled model would look like, what resources would be needed to physically build the model and provide reasoning and justification for the base utilised for their logarithmic scale(s).

The astronomical unit could be a suitable base for the logarithmic scale used for the respective distances between the planetary bodies and the sun, if students are finding difficulty determining one on their own.

1. If practical and plausible, recreate a physical model in the space identified on school grounds with your students.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Summarise**

* Students could be challenged to explore the same concepts using logarithms with a base other than 10, including the natural log .
* Students requiring additional support can continue working with logarithms that begin with a base of 10, rather than exploring logarithms with varied bases.

### Suggested opportunities for assessment

**Explore**

* Recording notes on observations of students requiring prompts to explore the concept of exponential and logarithmic relationships and those that do not.

**Summarise**

* Formative assessment records of student responses to the open-middle problem could be collated and notes taken of students still exploring problems resulting only in integer values through to students exploring problems involving non-integer and irrational values.

**Apply**

* Records of the justification and reasoning behind the choices made with regard to the base chosen for the logarithmic scale could be maintained. Whether students chose only one scale for both the distances and relative size of the planets or not, and why they made this choice, would serve as great opportunities to gauge how much students have engaged with connecting concepts and applying these in non-routine situations, helping to provide insight into their level of working mathematically.

## Appendix A

### Linear scale



## References

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