# Logging those sounds

Students will build an understanding of how to use a logarithmic scale to better understand and apply logarithmic laws and properties.

Students will need at least one digital device per pair to interact with Desmos or Polypad during this lesson.

## Visible learning

This learning episode incorporates Path content and assumes students are confident with related Core content.

### Learning intention

* To understand and apply logarithmic laws and properties.

### Success criteria

* I can use the addition law of logarithms.
* I can explain why the addition law works.
* I can use the subtraction law of logarithms.
* I can explain why the subtraction law works.
* I can use the power law of logarithms.
* I can explain why the power law works.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* establishes and applies the laws of logarithms to solve problems **MA5-LOG-P-01**

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## Activity structure

### Launch

1. Ask students the following questions:

Consider the following and make predictions about whether you would be able to hear:

* 1 centi-cube dropping onto a hard surface?
* The difference between 1 and 10 centi-cubes dropping onto a hard surface?
* The difference between 20 and 25 centi-cubes?
* The difference between 100 and 110 centi-cubes?

1. Place students into visibly random groups ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and provide them with access to enough centi-cubes (or similar) to conduct a practical experiment and test if their predictions were correct or not.

Students can test at what point the differences are perceptible by having one of the group members drop the centi-cubes onto the ground while the remaining group members have their eyes closed and attempt to listen to the changes between sounds.

1. In their randomly chosen groups, and using vertical, non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) (or an application such as [mentimeter.com](https://www.mentimeter.com/)), have students share their answers to the following question:

Thinking about how quiet we just found a single centi-cube is when it falls and hits a hard surface, what might be the smallest change in sound that we can hear?

1. Randomly select students to share their group’s responses with the class, looking to see if a collective response can be determined for both the quietest and loudest sounds.

### Explore

Students will need at least one digital device per pair to interact with Desmos or Polypad during this component of the lesson.

An alternative is provided in Appendices A and B for students without access to technology.

1. Provide students with access to the following Desmos activity ([bit.ly/hearlogarithms](https://bit.ly/hearlogarithms)), using pacing to limit their progress to screens 1 and 2.

If access to technology or Desmos is unavailable, provide students with Appendix A ‘Sounds’ which outlines a list of sounds in no particular order, and Appendix B ‘Blank number line’. Working in visibly random groups at vertical, non-permanent surfaces, students are to place each of the sounds on the number line, relative to the other sounds, in order from quietest to loudest.

1. In the Desmos activity, students will be presented with several labelled, moveable points with various sources of sound.
2. Students need to drag the points, putting the sounds in order from quietest to loudest, relative to the other sounds.
3. Display the collective student results for all to see using the embedded feature within the Desmos activity.

The following site, ‘Snapshots: Select and Sequence Student Work’ ([bit.ly/3sbxWRf](https://bit.ly/3sbxWRf)) provides a brief guide on how to use this feature in Desmos’ classroom function.

Making students anonymous, using another feature of Desmos, is a great way to avoid highlighting which students provided what results and can increase engagement levels.

1. Provide an opportunity for a whole class discussion of the results and randomly select students to speak about the results displayed on the screen compared to their own choices. Ensure students provide justification and reasoning for their responses when sharing their ideas and thoughts.

As part of this discussion, students should be encouraged to provide their reasons for where the sounds have been placed on the number line, including thoughts on how the sounds compare with one another and whether the graph represents a continuous scale beyond what is displayed, or if the scale is finite.

Students should also be encouraged to define where they have placed zero as a value, and whether or not there should be a place for negative numbers on this scale. This would encourage students to consider whether sounds could have a negative value (which is important when looking at ways to apply the decibel scale used later in this lesson).

1. Allow students time to work through screen 3 of the Desmos activity or Appendix C ‘Can you hear that?’ and Appendix D ‘Number line with pascal pressures values’.

The pascal (Pa) is the unit of measurement used to quantify, among other things, air pressure and sound.

1 pascal is equivalent to a force of 1 newton per square metre (and 1 newton is the amount of force necessary to provide a mass of one kilogram with an acceleration of one metre per second).

For a more comprehensive definition please refer to ‘What is Sound Pressure Level (SPL) and how is it measured?’ ([bit.ly/3sfnwzH](https://bit.ly/3sfnwzH)).

1. Students will now need to progress to screen 4, where they will see their original estimates and their updated responses, and will be required to undertake a notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) activity comparing these 2 representations.

**Students may notice:**

* The large range between the smallest and largest values.
* Each sound is related to one another by a relationship involving some multiple of 10.
* Difficulty distinguishing the differences between the values on the lower end of the scale.
* Even though the sounds towards the lower end can be distinguished just by listening to them, the scale doesn’t show a clear distinction between these values.

**Students may wonder:**

* What do the numbers actually measure?
* How do we compare the values in a meaningful way?
* Are any of these sounds dangerous or fatal, or is it just the source that is dangerous or fatal?
* Is the fact that these sounds are related by multiples of 10 of any significance?
* Can we do something to the values that has a relationship with 10?
* Is there an easier way to compare the results?
* Is it possible to compare the results on a number line without losing the impact of their differences?

1. Display the responses from students for the whole class to see, and randomly select students to share their thoughts on the results, with justification and reasoning, in a whole class discussion.

On a linear scale, the spectrum is far too broad to be able to list and compare sounds, so we need to look at comparing their relative magnitude instead – which is very similar to the way we process and hear sounds and volume with our ears naturally.

Students should be considering the following ideas during this discussion:

* What do these values add to our understanding, if anything?
* What is the benefit of comparing the sounds in this way?
* Does this scale help to provide a clearer picture of the comparative volume of these sounds? Why or why not?
* What would be a better way of comparing the relationship between these sounds?

1. Inform students that up until this point the sounds have been compared using a linear scale, which is ineffective at clearly showing the differences between values within a large range of values.
2. Continue by saying to the students that we will use a logarithmic scale instead, to compare the difference in magnitude of the values using the following formula:

Where indicates the pascals of pressure produced by the source of the sound

The decibel scale is derived from the baseline pascal value of a silent room, to which everything is being compared. This is important as practical applications of logarithms can only be used on dimensionless values, as logarithms are not a unit of measurement but can be used to view the change in magnitude of values that do have measurable units. Sounds with a greater pascal value will result in increasingly larger decibel values when entered into the provided formula.

1. Using this formula, have students progress to screen 5 in the Desmos activity (or if access to technology is limited provide Appendix E ‘The sound of silence’ for students to complete in their randomly selected groups from earlier) to determine the decibel value for each of the sources of sound.

As students are working through their solutions, they should be encouraged to undertake checks for reasonableness for their calculated responses, which may include the following:

* They should only have values greater than or equal to zero when calculating the logarithm with this formula, as none of the pascal measurements provided are less than the reference value in the denominator of the fraction.
* Using their knowledge that the formula utilises logarithms with a base of 10, once the fraction has been simplified, they can compare this with varying magnitudes of 10 to check their answers (for example, if the fraction simplifies to a value between 100 and 1000, their answer when the logarithm is calculated must fall between 2 and 3).

1. In a whole class discussion, randomly select students to share their results and provide reasons for the solutions at which they have arrived.
2. Pace students to screen 6 in the Desmos, following the instructions.

If Desmos is not available, provide students with a copy of Appendix F ‘Sounds on a log’ and have students place the sounds on this scale instead).

1. Pace students to screen 7 in the associated Desmos, following the instructions.

If access to technology is limited, students should undertake a notice and wonder activity comparing their work on Appendix B and their responses on Appendix F.

**Students may notice:**

* Their original estimation and the accurate representation are similar.
* The gaps between the sounds are easier to see and compare than the previous display.
* Sounds are mostly grouped together around the numbers on the scale.

**Students may wonder:**

* Why do we measure sound in 2 different ways?
* Why does the decibel scale better reflect our instincts when it comes to sound levels?
* What do decibels actually measure?

1. Explain to students that the decibel is a relative unit of measurement that expresses the ratio of 2 values on a logarithmic scale.

The decibel scale is used to measure a sound’s pressure or forcefulness, which is directly related to the amplitude of the sound, and we often refer to this as the volume.

1. The question was asked earlier ‘What would be the smallest change in sound that can be heard by the human ear?’. Inform students that in ideal situations this change is considered to be 1 decibel.
2. Provide students with a copy of Appendix G ‘I’m sure I heard something…’ and have them answer the questions provided:

1 decibel is considered the smallest change heard by the human ear.

Determine what this sound pressure level change (in Pa) would be for each of the 6 sources explored throughout this lesson.

1. When measuring sounds, the decibels recorded can be influenced by a range of factors. Ask students to list what these might include.

Factors impacting the decibels that students may come up with, but should be shown if not, include:

* the distance from the source
* the original pressure of the atmosphere in which the measurement is taken
* ambient sounds when measuring
* the quality and accuracy of the measuring device.

### Summarise

1. Use slides 2–21 from the *Logging those sounds* PowerPoint to provide an overview of the difference between linear and logarithmic scales.

These slides are intended to be moved through relatively quickly and are intended as an introduction to the visual representation of a logarithmic scale.

1. Use slides 22–33 from the *Logging those sounds* PowerPoint for explicit teaching of the law and properties of addition (or multiplication) and raising to a power of logarithms.

The explicit teaching technique used in the PowerPoint is ‘Your turn’. The first slide is a worked example which should be displayed for the students before using the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually explain to themselves what is happening in each step.
4. Students hold a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs, students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. Students should be provided with examples of these 2 logarithmic laws and properties using the *Multiplication law of logarithms* and the *Power law of logarithms* PowerPoints located at Logarithms (8 exercises!) ([bit.ly/3GJBYnS](https://bit.ly/3GJBYnS)).
9. Use slides 34–41 from the *Logging those sounds* PowerPoint for explicit teaching of the law and properties of subtraction (or division) of logarithms.
10. Students should be provided with examples of this logarithmic law and its properties utilising the *Division law of logarithms* PowerPoint located at Logarithms (8 exercises!) ([bit.ly/3GJBYnS](https://bit.ly/3GJBYnS)).
11. Students to make notes to their future forgetful selves for reference ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)).

### Apply

1. Provide students with a copy of Appendix H ‘Highway to the danger zone’.
2. Pose the following to the students:

Knowing that the decibel scale is a logarithmic scale of ratio, how could a negative decibel value be possible?

Students should be helped to arrive at the solution that if the baseline value was not considered to be the sound pressure level in pascals for a silent room, it would be possible to have a negative decibel reading and still hear sound.

The reason for this is to do with the optimal level at which the power is input into the amplifier and out to the speakers before there are risks of distortions to the sound and reduction in its quality.

More information around this concept can be found at ‘Guide to Negative Decibels | 4Cabling’ ([bit.ly/3KFvjgD](https://bit.ly/3KFvjgD)).

1. Have students create new baseline measurements based on a sound other than a ‘silent room’ and use the values in pascals for common sounds explored throughout this learning sequence to create questions for their classmates to explore and determine through the use of a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)).

## Assessment and differentiation

### Suggested opportunities for differentiation

**Apply**

* Students requiring additional challenges could be prompted to provide multiple different approaches, as outlined in the solutions provided, to solving the problems presented in Appendix H.

### Suggested opportunities for assessment

**Apply**

* Student responses to Appendix H and their accompanying justification could be used to help determine the depth of their Working Mathematically capability in this space.

## Appendix A

### Sounds

|  |
| --- |
| Source of sound |
| Hair dryer |
| Breathing |
| Total silence |
| Normal conversation |
| Rock concert |
| Jet plane taking off |

## Appendix B

### Blank number line



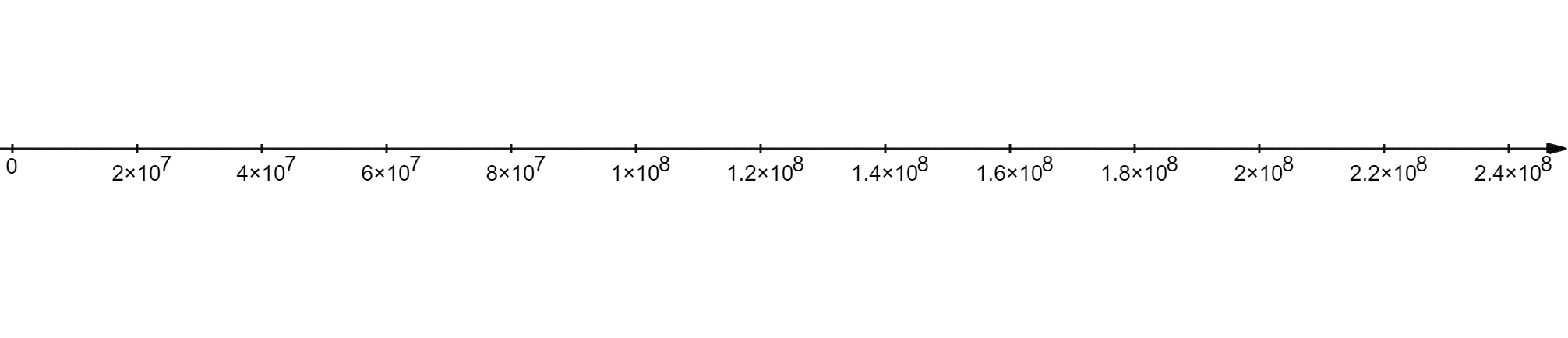
## Appendix C

### Can you hear that?

|  |  |
| --- | --- |
| Source of sound | Value in pascals |
| Total silence | 0.00002 Pa |
| Breathing | 0.0002 Pa |
| Normal conversation | 2 Pa |
| Hair dryer | 200 Pa |
| Jet plane taking off | 20 000 000 Pa |
| Rock concert | 200 000 000 Pa |

## Appendix D

### Number line with pascal pressure values



## Appendix E

### The sound of silence

|  |  |  |
| --- | --- | --- |
| Source of sound | Value in pascals | Value in decibels |
| Total silence | 0.00002 Pa |  |
| Breathing | 0.0002 Pa |  |
| Normal conversation | 2 Pa |  |
| Hair dryer | 200 Pa |  |
| Jet plane taking off | 20 000 000 Pa |  |
| Rock concert | 200 000 000 Pa |  |

## Appendix F

### Sounds on a log

A number line displaying evenly spaced increments of decibels, from zero through to two hundred. Each increment represents a difference of twenty decibels and is labelled accordingly.


## Appendix G

### I’m sure I heard something…

|  |  |  |  |
| --- | --- | --- | --- |
| Source of sound | Value in pascals | Value in decibels | Value in pascals for one decibel louder |
| Total silence | 0.00002 Pa |  |  |
| Breathing | 0.0002 Pa |  |  |
| Normal conversation | 2 Pa |  |  |
| Hair dryer | 200 Pa |  |  |
| Jet plane taking off | 20 000 000 Pa |  |  |
| Rock concert | 200 000 000 Pa |  |  |

1 decibel is considered the smallest change heard by the human ear.

In the table above, determine what this sound pressure level change (in Pa) would be for each of the 6 sources explored throughout this lesson.

## Appendix H

### Highway to the danger zone

Kenny was putting together a concert but was having some difficulties with the sound equipment when setting up. He knows solving the problem has something to do with logarithms, but he has always struggled when trying to calculate any **log in** his experience.

Using your knowledge of logarithmic laws and the decibel scale, help Kenny find the answers and provide justification for the following questions:

1. If pascals measure the ‘pressure’ of a sound, when a sound doubles in pressure how many decibels would this be (to the nearest decibel)?

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
|  |

1. For most people, sound is considered to double in loudness every 8 to 10 decibels. What would the range of pascals be for a sound that was twice the loudness of a normal conversation?

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
|  |

1. Using the formula for calculating the decibel, what would be the overall decibel value (to the nearest decibel) of adding together 5 sources of sound that produce a reading of 100 decibels independently (HINT: the answer is not 1000 decibels).

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
|  |

## Sample solutions

### Appendix E – the sound of silence

|  |  |  |
| --- | --- | --- |
| Source of sound | Value in pascals | Value in decibels |
| Total silence | 0.00002 Pa | **0** |
| Breathing | 0.0002 Pa | **10** |
| Normal conversation | 2 Pa | **50** |
| Hair dryer | 200 Pa | **70** |
| Jet plane taking off | 20 000 000 Pa | **120** |
| Rock concert | 200 000 000 Pa | **130** |

### Appendix G – I’m sure I heard something…

|  |  |  |  |
| --- | --- | --- | --- |
| Source of sound | Value in pascals | Value in decibels | Value in pascals for one decibel louder |
| Total silence | 0.00002 Pa | **0** | **0.000022 (6 d.p.)**  **Students may also respond that there is no difference at this level of sound, which would be acceptable with justification provided.** |
| Breathing | 0.0002 Pa | **10** | **0.00025 (5 d.p.)** |
| Normal conversation | 2 Pa | **50** | **2.51785 (5 d.p.)** |
| Hair dryer | 200 Pa | **70** | **251.78508 (5 d.p.)** |
| Jet plane taking off | 20 000 000 Pa | **120** | **25 178 508.24**  **(2 d.p.)** |
| Rock concert | 200 000 000 Pa | **130** | **251 785 082.4**  **(1 d.p.)** |

### Appendix H – highway to the danger zone

Kenny was putting together a concert but was having some difficulties with the sound equipment when setting up. He knows solving the problem has something to do with logarithms, but he has always struggled when trying to calculate any **log in** his experience.

Using your knowledge of logarithmic laws and the decibel scale, help Kenny find the answers and provide justification for the following questions:

1. If pascals measure the ‘pressure’ of a sound, when a sound doubles in pressure how many decibels would this be (to the nearest decibel)?

**Students may choose to use any of the sounds listed throughout this learning episode as an example, however the following solution provides the answer algebraically.**

**Let the sound pressure level being produced be equal to pascals and the equivalent decibel level be equal to .**

**When the sound pressure level is doubled, this would be equivalent to a sound producing pascals of pressure.**

The additional line of working shown above is not necessary to solve the problem but has been included to reinforce that the argument of a logarithm should always be a ratio and dimensionless.

**Therefore, when the sound pressure level being produced is doubled, this is a 3 decibel increase (to the nearest decibel).**

1. For most people, sound is considered to double in loudness every 8 to 10 decibels. What would the range of pascals be for a sound that was twice the loudness of a normal conversation?

**A normal conversation has been listed throughout this learning episode as .**

**Therefore, the range of decibels to be determined will be between and .**

* + 1. **At , let the sound pressure level being produced.**
    2. **At 60, let y the sound pressure level being produced.**

**Therefore, the range of pascals for the sound pressure level produced for something double the loudness of a normal conversation would be between and .**

1. Using the formula for calculating the decibel, what would be the overall decibel value (to the nearest decibel) of adding together 5 sources of sound that produce a reading of 100 decibels independently (HINT: the answer is not 1000 decibels).

**First, let’s determine the Pa of the sound pressure level for a sound of 100dB.**

**Students may approach this in several different ways, presented below are 2 possibilities.**

* + 2. **From the sources of sound explored in this learning episode it was seen that a hairdryer was producing a sound that registered and .  
        is greater than this source of sound.  
       Looking at the decibel scale, every is a difference of magnitude 10 for the pascals of pressure produced.  
       Therefore, if then .**

**Adding together the 5 sounds would result in a sound pressure level of .**

**Therefore, the overall decibels would be:**

## References

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