# Breeding like rabbits

Students will use online graphing software to explore the concept of exponential growth and decay.

Students will need at least one digital device per pair to interact with Desmos or Polypad during this lesson.

## Visible learning

This learning episode incorporates Path content and assumes students are confident with related Core content.

### Learning intentions

* To gain a deeper knowledge and understanding of exponential graphs and their features.
* To know how to transform exponential graphs using graphing applications.

### Success criteria

* I can graph an exponential relationship using graphing applications.
* I can use exponentials to describe and predict growth and decay.
* I can find the asymptote for an exponential and determine its equation.
* I can translate graphs of exponential relationships.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* interprets and compares non-linear relationships and their transformations, both algebraically and graphically **MA5-LIN-P-01**

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## Activity structure

### Launch

1. Students will need one laptop (or similar device) per pair that is able to access the internet.
2. Direct students to view and read through the webcomic ‘St Matthew Island reindeer comic about overpopulation: by Stuart McMillen’ ([bit.ly/stmatthewisland](https://bit.ly/stmatthewisland)).

This webcomic outlines a series of events, tracking the population of reindeer growing significantly quickly and then decreasing.

1. Students are to use the notice and wonder strategy ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) to reflect on the contents of the webcomic.

Students might notice:

* The reindeer population increased at different rates between the years that the island was visited.
* The drop-off in their population towards the end of their time on the island was significantly higher than their rate of population growth.

Students might wonder:

* Is it possible to calculate the growth rate of the population?
* About what aspects of the setting impacted on the initial growth rate and sudden decline in the population?

1. Initiate a class discussion and using a Pose-Pause-Pounce-Bounce question strategy (PDF 200 KB) ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) (or similar), have the students share their responses and thoughts regarding the webcomic, particularly related to the growth-rate of the reindeers.
2. As a prompting question leading into the Explore aspect of this lesson, ask students to consider if there are any patterns in the changes to the population over time.

### Explore

#### Creating a model

1. Students will need one laptop (or similar device) per pair that is able to access the internet.
2. Provide students with a copy of Table 1 from Appendix A ‘Reindeers on St Matthew Island’.

If students require additional support interpreting the data, provide them with a copy of Table 2 from Appendix A which places the information into a more defined table of values.

1. Students will need to use a graphing application (such as Desmos – [desmos.com/calculator](https://www.desmos.com/calculator)) to plot these data points.
2. Students will need to add an item and select the ‘table’ option for this component.

To gain a better understanding of plotting points using Desmos, please refer to the site ‘Graphing and Connecting Coordinate Points' <https://help.desmos.com/hc/en-us/articles/4405411436173-Graphing-and-Connecting-Coordinate-Points>.

1. Students are to answer the following prompts:
2. Using your knowledge of graphs and linear relationships, determine the gradient of each of the line segments made by joining the following points:
   * 1. and
     2. and
     3. and
     4. and .
3. Would you say that the relationship between the first 3 points could be described as increasing or decreasing compared to one another? Justify your response.
4. Would you say that the relationship between the final 3 points is increasing or decreasing compared to one another? Justify your response.

Students may require additional support to identify that the positive gradients of the lines created in part a) i. and a) ii. could help to justify that the relationship between the 3 points is increasing, just as the negative gradient of the lines created in part a) iii. and a) iv. could help justify that the relationship between the final 3 points is decreasing.

These points do not prove that the functions are increasing or decreasing, but are a good place to start building these concepts for the students leading into their future studies in Mathematics.

1. Initiate a whole class discussion and randomly select students to share their pair’s responses to parts a, b and c.
2. Once students have engaged in the whole class discussion, ask them the following:

Does a relationship exist between these data points? In your pairs, justify your response.

1. Using a graphing application, ask students to graph the following functions on the same set of axes:
2. (using Desmos this would be y=x^2)
3. (using Desmos this would be y=x^(-2) )
4. (using Desmos this would be y=2^x)
5. (using Desmos this would be y=2^(-x) ).

Looking at these 4 functions, which one would best suit the relationship seen between the first 3 points from the reindeer data? Justify your response.

1. Initiate a whole class discussion for students to share their pair’s responses and justifications.

Ensure that students are guided to the conclusion that the function (the increasing exponential function) best models the relationship from the data plotted, even though it does not map the points exactly.

1. Inform students that the function that best suits the shape of the relationship is referred to as an ‘exponential function’.
2. Show students the video ‘Learn Desmos: Sliders’ (0:57) ([bit.ly/desmos\_sliders](https://bit.ly/desmos_sliders)).
3. Provide students with a copy of Appendix B ‘Manipulating functions’ and have them work through the steps outlined.
4. Having now had the opportunity to explore many ways to transform the function, have students answer the following:

Looking at the ways to transform the exponential function, which transformations or combination of transformations, creates a graph that best suits the relationship seen between the first 3 points of the reindeer data? Justify your response.

Students ready to be challenged with more complex concepts could explore how non-integer values impact the transformations.

Students will find it is not possible to match the path exactly for 2 main reasons:

* + The data does not truly reflect an exponential relationship but does resemble one.
  + To achieve the closest possible representation, students would need to use non-integer values
  + One of the closest exponential functions that would fit these data points would be
  + As a point of interest, if only the first 3 data points are included in the table, typing the function on a new line in Desmos will provide an approximation of the function that best fits the data.

1. Draw students’ attention to the final 3 data points and ask them the following:

Looking at the relationship between the last 3 points of data, determine which combination of function transformations would best suit the shape of the decreasing relationship seen here.

Students ready to be challenged with more complex concepts could again explore how non-integer values impact the transformations.

Students will find it is not possible to match the path exactly for several reasons:

* + The data does not truly reflect an exponential relationship but does resemble one.
  + To achieve the closest possible representation, students would need to use non-integer values.
  + Students have yet to explore the concept of translating exponential functions horizontally.

#### Developing a more sophisticated model

1. Use a whole class discussion to decide on the minimum number of reindeers that must survive if the population is to grow.

Students should determine that one male and one female would be needed, which is a minimum of 2 specimens.

1. Using a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to discuss how they could adjust the functions to show that the population should never be lower than 2.
2. Inform students that the minimum number of reindeers would need to be an asymptote.

Provide students with the definition of an asymptote as:

A line (or a curve) whose distance from a curve approaches zero as or gets infinitely large or infinitely small. For example, the curve has an asymptote .

1. Ask students to engage in a Think-Pair-Share exercise to determine the following:

Why does have an asymptote of and have an asymptote of ? Provide reasoning and justification for your answer.

1. Have students share their responses with the class as part of a whole class discussion.
2. Students should edit the graphs they have been building in Desmos and adjust their functions to reflect an asymptote of 2.
3. Inform students that a function that could fit closely with the data provided would be and that this curve has an asymptote of .

### Summarise

1. To support student understandings of this concept they are to explore the ‘Marbleslides: Exponentials’ Desmos activity ([bit.ly/marbleslidesexponentials](https://bit.ly/marbleslidesexponentials)).

In this activity, students will be required to manipulate the exponential functions within the activity to change the direction of the falling marbles and collect all of the stars on each slide.

1. Students will need to create notes to their future forgetful selves ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)). Students should be encouraged to consider including:
2. The algebraic form for an exponential function.
3. Definitions for the specific terminology utilised in defining and describing exponential functions, including -intercept and asymptote.
4. Examples of the features of an exponential function, with a diagram to help support and show their understanding.
5. Have students access the following Desmos activity, ‘Two Truths and a Lie: Exponentials’ ([bit.ly/twotruthslie\_exponentials](https://bit.ly/twotruthslie_exponentials)).

In this activity, students will be required to create an exponential curve, write 2 true statements about it, add one false statement and have their peers determine which statements are true.

### Apply

1. Highlight to students that for each reindeer data set, there was no exact function that could be found to fit the data, but the use of exponential functions provided a good representation.
2. Have students undertake a Think-Pair-Share to answer the following question:

What would contribute to the population size of the reindeer not fitting exactly onto an exponential function?

1. Emphasise that mathematical modelling is a great way to predict the outcome of phenomena such as population growth or decay, but the external factors in real-world settings often cause unpredictability in the outcomes we are hoping to see or expect.
2. Have students access the PhET simulator for Natural selection – ([bit.ly/phetnaturalselection](https://bit.ly/phetnaturalselection)).
3. Students will need to select the **Lab** option and begin by adding a mate – this will start the simulator.
4. Students are required to select the **Data probe** from the table at the bottom left of the screen. This will create a slidable bar, that when dragged over the data line on the graph, will show the population of rabbits at that point.
5. Students will need to create a table of values to record the generation () and population size () for the rabbits.
6. From this table of values, students will determine if the growth rate represented is exponential and then calculate the exponential function that best fits the model.
7. Present students with the following statement:

To begin the population, there must be at least 2 rabbits. This means that the exponential graph that reflects the growth rate must have an asymptote of .

Do you agree or disagree with this statement? Justify your response.

As an argument in favour of this point, students may suggest that there will always have to be at least 2 rabbits for the population to continue, so this would create an asymptote of .

As an argument against this point, students may suggest that there is no asymptote at , but instead there is a intercept of , as it is entirely possible for the population to fall to , just as it did with the scenario about the reindeers.

These are only examples and students may determine additional responses and reasoning to these.

1. Once students have explored this initial model (which will eventually stop once the rabbits have taken over the world), have them restart the model and begin introducing other factors, such as limited food, wolves, and dominant and recessive genes (fur, ears and teeth).
2. Tracking and recording the data each time, students are to work towards introducing the correct combination of mitigating factors to cause the increasing exponential function to become a decreasing exponential function, if possible.

Inform students that modelling of this type is conducted to track and monitor actual population growth in species and is helpful in determining and establishing conservation programs to re-introduce species into the wild.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* If students require additional support interpreting the data, provide them with a copy of Table 2 from Appendix A which places the information into a more defined table of values.
* For students requiring additional support, they may need to be provided with the process for determining gradients from 2 given points as outlined subsequently:

* Students requiring additional support could instead plot each of the points and use the method to determine the gradients of each line segment created. This could be used to help justify their reasoning about the nature and behaviour of the changes in population.
* Students ready to be challenged with more complex concepts could explore how non-integer values impact the transformations.

### Suggested opportunities for assessment

**Explore**

* Throughout this component, students will be providing justification for their decisions and communicating their reasoning – these can be used as a formative assessment to help gauge the depth of student’s ability to work mathematically through these tasks.

**Summarise**

* Monitoring of the statements created, and their accuracy, as students engage with the ‘Two truths and a lie’ activity can be utilised as a formative assessment of student understanding.

## Appendix A

### Reindeers on St Matthew Island

Table 1 – population of reindeers on St Matthew Island

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| Calendar year observation was made | Size of reindeer population |
| 1944 (0 years) | 29 |
| 1957 (13 years from 1944) | 1 350 |
| 1963 (19 years from 1944) | 6 000 |
| 1966 (22 years from 1944) | 42 |
| 1980 (36 years from 1944) | 0 |

Table 2 – table of values representing population of reindeers on St Matthew Island

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## Appendix B

At this stage, if students have not determined as the function that best suits the information provided, ensure they are given that information before proceeding with this Appendix.

### Manipulating functions

Using the slider feature of Desmos, spend time adjusting the following aspects of the exponential function to see if you can create a better fit with the data points provided:

1. Adjust the value of the base to alternate, positive index values, recording what impact this has on the function (for example , , and so on).

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1. Introduce an integer constant to the function, recording what impact this has on the function (for example , , , and so on).

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1. Introduce an integer coefficient to the function, recording what impact this has on the function (for example , , and so on).

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1. Use a combination of these adjustments to attempt to create a function that best suits the original 3 data points that were plotted, recording what impact this has on the function (for example , and so on).

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## References

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