# Uncovering shape secrets

Students use the concept of perimeter to delve into solving equations involving 2 steps using bar models.

## Visible learning

### Learning intention

* To be able to solve equations.

### Success criteria

* I can explain the difference between an expression and an equation.
* I can solve equations involving 2 steps.
* I can represent the perimeter of a shape as an equation.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* applies knowledge of the perimeter of plane shapes and the circumference of circles to solve problems **MA4-LEN-C-01**
* solves linear equations of up to 2 steps and quadratic equations of the form
**MA4-EQU-C-01**

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## Activity structure

Please use the associated PowerPoint *Uncovering shape secrets*to display images in this lesson.

### Launch

1. Assign students into visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and have them move to a vertical non-permanent surface ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).
2. Inform students that we will be looking at a rectangle where the length is 3 centimetres longer than the width.
3. Display slide 2 from the PowerPoint *Uncovering shape secrets.* This shows the rectangle from Figure 1, where students are prompted to find the missing value , the width of the rectangle, if we know the perimeter is 66 units.

Figure 1 – a rectangle with a length that is 3 centimetres longer than its width



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The aim of this activity is not for students to find the correct answer but to explore strategies on how to approach it.

1. Ask students assessing and advancing questions to further student thinking.

Assessing questions draw out students’ thinking about a problem and what methods they have tried so far.

Advancing questions are intended to help move students’ thinking forward toward the lesson goals. We want to draw their attention to something they may not have noticed or considered yet.

Question suggestions are included below. Students can be prompted using these assessing and advancing questions to help guide their thinking.

Table 1 – assessing and advancing questions

|  |  |
| --- | --- |
| Assessing questions | Advancing questions |
| Can you explain the question to me? | Can you put some values in for w to see what the solution is?  |
| What have you tried? | How could you verify your solution? |
| What does represent? | How do you find the perimeter of a shape? |

1. In a class discussion, ask students what they found easy and difficult about this problem.
2. State to students that they were attempting to solve an equation. Explain the importance of being able to solve equations.

It is important to be able to solve equations as it is one of the most prevalent problem-solving skills we have and appears, not always obviously, in real life. It is important to be able to effectively communicate solution strategies in mathematics, and giving students a way to communicate mathematically allows students to reason and show understanding of problems.

### Explore

1. Display slide 3 from the PowerPoint *Uncovering shape secrets.* This slide shows Kyran’s approach to solving the problem using guess and refine.
2. Explain to students that Kyran was evaluating an expression to try and find his solution.
3. Ask students to continue his method to get closer to the solution.
4. Display slide 4 from the PowerPoint *Uncovering shape secrets*. This slide shows Alex’s approach to solving the problem using the simplified equation formed from the perimeter of the rectangle.
5. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), students should discuss the following:
* Where did Alex get the perimeter expression from?
* What would Alex’s next step be to find the answer?
* What are the advantages and disadvantages of Kyran and Alex’s approaches?
1. State to students that we will be exploring Alex’s approach, which was the simplified equation. Before we can work with this, we need to establish what an equation is.
2. Display slide 5 from the PowerPoint *Uncovering shape secrets.* This slide shows examples of expressions and equations side by side.
3. In a Think-Pair-Share, ask students what they think is the difference between an expression and an equation.

An expression is 2 or more numbers or variables connected by operations. Expressions do not include an equal sign.

An equation is a statement that includes the ‘=’ symbol. Equations are used to show the equality of 2 expressions.

1. Acknowledge to students that the situation in the Launch activity became an equation when we were told the total perimeter was 66. This gave us a need to include the ‘=’ symbol.
2. Display Figure 2, available on slide 6 of the PowerPoint *Uncovering shape secrets*, showing bars representing the lengths on each of the sides of the rectangle.

Figure 2 – rectangle with bars shown on edges



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Students were introduced to bar models to solve one-step equations in Lesson 3 – I see triangles of Unit 6 – triangles and quadrilaterals.

1. Display Figure 3, available on slide 7 of the PowerPoint *Uncovering shape secrets*, showing a bar model representation of the equation for the original rectangle.

Figure 3 – bar model representing an equation



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1. In a Think-Pair-Share, ask students the following questions:
* What are the similarities and differences between the 3 representations?
* What parts of each representation do you think are connected?
* What do the bars represent?
* Does the length of the unknown matter?
* What is used to represent the equal sign?
1. Display slide 8 of the PowerPoint *Uncovering shape secrets*, showing 3 different versions of the bar model representation of the equation for the original rectangle. Ask students to identify similarities and differences between the 3 models.
2. Distribute Appendix A ‘Match them up’ to each pair of students. Ask students to match the algebraic equations with their bar model representation.
3. In a class discussion, ask students to explain how the diagrams relate to the equations and if they would change them in any way.

To help with solving equations, students may suggest putting the bars labelled with x together, or students could suggest moving things around, so the pronumerals and numbers are together.

#### Bar models

This lesson can be modified to use Algebra tiles in place of bar models. An example of solving equations with Algebra tiles on Polypad (<https://mathigon.org/polypad>) can be seen in the video ‘Solving a 2-step equation with Polypad’ (<https://bit.ly/EquationPolypad>).

1. Using the bar models from slide 8 of the PowerPoint *Uncovering shape secrets,* students are to try to find the solution using the bar model.

For students who have already found the answer to the Launch problem, given them a similar equation to solve.

1. Show slide 9 from the PowerPoint *Uncovering shape secrets*. This slide shows students an equation being solved using bar models. Ask students what they notice/wonder.
2. Initiate a Pose-Pause-Pounce-Bounce question strategy (PDF 557KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)), to share student’s understanding of solving equations using bar models.
3. By working in different visibly random groups of 3 on vertical non-permanent surfaces, students are to return to Appendix A ‘Match it up’ and solve the equations using the bar models.

These solutions will be used again so ensure students have adequate space to solve all equations or document their solutions elsewhere before erasing.

1. Walk around the room and look at student work. Join groups together that have produced different solutions to the same problem and state ‘At least one of you is incorrect.’ Leave the groups to debate and justify the correct answer.
2. Ask students to discuss in their groups, what is common about the order of the steps students used to solve the bar models, and to think why they may have done that.

Students could make the connection that this is the reverse of the order of operations. Using prompting questions such as, ‘Is there another mathematical concept we have learnt where you must go in a specific order?’ could help students make the connection.

1. Conduct a class discussion to share thoughts on the order to solve equations and why.
2. Ask students how they could tell if they have the right solution.

The purpose of this question is for students to realise that by substituting their answer into the variables, the equation should still be true.

1. Students are to return to their boards and check their solutions.

### Summarise

1. Introduce the term inverse operation: The operation that reverses the effect of another operation. Use examples from their work to clarify its meaning.
2. Use slides 10–17 from the PowerPoint *Uncovering shape secrets* for the explicit teaching of solving equations with 2 steps.

The explicit teaching technique used in the PowerPoint is ‘Your turn’. The first slide is a worked example which should be displayed for the students before using the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually explain to themselves what is happening in each step.
4. Students hold a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs, students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. Have students return to their problems on their vertical non-permanent surfaces and see if they can change their bar models from Appendix A into algebraic representations to communicate their thinking.
9. As a class, ask groups to discuss how they connected to the algebraic representation. Challenge them to use the terminology inverse and variable when explaining their answer.
10. Individually, students are to attempt the Open Middle problem ‘Two-step equations 3’ (<https://www.openmiddle.com/two-step-equations-3/>) to find the largest and smallest possible solutions using the numbers 1 to 9 to fill the boxes.
11. By selecting random students, discuss the solution students found and the strategies they used to get to the solution they found.

### Apply

#### Puzzling perimeters

This activity uses the concepts of simplifying expressions and solving equations for the perimeter of varying shapes.

1. Issue each student a copy of Appendix B ‘Puzzling perimeters’.
2. By working in visibly random groups of 3 on vertical non-permanent surfaces, ask students to solve the problems in any order.
3. Invite a student from each group to share their solution to a problem, explaining their method and justifying their answer.

#### Worded problems

In this activity, students are to translate and solve worded problems involving the perimeter of quadrilaterals and triangles.

1. Issue each student a copy of Appendix C ‘Worded problems’.
2. By working in visibly random groups of 3 on vertical non-permanent surfaces, ask students to solve the problems in any order.
3. Invite a student from each group to share their solution to a problem, explaining their method and justifying their answer.

#### Perimeter challenge

Activity adapted from the NRICH activity ‘Perimeter challenge’ (<https://nrich.maths.org/11119>). Sample solutions to this problem can be found on the site.

1. In new visibly random groups of 3 at non-permanent surfaces Distribute Appendix D ‘Perimeter challenge’. The sheet displays the following diagram.



1. State that all the rectangles are identical and ask students to find the dimensions.
2. Walk around the room and circle student work that shows different approaches to solving the equations. Join groups together that have produced different solutions to the same problem and state ‘At least one of you is incorrect.’ Leave the students to debate which answer is correct and to justify their reasoning.
3. Initiate the sharing of solutions by highlighting at least one thing done well in the approach of each group in the class.

When sharing student solutions, it is best to start with the most basic solution and then increase in complexity. You do not need to highlight the entire solution, rather mathematical strategies that work towards a solution.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students may benefit from first simplifying algebraic expressions to check student understanding of like terms.

**Explore**

* If students find it difficult to add numbers mentally, they can be allowed to use a calculator.
* Visual representations, like bar models, are used to assist students with their understanding of solving equations.

**Summarise**

* Open middle problems enable students to use the guess and refine method to approach problems, that allows an access point for all students. For those still not confident, have them focus on one type of equation at a time.

**Apply**

* To enable students who are not confident in simplifying using like terms, give the equations to students to match to each shape in Appendix B.
* Ask students to draw diagrams for Appendix C and then compare as a whole class before solving the problems.
* Asks students to make their own worded problem in Appendix C.

### Suggested opportunities for assessment

**Explore**

* Check student understanding of substitution by monitoring responses from students to Kyran’s approach.
* Monitor responses in class discussions to check for student understanding of solving equations using bar models.

**Summarise**

* Monitor student responses in the ‘Your turn’ section to check for understanding.
* Students will demonstrate their working mathematically skills in discussions and justificationswhen completing the open middle problem.
* Create an exit ticket where students need to choose 2 equations they have solved from the open middle problem that best show their progress.

**Apply**

* Appendix B could be collected and used as summative assessment for this unit of learning.

## Appendix A

### Match them up

|  |  |
| --- | --- |
|  |  |
|  |  |
|  | An image of a bar model made in Desmos. The model is made up of 2 rectangles lined up end to end, opposing one other rectangle on the long side. The 2 rectangles in a line are blue and of the same length, labelled with an x. The single long rectangle that opposes these is red and is labelled with a 17. The single long rectangle is longer than the 2 rectangles in a line put together, and the difference between these lengths is marked with an arrow, labelled with the number 5.  |
| An image of a bar model made in Desmos. The model is made up of 3 rectangles lined up end to end, opposing one other rectangle on the long side. The three rectangles in a line include 2 blue ones of the same length, labelled with an x, and one orange one of a different length, labelled with a 5. The single long rectangle that opposes these is red and is labelled with a 57. The single long rectangle is the same length as the 3 rectangles in a line put together.  | An image of a bar model made in Desmos. The model is made up of 1 long orange rectangle labelled with a 17, and another shorter red rectangle labelled with a 5. The shorter red rectangle is lined up with the orange rectangle, and there are two lengths indicated by arrows and marked with an x that show the difference between the two rectangles.  |
| An image of a bar model made in Desmos. The model is made up of 3 rectangles lined up end to end, opposing one other rectangle on the long side. The three rectangles in a line include 2 blue ones of the same length, labelled with an x, and one orange one of a different length, labelled with a 5. The single long rectangle that opposes these is red and is labelled with a 17. The single long rectangle is the same length as the 3 rectangles in a line put together.  | An image of a bar model made in Desmos. The model is made up of 3 rectangles lined up end to end, opposing one other rectangle on the long side. The three rectangles in a line include 2 blue ones of the same length, labelled with an x, and one orange one of a different length, labelled with a 5. The single long rectangle that opposes these is red and is labelled with a 17. The single long rectangle is the same length as the 3 rectangles in a line put together.  |

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## Appendix B

### Puzzling perimeters

1. Find the value of if the perimeter of the rectangle is equal to 55 units.



1. Find the value of if the perimeter of the rectangle is equal to 64 units.



1. Find the value of if the perimeter of the triangle is equal to 22 units.



1. Find the value of if the perimeter of the pentagon is equal to 37 units.



1. Find the value of if the perimeter of the pentagon is equal to 471 units.



1. Find the value of if the perimeter of the shape is equal to 288 units.



## Appendix C

### Worded problems

1. A square and equilateral triangle have the same perimeter. If the length of one side of the triangle is 4 units, find the side length of the square.
2. The length of a rectangle is 3 times as long as its width. Find the length and width of the rectangle if the perimeter is 56 units.
3. A kite has a perimeter of 2688mm. Its longer sides are 10% longer than the shorter sides. Find the length in millimetres of one of the shorter sides.
4. The lengths of adjacent sides of a parallelogram are consecutive numbers. If the perimeter is 42 units find the length of the longest side of the parallelogram. Hint: you may need to ask your teacher what the words adjacent and consecutive mean.
5. The length of a rectangle is 2 cm less than 7 times its width. The perimeter is 60 cm. What are the dimensions of the rectangle?
6. The first side of a triangle is 8 cm shorter than the second side. The third side is 4 times larger than the second side. The perimeter is 52 cm. What are the lengths of the triangle?

## Appendix D

### Perimeter challenge

Given all the rectangles are identical, find the dimensions of a rectangle.



## Sample solutions

### Appendix A – match them up

|  |  |  |
| --- | --- | --- |
| Equation | Bar model | Solution |
|  | An image of a bar model made in Desmos. The model is made up of 3 rectangles lined up end to end, opposing one other rectangle on the long side. The three rectangles in a line include 2 blue ones of the same length, labelled with an x, and one orange one of a different length, labelled with a 5. The single long rectangle that opposes these is red and is labelled with a 17. The single long rectangle is the same length as the 3 rectangles in a line put together.  |  |
|  | An image of a bar model made in Desmos. The model is made up of 3 rectangles lined up end to end, opposing one other rectangle on the long side. The three rectangles in a line include 2 blue ones of the same length, labelled with an x, and one orange one of a different length, labelled with a 5. The single long rectangle that opposes these is red and is labelled with a 17. The single long rectangle is the same length as the 3 rectangles in a line put together.  |  |
|  | An image of a bar model made in Desmos. The model is made up of 2 rectangles lined up end to end, opposing one other rectangle on the long side. The 2 rectangles in a line are blue and of the same length, labelled with an x. The single long rectangle that opposes these is red and is labelled with a 17. The single long rectangle is longer than the 2 rectangles in a line put together, and the difference between these lengths is marked with an arrow, labelled with the number 5.  |  |
|  | An image of a bar model made in Desmos. The model is made up of 3 rectangles lined up end to end, opposing one other rectangle on the long side. The three rectangles in a line include 2 blue ones of the same length, labelled with an x, and one orange one of a different length, labelled with a 5. The single long rectangle that opposes these is red and is labelled with a 57. The single long rectangle is the same length as the 3 rectangles in a line put together.  |  |
|  | An image of a bar model made in Desmos. The model is made up of 1 long orange rectangle labelled with a 17, and another shorter red rectangle labelled with a 5. The shorter red rectangle is lined up with the orange rectangle, and there are two lengths indicated by arrows and marked with an x that show the difference between the two rectangles.  |  |

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### Appendix B – puzzling perimeters

1. units
2. units
3. units
4. units
5. units

### Appendix C – worded problems

1. Side lengths of the square are 3 units.
2. Width = 7 units
Length = 21 units
3. mm
4. Longer side is 11 units.
5. cm
6. cm

First side =

Second side =

Third side =

## References

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