# That’s ab-SURD

Students explore fractional powers and their relationship to surds.

## Visible learning

This lesson incorporates Path content. Most students should be able to engage with the Launch and Explore sections.

### Learning intention

* To understand that a number with a fractional index can be written as a surd.

### **Success criteria**

* I can use known index laws to explain the meaning of fractional indices.
* I can express a surd as a number with a fractional index.
* I can express a number with a fractional index as a surd value.
* I can solve problems that involve fractional indices.

### Syllabus outcomes

A student:

* describes and performs operations with surds and fractional indices **MA5-IND-P-02**
* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**

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## Activity structure

Students will begin by finding a gap in the use and application of the index laws, before exploring how to use fractional indices.

### Launch

1. Students will complete the table in Appendix A ‘Indices Revision Table’ by working in visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) on vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).

Except for the final row, students should have seen examples of all the concepts addressed. This activity could be used to support formative assessment.

1. Initiate a sharing of solutions and reasoning using the Pause-Pose-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)).

The final row of the table introduces the concept of a surd but provides no insight into how to represent this value in its exponent form. The rest of the lesson will explore this concept.

### Explore

1. Provide students with a copy of Appendix B ‘Table of values’ and have them complete the missing values where they can. Students should be working in visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and using a vertical, non-permanent surface ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) or mini-whiteboards ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)).

The empty space in the first row between and , is designed to make students predict what index can be used to represent this value.

1. Provide students with access to the Desmos graph *‘*Fraction Powers’ (<https://bit.ly/FractionPowers>) so they can check their answers and explore the other bases listed in Table 2. Students will need to use slider ‘**a**’ to adjust the base and they can then read off any interesting points on the graph, by adjusting slider ‘**p**’.

Where access to digital devices is limited or unavailable, the teacher could project the graphs onto a screen for students to view or teachers could print the required graphs for students to use.

1. Select students to share their answers for the missing cells in Tables 1 and 2. Ensure they provide their reasoning and justification for their solution.
2. Before confirming if they are correct, ask students to complete questions 5 to 8 from Appendix B.
3. Conduct a whole class discussion using the Pose-Pause-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) to guide students to the idea that a power of is equivalent to a square root, and a power of a is equivalent to a cube root and so on.

### Summarise

1. Use slides 2–9 from the *That’s ab-SURD!* PowerPoint for explicit teaching of the proof linking fractional indices to surds.

The explicit teaching technique used in the associated PowerPoint is ‘Your turn.’ The first slide is a worked example which should be displayed for the students and then use the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually think and explain to themselves what is happening in each step.
4. Students hold up a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs, students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. Ask students to write a rule for fractional index in their own words. Encourage students to then convert this to a rule using a variable (algebraic) base and a 1 in the numerator position.

Students’ rule should resemble the following:

1. Use the *That’s ab-SURD!* PowerPoint slides 10–13 for explicit teaching of how to express a number with a fractional index as a surd.
2. Students will complete ‘Fractional indices-unit fractions’ from Appendix C. They can either work through these questions individually or in their visibly random groups of 3 at vertical non-permanent surfaces.
3. For students who are ready to explore fractional indices further, direct them to Appendix B ‘Tables of values – challenge’. This activity allows students to explore indices with non-unit fractions as powers.
4. Use slides 14–17 from the *That’s ab-SURD!* PowerPoint for explicit teaching of how to express a number with a non-unit fractional index as a surd.
5. Ask students to write a rule for fractional indices involving non-unit fractions, in their own words. Encourage students to then convert this to a rule using a variable (algebraic) base.

Students’ rule should resemble the following:

1. Students could now complete ‘Fractional indices-challenge questions’ from Appendix C. Students can either work through these questions individually or in their visibly random groups of 3 at vertical non-permanent surfaces.

### Apply

1. Using their knowledge of fractional indices and the index laws, ask students (in their groups from earlier) to create solutions to the following open-middle problems:

* Fraction exponents (Fractional indices with numbers other than 1 in the numerator) –([bit.ly/OMFractionExponents](https://bit.ly/OMFractionExponents)).
* Rational exponents (Fractional indices with numbers other than 1 in the numerator) – (<bit.ly/OMRationalExponents)>.

There are many possible answers to these problems. Some possible solutions are shown on the website.

1. Display student solutions around the room and have students conduct a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)). Ask students to check one another’s solutions.

Students should be encouraged to use a strategy such as checking for reasonableness when undertaking this activity. For instance, for fractional powers, if the final answer is greater than the base, this is an unreasonable solution.

1. Ensure students are still in their visibly random groups from the Launch of this lesson and provide them with a copy of Appendix D ‘Problems for students’.
2. Students are to begin by only completing the second column ‘Solutions (part a)’. They need to find as many solutions as possible with unit fractions as the power.
3. Students will then use their integer answers from the second column and represent each of these in index form with non-unit fractions as the power.

For example, the answers in the first row were 49 and 7. We could represent these as  **or . There is no need to simplify the fractions.**

## Assessment and Differentiation

### Suggested opportunities for differentiation

**Launch**

* Teachers may choose to challenge students by providing more difficult questions.
* Working in visibly random groups of 3 will provide support to students who are still becoming familiar with the index laws.

**Explore**

* Students can use the Desmos graph to help them calculate values in the table.
* The activities in Appendix C are based on Variation theory and have been designed for students to notice the difference between one question and the next and to predict how this change will affect the answer. As a minimum, students should consider whether the answer will be higher or lower than their previous answer.
* After completing Appendix C, students could create their own questions which explore.

**Apply**

* The Open middle problems can be solved in multiple ways. Students should be encouraged to find as many solutions as possible.
* The problems from Appendix D could be given as a relay. Students would complete one problem before being given the next. This may reduce mathematical anxiety from students thinking they have to complete all of the problems. Different students could be given different numbers to work on if some students need support.

### Suggested opportunities for assessment

**Explore**

* The predicted answer column in Appendix C is a good opportunity for teachers to identify student misconceptions.

**Apply**

* To provide additional support, student can continue to focus on evaluating problems involving only numerical bases.
* To provide a more challenging opportunity for your students, have them work on evaluating problems involving algebraic bases.

Appendix A

### Indices revision table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Base | Power | Numerical form | Expanded form | Index form |
| 10 | 2 | 100 | 10 x 10 |  |
|  | 4 | n/a | x x x |  |
| 17 |  | 289 |  |  |
|  | 5 | 243 |  |  |
|  |  |  |  |  |
|  | 2 |  |  |  |
|  | 4 |  |  |  |
|  | -6 | 1 |  |  |
|  |  | n/a |  |  |
|  | 0 |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| 9 |  | 3 |  |  |

## Appendix B

### Tables of values

1. Use your knowledge of indices to fill in the blank cells in Table 2.

Table – table of values for base 4

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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1. Use the Desmos graph ‘Exploring indices’ (<https://bit.ly/FractionPowers>) to check your answers in Table 2.
2. Use the Desmos graph to find which power gives the required answers for the following bases.

Table – table of values for various bases

|  |  |  |
| --- | --- | --- |
| 1. Base | 1. Required Answer | 1. Power |
| 1. 9 | 1. 3 |  |
| 1. 16 | 1. 4 |  |
| 1. 25 | 1. 5 |  |
| 1. 36 | 1. 6 |  |

1. What do you notice about the numbers in Table 3. What is the relationship between the base and the required answer?
2. Use your knowledge of indices to fill in the blank cells in Table 4.

Table – table of values for base 8

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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1. Use the Desmos graph *‘*Exploring indices’(<https://bit.ly/FractionPowers>) to check your answers in Table 4.
2. Use the Desmos graph to find which power gives the required answers for the following bases.

Table – table of values for various bases

|  |  |  |
| --- | --- | --- |
| 1. Base | 1. Required Answer | 1. Power |
| 1. 16 | 1. 2 |  |
| 1. 27 | 1. 3 |  |
| 1. 81 | 1. 3 |  |
| 1. 32 | 1. 2 |  |

1. What do you notice about the numbers in Table 5. What is the relationship between the base and the required answer?

### Tables of values – challenge

1. Use your knowledge of indices to fill in the blank cells in Table 6.

Table – table of values for base 4

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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1. Use the Desmos graph ‘Exploring indices’ to check your answers in Table 6.
2. Use your knowledge of indices to fill in the blank cells in Table 7.

Table – table of values for base 8

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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1. Use the Desmos graph ‘Exploring indices’ to check your answers in Table 7.
2. What do you notice about the numbers in Tables 6 and 7. What is the relationship between the base, power and the required answer?
3. Test your theories by creating your own indices and solutions and checking your answers by using the Desmos graph.
4. Students could be further challenged by considering if there are any numbers that they could not reach by using indices.

## Appendix C

### Fractional indices – unit fractions

Complete the table by considering:

* How has each question changed from the previous questions?
* What effect do you think that change will have on the answer?
* Calculate the actual answer.

|  |  |  |  |
| --- | --- | --- | --- |
| Question | How question has changed from previous question | Predicted answer | Actual answer |
|  | NA |  |  |
|  | Base is doubled |  |  |
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### Fractional indices – challenge questions

Complete the table by considering:

* How has each question changed from the previous questions?
* What effect do you think that change will have on the answer?
* Calculate the actual answer.

|  |  |  |  |
| --- | --- | --- | --- |
| Question | How question has changed from previous question | Predicted answer | Actual answer |
|  | NA |  |  |
|  | Power is doubled |  |  |
|  |  |  |  |
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## Appendix D

### Problems for students

1. In the column ‘Solutions (part a)’, use fractional indices to find as many different integer values as possible.

|  |  |  |
| --- | --- | --- |
| Problem | Solutions (part a) | Solutions (part b) |
|  | **and** | **or** |
|  |  |  |
|  |  |  |
|  |  |  |
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|  |  |  |
|  |  |  |

1. Use the integer answers from the second column and represent each of these in index form with non-unit fractions as the power.

For example, the answers in the first row were 49 and 7. We could represent these as or . There is no need to simplify the fractions.

## ****Sample solutions****

### **Appendix A – indices revision table**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Base | Power | Numerical form | Expanded form | Index form |
| 10 | 2 | 100 | 10 x 10 |  |
|  | 4 | n/a | x x x |  |
| 17 | 2 | 289 | 17 x 17 | 172 |
| 3 | 5 | 243 |  |  |
| 6 | 7 | 279936 |  |  |
|  | 2 | 15625 |  |  |
|  | 4 | n/a |  |  |
| 1 | -6 | 1 |  |  |
|  | -4 | n/a |  |  |
| **multiple** | 0 | 1 | n/a |  |
| 3 | -4 |  |  |  |
| 3 | -4 |  |  |  |
|  | 0 | 1 | n/a |  |
| 9 |  | 3 | n/a |  |

### Appendix B – tables of values

1. Numerical base of 4

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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|  |  |  |
| --- | --- | --- |
| Base | Required Answer | Power |
| 9 | 3 |  |
| 16 | 4 |  |
| 25 | 5 |  |
| 36 | 6 |  |

Students might notice that the required answer is the square root of the base.

Numerical base of 8

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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|  |  |  |  |  |  |  |  |  |  |

|  |  |  |
| --- | --- | --- |
| Base | Required answer | Power |
| 16 | 2 |  |
| 27 | 3 |  |
| 81 | 3 |  |
| 32 | 2 |  |

Students might notice that the denominator of the power indicates the root of the base. Students might find it easier to think will have a power of .

### Appendix B – tables of values – challenge

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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Students might notice that there is an inverse relationship between the result of positive and negative powers.

Students might notice that the result of is equivalent to the result of .

### Appendix C – fractional indices

Students will have multiple different answers in the ‘Predicted answer’ column.

|  |  |  |  |
| --- | --- | --- | --- |
| Question | How question has changed from previous question | Predicted answer | Actual answer |
|  | NA | 10 | 10 |
|  | Base is doubled | 20 | 14.1 |
|  | Base is doubled | 40 | 20 |
|  | Base is divided by 100 | 0.4 | 2 |
|  | Base is halved | 1 | 1.4 |
|  | Base is divided by 2 | 1 | 1 |
|  | Base is multiplied by 8 | 4 | 2.8 |
|  | Index is smaller | 3 | 2 |
|  | New question | 3 | 3 |
|  | Base is multiplied by 1000 | 3000 | 30 |
|  | New question |  |  |
|  | New question | 4 | 4 |
|  | Base is divided by 100 | 0.4 | 0.4 |
|  | Base is a pronumeral | na | or |
|  | Base is multiplied by 8 |  |  |

### Appendix C – fractional indices – challenge questions

Students will have multiple different answers in the ‘Predicted answer’ column.

|  |  |  |  |
| --- | --- | --- | --- |
| Question | How question has changed from previous question | Predicted answer | Actual answer |
|  | NA |  | 2 |
|  | Power is doubled |  |  |
|  | Power is doubled |  |  |
|  | New question |  |  |
|  | Power is doubled |  |  |
|  | Base multiplied by 1000 | 9000 | 900 |
|  | Power doubled from a previous question |  |  |
|  | New question |  |  |
|  | Index changed to negative |  |  |
|  | Bases changed |  |  |
|  | Index changed |  |  |
|  | Powers swapped | Same as before |  |
|  | Indices changed |  |  |
|  | Powers swapped |  |  |

### Appendix D – problems for students

|  |  |  |
| --- | --- | --- |
| Problem | Solutions (part a) | Solutions (part b) |
|  | **and** | **or** |
|  | and | and |
|  | and | and |
|  | , and | , , , , and |
|  | , and | , , , , and |
|  | , and | , , , , and |
|  | , , and | , , , , , , , , , , and |

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