# Growing with power

Students investigate the multiplication and power of a power index laws to apply them to algebraic bases.

## Visible learning

### Learning intentions

* To be able to use the multiplication index law to simplify algebraic expressions.
* To be able to use the power of a power index law to simplify algebraic expressions.

### Success criteria

* I can identify the power and the base of a number written in index form.
* I can simplify algebraic expressions using the multiplication index law.
* I can simplify algebraic expressions using the power of a power index law.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases **MA5-IND-C-01**

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-k-10-2022) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

## Activity structure

### Launch

1. Hand students a copy of Appendix A ‘Two truths, one lie’ ([bit.ly/TwoTruthsOneLieStrategy](https://bit.ly/TwoTruthsOneLieStrategy)).
2. Students will look at the expression in the first column and identify which one of the 3 answers is a lie.
3. Have students engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to find the incorrect expression in each of the problems.
4. Challenge students to explain why each of the ‘lies’ is incorrect.

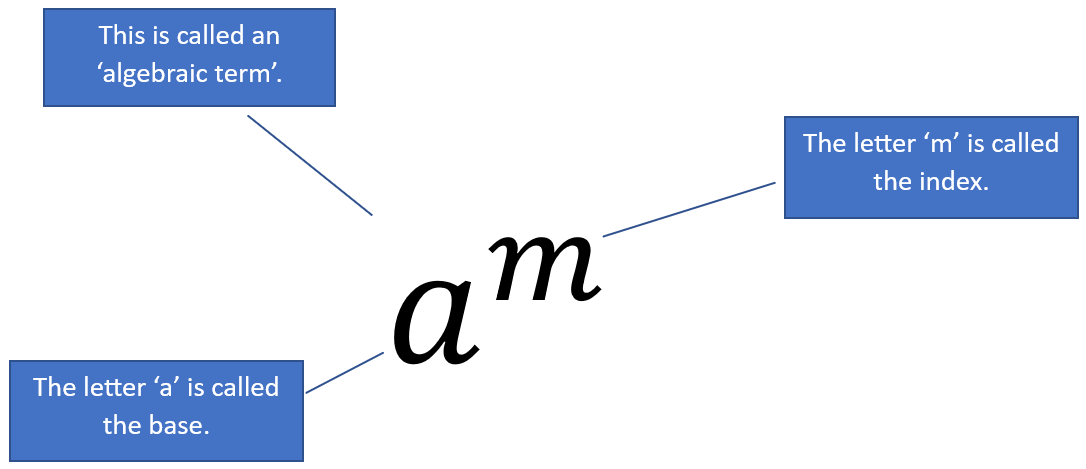
Allow students to explain the ‘lies’ in their own way without intervention or support. The Apply section of this lesson uses generalised algebraic rules to improve these explanations.

## Multiplication of indices

### Explore

1. Begin the lesson by writing the term on the board.
2. Pose the following questions:
3. What can you see?
4. What do you notice?
5. What do you already know?
6. What do you wonder?
7. Add the labels to the term.

Figure 1 – annotation of the term am



Students benefit from being able to communicate effectively by using the appropriate language. Discuss with students that the index can also be called ‘exponent’ and ‘power’ and that the plural of ‘index’ is ‘indices’.

1. Write the following expressions on the board:
2. Have students engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to identify any patterns in these expressions. After a short time, reveal the solutions below and ask for further contributions from students.

It is important at this stage that students recognise that the base of each exponent is the same within each question.

1. Model writing each index in expanded form. Use different coloured whiteboard markers to indicate different values rather than brackets.
2. Have students develop a process in pairs to explain what is happening and apply this process to the other 4 examples.

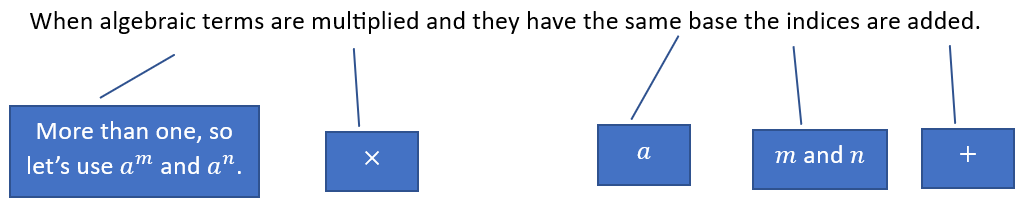
### Summarise

1. Ask students to think about how they might write the process of multiplying algebraic indices with the same base in words, so that it reads like instructions and would make sense to someone reading it. Have students work in pairs to write the instructions.
2. Ask students to read out their instructions and use the student responses to create a written version on the board. Ask students the question: ‘Are there any words we used that can be represented by a mathematical symbol?’.

An example of the instructions written on the board may be: ‘When algebraic terms are multiplied and they have the same base, the indices are added’. While the wording may be slightly different, the intention should be the same.

1. Discuss how components of the instruction can be substituted with mathematical symbols to make it more compact. An example is shown below.

Figure 2 – sentence annotated with mathematical symbols



1. Write the formal law on the board: .
2. Propose the following questions for discussion:
3. What do you think?
4. Which is easier to remember, the law or the written instruction?
5. Which is easier to write, the law or the instruction?
6. Why is it called a law?

Discussion points may include the idea that the words may be easier to remember, but the algebra is easier to write and that the terms law, rule and instruction are all valid names.

1. Have students write notes to their future forgetful self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)). These notes should include the rule, where it came from and some typical examples that are found.
2. Provide students with the following 5 ‘Your turn’ questions to consolidate their learning. Simplify:

These 5 quick questions increase in difficulty level moving from numbers to pronumerals and introducing multiple pronumerals and numbers.

## Power of a power

### Explore

1. Write the following 5 expressions on the board:
3. Have students engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to discuss what they notice about these expressions. After a short time, reveal the solutions below and ask for further contributions from students.
4. Simplify the 5 questions on the board without discussion.

It is important at this stage that students recognise that the index outside the brackets creates a repeated product of the term inside the bracket.

1. Model writing the expanded expression. Again, use different coloured whiteboard markers to indicate different parts.
2. Have students develop a process in pairs to explain what is happening and apply this process to the other 4 examples.

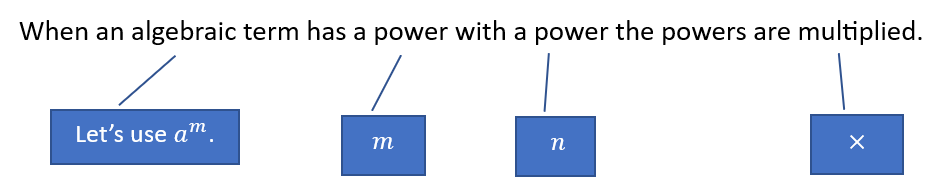
### Summarise

1. Ask students to think about how they might write the process in words so that it reads like a instructions and would make sense to someone reading it. Have students work in pairs to write the instructions.
2. Ask students to read out their instructions and use the student responses to create a written version on the board. Ask students the question: ‘Are there any words we used that can be represented by a mathematical symbol?’.

An example of the student generated instruction written on the board may be: ‘When an algebraic term has a power with a power, the powers are multiplied’. While the wording may be slightly different, the intention should be the same.

1. Discuss how components of the instructions can be substituted with mathematical symbols to make it more compact. An example is shown below.

Figure 3 – sentence annotated with mathematical symbols



1. Write the formal law on the board: .
2. Propose the following questions for discussion:
3. What do you think?
4. Which is easier to remember, the law or the written instruction?
5. Which is easier to write, the law or the instruction?

Discussion points may include the idea that the words may be easier to remember, but the algebra is easier to write and that the terms ‘law’, ‘rule’ and ‘instruction’ are all valid names.

1. Have students write notes to their future self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)). These should include the rule, where it came from and some typical examples that are found.
2. Provide students with the following 5 ‘Your turn’ questions to consolidate their learning. Simplify:

These 5 quick questions have increased in difficulty level moving from numbers to pronumerals and introducing multiple pronumerals and numbers.

### Apply

#### Using the formula

1. Display the statement below to students, which explains why the ‘lie’ in the first problem of Appendix A ‘Two truths, one lie’ is incorrect.  
    because where the base does not change.’
2. Emphasise that this explanation uses the general rule that we established earlier.
3. Have students write an explanation using their generalised index laws for why each of the remaining ‘lies’ in Appendix A are incorrect.

#### Codebreaker

1. The codebreaker activity in Appendix B ‘Codebreaker’ provides students with an activity which will help them apply both the multiplying law and power of a power law.  
   Print a copy for each student and distribute.

Teachers can choose to challenge students with Appendix C ‘Codebreaker with negative and fractional indices’. Students need only to be able to operate with fractions and negative numbers to complete this activity.

1. When students have completed the Codebreaker activity, the joke should read:  
   ‘There are 10 kinds of people in this world; those who understand binary and those who don't.’

The joke used in the codebreaker activity links informally to the launch activity on binary code in lesson one of the indices unit *Invaluable indices.*

#### Power Stack

1. Present the following scenario to students:

Kimberly wants to define as but Nermeen thinks that such a stack of powers should be defined as .

1. Ask students to consider the following questions:
2. Does it matter? Do you get the same answer either way?
3. Would this be true for other numbers?
4. Can you write both Kimberly’s and Nermeen’s definitions using algebra?
5. Provide a reason to explain who you agree with.
6. What if we had an extra 3 in our stack?

Adapted from NRICH ‘Power Stack’

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore and summarise**

* There are regular checkpoints for teachers where specific realisations and conclusions are identified as being essential before progressing. Teachers should use questioning techniques to verify student understanding of each of these points before continuing.

**Apply**

* The Appendix C ‘Codebreaker with negative and fractional indices’ is almost the same as Appendix B. However, Appendix C contains 2 expressions which include negative indices and an expression with a fractional index to extend students.

### Suggested opportunities for assessment

**Launch**

* **The ability to identify and explain why one of the answers in Appendix A is incorrect gives evidence of student understanding of the related index law.**

**Summarise**

* Monitor student responses during discussion to assess their understanding of the 2 index laws and how they operate.
* Use the consolidation questions in step 7 of each section to verify student understanding of the key ideas in each generalisation.

**Apply**

* Monitor student responses during the codebreaker activity. Some students may find it difficult to form the words in the joke.

## **Appendix A**

### Two truths, one lie

For each of the 4 questions, can you figure out which of the following statements are true and which one is the lie?

## **Appendix B**

### Codebreaker

Simplify each expression, writing your answer in the cell below the expression, and use the answer to determine each letter in the code below to spell out a joke.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **a** | **b** | **d** | **e** | **f** | **h** |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **i** | **k** | **l** | **n** | **o** | **p** |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **r** | **s** | **t** | **u** | **w** | **y** |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Get ready to laugh!

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  | **1** | **0** |
|  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | ; |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | ‘ | \_\_ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## **Appendix C**

### Codebreaker with negative and fractional indices

Simplify each expression, writing your answer in the cell below the expression, and use the answer to determine each letter in the code below to spell out a joke.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a | b | d | e | f | h |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | k | l | n | o | p |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| r | s | t | u | w | y |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Get ready to laugh!

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  | **1** | **0** |
|  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | ; |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \_\_ | \_\_ | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ |  | \_\_ | \_\_ | \_\_ | ‘ | \_\_ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Sample solutions

### Appendix A – Two truths, one lie

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | 1. **lie** |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | 1. **lie** |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1. **lie** |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1. **lie** |  |  |

### Apply explanations

1. because where the base does not change.
2. because where the powers add together.
3. because when there is a power to a power and .
4. because when there is a power to a power and equivalent to .

### Appendix B and C – codebreaker solutions

|  |  |  |  |
| --- | --- | --- | --- |
| Letter | Appendix B | Appendix C | Solution |
| a |  |  |  |
| b |  |  |  |
| d |  |  |  |
| e |  |  |  |
| f |  |  |  |
| h |  |  |  |
| i |  |  |  |
| k |  |  |  |
| l |  |  |  |
| n |  |  |  |
| o |  |  |  |
| p |  |  |  |
| r |  |  |  |
| s |  |  |  |
| t |  |  |  |
| u |  |  |  |
| w |  |  |  |
| y |  |  |  |

## References

This resource contains NSW Curriculum and syllabus content. The NSW Curriculum is developed by the NSW Education Standards Authority. This content is prepared by NESA for and on behalf of the Crown in right of the State of New South Wales. The material is protected by Crown copyright.

Please refer to the NESA Copyright Disclaimer for more information [https://educationstandards.nsw.edu.au/wps/portal/nesa/mini-footer/copyright](https://aus01.safelinks.protection.outlook.com/?url=https%3A%2F%2Feducationstandards.nsw.edu.au%2Fwps%2Fportal%2Fnesa%2Fmini-footer%2Fcopyright&data=05%7C01%7CCaitlin.Pace1%40det.nsw.edu.au%7C9c2c1a9f59c94d2df30708dafa7edb23%7C05a0e69a418a47c19c259387261bf991%7C0%7C0%7C638097720042599463%7CUnknown%7CTWFpbGZsb3d8eyJWIjoiMC4wLjAwMDAiLCJQIjoiV2luMzIiLCJBTiI6Ik1haWwiLCJXVCI6Mn0%3D%7C3000%7C%7C%7C&sdata=BzQh0UsffVZE3eO22b2Xba3p0VMOBZSHfS21FGHXtZM%3D&reserved=0).

NESA holds the only official and up-to-date versions of the NSW Curriculum and syllabus documents. Please visit the NSW Education Standards Authority (NESA) website <https://educationstandards.nsw.edu.au/> and the NSW Curriculum website [https://curriculum.nsw.edu.au/home](https://curriculum.nsw.edu.au/).

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-k-10-2022) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

**© State of New South Wales (Department of Education), 2023**

The copyright material published in this resource is subject to the *Copyright Act 1968* (Cth) and is owned by the NSW Department of Education or, where indicated, by a party other than the NSW Department of Education (third-party material).

Copyright material available in this resource and owned by the NSW Department of Education is licensed under a [Creative Commons Attribution 4.0 International (CC BY 4.0) licence](https://creativecommons.org/licenses/by/4.0/).

[](https://creativecommons.org/licenses/by/4.0/)

This licence allows you to share and adapt the material for any purpose, even commercially.

Attribution should be given to © State of New South Wales (Department of Education), 2023.

Material in this resource not available under a Creative Commons licence:

* the NSW Department of Education logo, other logos and trademark-protected material
* material owned by a third party that has been reproduced with permission. You will need to obtain permission from the third party to reuse its material.

**Links to third-party material and websites**

Please note that the provided (reading/viewing material/list/links/texts) are a suggestion only and implies no endorsement, by the New South Wales Department of Education, of any author, publisher, or book title. School principals and teachers are best placed to assess the suitability of resources that would complement the curriculum and reflect the needs and interests of their students.

If you use the links provided in this document to access a third-party's website, you acknowledge that the terms of use, including licence terms set out on the third-party's website apply to the use which may be made of the materials on that third-party website or where permitted by the *Copyright Act 1968* (Cth). The department accepts no responsibility for content on third-party websites.