# A4 cylinders

Students investigate the 2 cylinders formed by curling an A4 sheet of paper in 2 different ways. They compare the volume of each cylinder and deduce and apply strategies for finding the surface area of cylinders under a variety of conditions.

## Visible learning

### Learning intentions

* To be able to develop the formula to find the surface area of a cylinder.
* To be able to apply the formula to find the surface area of a cylinder.

### Success criteria

* I can compare the size and shape of 2 cylinders.
* I can determine the net formed from cylinders, both closed and open.
* I can use the formula to find the surface area of a cylinder.
* I can explain how to develop the formula for the surface area of a cylinder.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* simplifies algebraic fractions with numerical denominators and expands algebraic expressions **MA5-ALG-C-01**
* solves linear equations of up to 3 steps, limited to one algebraic fraction **MA5-EQU-C-01**
* solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids **MA5-ARE-C-01**

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## Activity structure

### Launch

1. Prepare 4–6 cylindrical items that are similar in size. This could include spread jars, cans, or cups.
2. Display each item for students to view.
3. Ask students to consider which item would have the greatest volume. Then ask students to decide on how they would rank the items from least to greatest volume.
4. Collect some predictions from students whilst asking them, how they came to their decisions. Was there a strategy?
5. Ask students how they could know for sure what the order was in terms of least to greatest volume. Students may discuss that they could look at the capacity of each item and/or calculate the volume.

Stage 4 content of finding the volume of a cylinder could be reviewed. An optional task would be to calculate the volume of each of these cylindrical items.

1. Ask students to consider the surface area of each item and pose the question:
2. If we were to rank these items from least to greatest surface area would the order remain the same?

Have a class discussion about this. A definite answer should not be formed, nor should calculations be conducted. This should be a discussion to spark interest.

### Explore

A PowerPoint *A4 Cylinders* has been created to use throughout this activity, which includes solutions.

1. Demonstrate how to fold an A4 piece of paper 2 ways. The image below is on slide 3 of the PowerPoint to help students visualise what is occurring.

Figure 1 – A4 paper rolled 2 ways

A4 sheet of paper flat with a
diagram showing the paper curled up long ways and a diagram showing the paper curled up the short side.

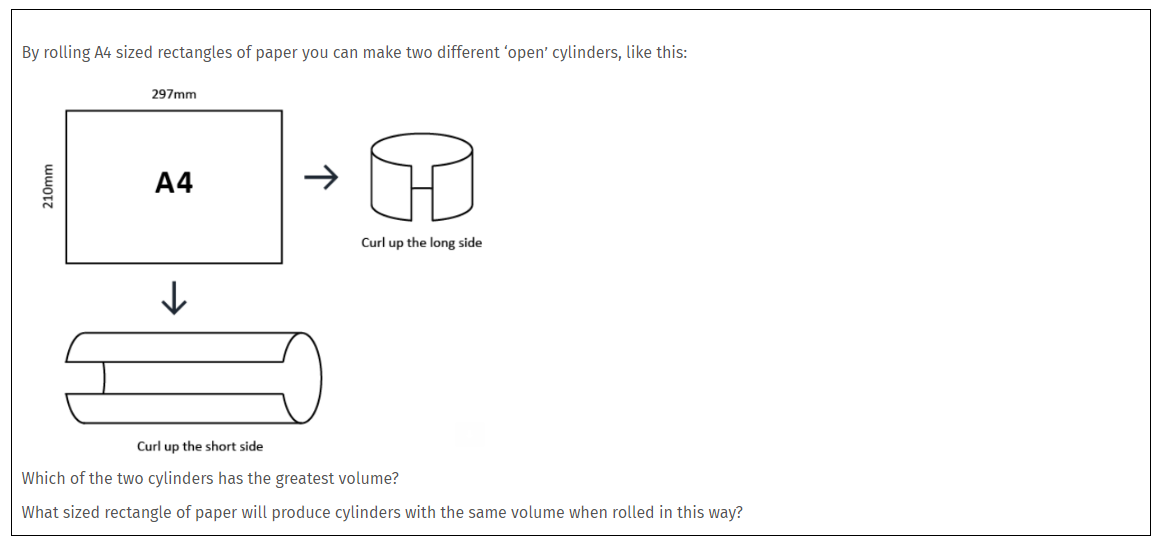
Image from [NZ Maths](https://nzmaths.co.nz/resource/a4-cylinders) and is licensed under the [Creative Commons Attribution 3.0 New Zealand licence](https://creativecommons.org/licenses/by/3.0/nz/).

1. Ask students to consider which of the 2 cylinders has the greatest volume. No measurements should be provided, and no calculations should be performed. This is an initial guess. Collect results using a Mentimeter poll ([mentimeter.com/](https://www.mentimeter.com/)).
2. Using a questioning strategy such as Pause-Pose-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) ask students to explain why they think one cylinder will have a greater volume than the other.
3. In randomly selected groups of 3, students will create a notice and wonder list ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) on the similarities and differences between the 2 cylinders. These thoughts could be put on Vertical Non-Permanent Surface (VNPS) ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)). A gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) and a brief class discussion should be done following this to share some responses.

The aim is for students to notice that each open cylinder created has the same surface area, despite having different volumes. Students should also notice that they’ll need the radius of each cylinder to calculate the volumes.

1. Students will work in their randomly assigned groups of 3, where they will be slowly given information or further questions to analyse the 2 different cylinders formed. Each group will need at least one A4 sheet of paper and a ruler if requested.
2. Provide the dimensions of the A4 sheet of paper, 210 mm by 297 mm, and ask each pair/group to come up with mathematical reasoning and calculations to support their decision of which cylinder will have the largest volume, the landscape or portrait orientation. Students may need to be reminded of how to calculate the volume of a cylinder and what portrait and landscape orientation means.

Figure 3 – measurements of an A4 sheet of paper



Some students may find it easier if these measurements are converted to centimetres (21 cm by 29.7 cm) or in centimetres and rounded to the nearest whole number (21 cm by 30 cm).

Monitor student progress as they try to calculate the volume of each cylinder formed. They will likely struggle or make errors when finding the length of the radius. This is okay, try not to correct them, instead ask questions to make them consider their strategy.

1. Ask the question again, ‘Which of the 2 cylinders formed has the greatest volume?*’* and collect results using a Mentimeter poll ([mentimeter.com/](https://www.mentimeter.com/)) or similar.
2. Collect the results for the volume of each cylinder from the students. These answers could be quite different, and some time may need to be spent on how they found the length of the radius. For students who measured the distance across the cylinder, it is worthwhile demonstrating how to accurately calculate the radius, using the circumference of the circle and revising their algebraic skills, using the PowerPoint slides 5–14. Students may also have used trial and error to determine the radius.

If multiple groups measured the distance across the cylinder to calculate the surface area of the circle, discuss why the answers are different. This should lead into a discussion that examines the effect that truncating or rounding during calculations has on the accuracy of the results.

1. Ask each group to calculate the surface area of each of the open cylinders. They should easily see that each cylinder has the same surface area, by calculating the area of the A4 sheet of paper.
2. Ask the students to consider:
3. What if each of the cylinders was closed at one end?
4. Would the surface areas of the 2 cylinders change by the same amount?
5. What is the difference between the surface of the open and closed cylinders?

Students are then to perform calculations to see which cylinder would have the greatest surface area.

1. Ask the students to consider:
2. What if each of the cylinders were closed at both ends, that is at the bottom and top?
3. Would the surface areas of the 2 cylinders change by the same amount?
4. What is the difference between the 2 surface areas?
5. Students are then to perform calculations to see which cylinder would have the greatest surface.
6. Students inquire how to find the surface area of any closed cylinder, by considering questions such as:
7. What information or dimensions are required to find the surface area of a cylinder?
8. What shapes make up the net of a cylinder?
9. How do you find the area of each face?
10. Can you establish a formula to use to find the surface area of any cylinder?
11. After some discussion, develop the formula for the surface area of a cylinder, using the A4 Cylinders PowerPoint slides 15–24. Suggested probing questions include:
12. How do we find the area of a circle?
13. How many circles are there on a closed cylinder?
14. The dimensions of the rectangle (sheet of paper) need to be found in terms of characteristics of the circles, can you describe a side of the rectangle in terms of the circle?

Students should recognise that one length of the rectangle represents the circumference of the circle.

1. What is the formula for the circumference of a circle?
2. Does the remaining side of the rectangle have any relationship to the circles?
3. What is the formula to find the surface area of any cylinder?

### Summarise

1. Students will be explicitly shown how to apply the formula of the surface area of a cylinder to a range of different cylinders. For example, closed, open at one end, open at both ends and some drawn as nets rather than a 3-dimensional solid. The *A4 Cylinders* PowerPoint slides 26–32 can be used for these worked examples.
2. Students then make notes to their future forgetful self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) on how to apply the surface area of a cylinder formula.

### Apply

#### Revisiting the A4 cylinder

1. After discovering that the volume changes depending on which way the A4 sheet of paper is rolled, ask students to consider what sized paper would give the same volume regardless of how it is rolled?
2. Students should work in pairs to consider this question. They may come up with the answer without calculations or may use algebraic techniques to discover that the sheet of paper needs to be a square.

Solutions for this can be found at the conclusion of this document, in ‘Sample solution’.

#### Soft drink can design

1. Students are to design 2 different soft drink cans that have the same volume but different surface areas. Each can will have a different radius and height.

Students may like to determine a particular capacity of their soft drink can to work towards so Stage 4 content of converting between metric units of volume and capacity should be revised. That is, .

1. Students are to analyse each can and determine which can would keep their drink coldest for the longest amount of time and why.

This activity requires extensive algebraic skills and might be best explored as a class.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* This activity provides an opportunity to revise Stage 4 content of parts of a circle, area of a circle, circumference of a circle and the volume of a cylinder. Some students will require more revision than others.

**Explore**

* Students may need the dimensions of the A4 sheet of paper converted to centimetres for them or converted and rounded to whole numbers.
* Students may need to be reminded of how to calculate the volume of a cylinder.
* Students may need assistance to find the radius of each of the cylinders. Are they measuring it once they’ve formed the cylinder or are they using the circumference formula to accurately find the radius.

**Summarise**

* Some students may benefit from having physical nets of the cylinders when applying the formula to find the surface area.

**Apply**

* This task could be done as a whole class activity.

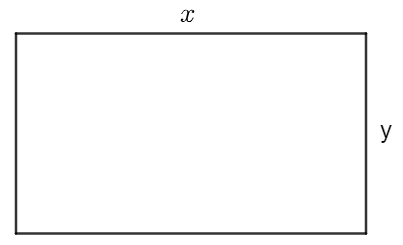
### Suggested opportunities for assessment

* Monitor student discussions to check for common misconceptions.
* Record student explanations of how they calculated the surface area, prior to being explicitly shown the formula.
* Collect student soft drink can designs to check for understanding.

## Sample solution

### Apply – revisiting the A4 cylinders

Figure 4 – rectangle labelled x by y



Consider the volume if the side labelled became the circumference of the end circles.

Consider the volume if the side labelled became the circumference of the end circles.

Let each volume equal each other.

Multiple each side by 2 and divide by common terms.

Students may also have just recognised that each sheet of paper needed to be a square.

## References

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