# Cube houses

Students investigate the surface area of prisms by comparing different house designs made from 4 cubes.

## Visible learning

### Learning intention

* To be able to calculate the surface area of prisms.

### Success criteria

* I can compare the size and shape of 2 prisms.
* I can determine the area of the faces of a prism.
* I can calculate the surface area of a rectangular prism.
* I can calculate the surface area of a triangular prism.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* simplifies algebraic fractions with numerical denominators and expands algebraic expressions **MA5-ALG-C-01**
* solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids **MA5-ARE-C-01**

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## Activity structure

### Launch

1. Explain to students that in Holland there is a famous architectural structure called ‘The Cube Houses’. The structure is composed of cubes and the houses are located close to the city centre of Rotterdam.
2. Ask students to explain what they imagine the houses might look like (do not show them any pictures at this point). Can they draw what they are imagining?
3. Have a few students explain what they are imagining and share some of the students’ drawings.
4. Following this, show students images of Rotterdam’s Cube Houses and compare with the images they have drawn.

Figure 1 – The Cube Houses



‘[File:Rotterdam Cube House street view](https://openverse.org/image/cf36e43a-7544-4519-a244-1f30d5626652?q=the%20cube%20houses)’ by Raul Ayres is licensed under [CC BY-SA 3.0](https://creativecommons.org/licenses/by-sa/3.0/).

### Explore

#### Equipment

* Centicubes (4 per student).
* Isometric paper.
* Appendix B ‘Cube houses investigation’*.*

If cubes and/or isometric paper is not available, students can use virtual manipulatives using [toytheater.com/cube/](https://toytheater.com/cube/).

#### Method

1. Pose the following questions to students:
2. What if we had a street of houses made from cubes?
3. What might this look like?
4. Would all the houses look the same?
5. What design considerations would we need to consider?
6. The students are going to become architects, with a challenge to build as many different houses as possible from 4 cubes. The orientation of the house on the ‘block of land’ determines if 2 houses are the same. For example, the 2 houses below are considered different house designs.

Figure 2 – four cube houses



Images created using the free virtual manipulatives at [toytheater.com/cube/](https://toytheater.com/cube/).

1. In randomly assigned pairs or groups of 3, students will use 4 centicubes to explore the different house designs, recording their designs on the isometric paper.
2. Ask each group to share a design until all 15 designs are shared with the class. All design options can be found in Appendix A ’Cube house solutions’.
3. Once students have discovered all possible house designs, ask them to consider:
4. What is the same about each of the houses formed?
5. What is different about each of the houses formed?

Students may discuss that some houses are single storey whilst others are 2, 3 or 4 storeys, or that some houses take up more land space than others. However, this task is leading students to discover that the volume is the same for each prism formed, , but the surface area is different for each prism.

1. The architects now need to consider the most cost-efficient houses, by investigating the cost to build each of the houses. Below is a list of the costs to build each section of the house.

|  |  |
| --- | --- |
| Section | Cost per square |
| Floor | $6 000 |
| External wall | $3 000 |
| Roof | $5 000 |

Ask the class to consider and discuss: Will all houses cost the same, since they are all made from 4 cubes?

1. In their groups, students are to select their favourite 3–5 houses and determine the cost to build each of them. They can then select their favourite design, by considering the visual design of the house, as well as the cost to build the house.
2. Following this, select a few groups to share their findings with the class.

This discussion should then lead into how they found the cost to build, that is they were finding the surface area of the house. Clearly define surface area of solids to students as the measure of the total area of the surface(s) of a 3-dimensional shape or object.

### Summarise

1. Use the *Cube Houses* PowerPoint for explicit teaching of calculating the surface area of rectangular and triangular prisms.

The explicit teaching technique used in the PowerPoint is ‘Your turn’. The first slide is a worked example which should be displayed for the students and then use the following steps.

1. Reveal the question to the students and its solution.
2. Students read in silence.
3. Students individually think and explain to themselves what is happening in each step.
4. Students hold a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs, students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class, using a technique such as Pause-Pose-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)).
8. Students are then to complete the faded worked examples in Appendix C ‘faded worked examples’.

### Apply

#### Home office space renovation

1. Explain to students that they have been given a double garage that is being converted into an office space. Students will be given an image of the space, see Appendix D ‘Home office space renovation’. The image shows the measurements for a door and a window, and they will need to determine the amount of paint required if the walls and ceiling need to be painted.

Teachers are to note that this is the first time students have been asked to calculate the surface area of a composite solid. This task is for students to investigate and challenge themselves. Surface area of composite solids will be explored in more detail in future lessons.

1. Following this, students will investigate 2 different styles of roof replacements to determine the most cost-effective design.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students may struggle to draw in 3-dimensions. Some may benefit from isometric paper, others may benefit from just explaining their visualisations.

**Explore**

* Some students may need assistance with drawing their house designs on the isometric paper. These students may benefit from using the suggested virtual manipulatives.
* Encourage students to re-construct their house designs to assist with calculating the cost of building. Some students may struggle to visualise which faces represent the floor, and how many external walls there are.

**Summarise**

* Having physical nets and 3-dimensional objects can assist students with visualisation when calculating the surface area.
* Some students may need to revise calculating the hypotenuse using Pythagoras’ theorem.

**Apply**

* Some students could be extended to explore different styles of roof designs. They could then calculate the cost of painting the door and window frames. Discuss the addition of a bathroom in one corner and how this would affect the amount of paint needed for the internal walls.
* Students could be challenged by renovating a room with various shaped surfaces.

### Suggested opportunities for assessment

* Monitor reasons provided for students’ choice of a favourite house design to clarify any misconceptions.
* Monitor conversations occurring in group work for formative assessment.
* Teachers could collect the faded worked examples to check for understanding.
* The activity in the ‘Apply’ section could be turned into a summative assessment task.

## **Appendix A**

### Cube house solutions

Below are the 15 possible house designs.

|  |  |  |
| --- | --- | --- |
| 4 cubes in a row, standing up. | 4 cubes in a row, lying down | 3 cubes on top of each other with the last cube next to it.  |
| 3 cubes in a row and 1 off the first cube. All at the same level.  | 3 cubes in a row and 1 off the last cube. All at the same level.  | 3 cubes in a row and 1 on top of the last cube.  |
| 3 cubes in a row and 1 on top of the middle cube.  | 3 cubes in a row and 1 off the middle cube. All at the same level.  | 2 cubes in a row, joined by 2 more to form a square of cubes |
| 2 cubes in a row and 2 more cubes in a row off the last cube. | 2 cubes in a row and 2 more cubes in a row off the first cube. | 2 cubes with 2 cubes on top forming a second level. |
| 2 cubes standing up and 1 cube next to the bottom cube and the other behind that cube. | 2 cubes next to each other, 1 cube behind the first cube and on top of that cube. | 3 cubes joined like a corner and 1 cube on top of the middle cube. |

Images created using the free virtual manipulatives at [toytheater.com/cube/](https://toytheater.com/cube/)

## Appendix B

### Cube houses investigation

1. You have become an architect, with a challenge to build as many different houses as possible from 4 cubes.

The orientation of the house on the ‘block of land’ determines if 2 houses are the same. For example, the 2 houses below are considered different house designs.

Figure 3 – 4-cube houses



Images created using the free virtual manipulatives at [toytheater.com/cube/](https://toytheater.com/cube/)

1. In your group use 4 centicubes to explore as many different house designs as possible, recording your designs on the isometric paper provided.
2. Consider the following:
3. What is the same about each of the houses formed?
4. What is different about each of the houses formed?
5. You, as the architect now need to consider the most cost-efficient houses, by investigating the cost to build each of the houses. Below is a list of the costs to build each section of the house.

|  |  |
| --- | --- |
| Section | Cost per square |
| Floor | $6 000 |
| External wall | $3 000 |
| Roof | $5 000 |

Will all houses cost the same, since they are all made from 4 cubes?

1. In your group, select your favourite 3–5 houses and determine the cost to build each of them. Be sure to consider the visual design of the house, as well as the cost to build the house. Justify your decision with mathematical reasoning.

## Appendix C

### Faded worked examples – rectangular prisms 1

Calculate the surface area of each rectangular prism.

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| Rectangle prism with length 45cm, height 15cm and width 25cm | Square based prism, with square of side length 6.3cm and the prism has a height of 7.1cm.  | Rectangular prism with width 40mm, length 50mm and height 20mm |
| Front face | Front face  | Front face |
| Back face | Back face | Back face |
| Right face | Right face | Right face |
| Left face | Left face | Left face  |
| Bottom face | Bottom face | Bottom face |
| Top face | Top face | top face |
| Surface area | Surface area | Surface area |

### Faded worked examples – rectangular prisms 2

Calculate the surface area of each rectangular prism.

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| A net of a rectangular prism. The rectangular cross section has dimensions 9cm by 15cm. And the prism has a height of 27cm. The faces that make up the height of the prism are labelled 1,2,3,4 respectively.  | A net of a square based prism. The square cross section has side length 6cm. And the prism has a height of 11cm. The faces that make up the height of the prism are labelled 1,2,3,4 respectively.  | A net of a rectangular prism. The rectangular cross section has dimensions 3cm by 3.5cm. And the prism has a height of 1.7cm.  |
| Small rectangle left | Square face top  | Big rectangle top left |
| Small rectangle right (same as the other small rectangle) | Square face bottom | Big rectangle middle right |
| Rectangle 1 | Rectangle 1 | Rectangle 1 |
| Rectangle 2 | Rectangle 2 | Rectangle 2 |
| Rectangle 3 (same as rectangle 1) | Rectangle 3 | Rectangle 3 |
| Rectangle 4 (same as rectangle 2) | Rectangle 4 | Rectangle 4 |
| Surface area | Surface area | Surface area |

### Faded worked examples – triangular prisms 1

Calculate the surface area of each triangular prism.

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| Triangular prism with height 5cm, Width 12cm, length 9cm and sloping side of 13cm | Triangular prims with base of 16cm by 21cm and sloping sides of 10cm and height of 6cm | A triangular prism. The triangular cross section is an equilateral triangle with side length 8.6cm and a perpendicular height of 7.4cm. The prism has a height of 14.2cm.   |
| Front triangular face | Front triangular face | Front triangular face |
| Back triangular face (same as front) | Back triangular face | Back triangular face |
| Right/slope rectangle face  | Right/slope rectangle face | Right/slope rectangle face |
| Left rectangle face  | Left /slope rectangle face | Left/slope rectangle face |
| Bottom rectangle face | Bottom rectangle face | Bottom rectangle face |
| Surface area | Surface area | Surface area  |

### Faded worked examples – triangular prisms 2

Calculate the surface area of each triangular prism.

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| A net of a right angled triangular prism. The triangular cross section has a base of 4cm and a height of 3cm. The prism has a height of 9cm. The rectangular faces are labelled 1,2,3.  | The net of a triangular prism. The triangular cross sections are isosceles triangles with equal side lengths of 15cm and the remaining side 24cm. The prism has a height of 17cm. The rectangular faces are labelled 1,2,3. | The net of a triangular prism. The triangular cross section is an equilateral triangle with side length 4.6cm. The height of the prism is 9.2cm.  |
| Pythagoras theorem (hypotenuse) | Pythagoras theorem (height of triangle) | Pythagoras theorem |
| Triangular face | Triangular face | Triangular face |
| Rectangle 1 | Rectangle 1 | Rectangle 1 |
| Rectangle 2 | Rectangle 2 | Rectangle 2 |
| Rectangle 3 | Rectangle 3 | Rectangle 3 |
| Surface area | Surface area  | Surface area |

## Appendix D

### Home office space renovation

Below is an image of a double garage that has been converted into a home office space. Measurements are shown for the room, door, and window.



1. Determine the amount of paint required if the walls and ceiling need to be painted. Assume that 1 litre of paint covers 15 square metres.
2. The roof of the new home office space needs replacing. Below are 2 different designs that the owner needs to choose between. The cost of replacing a roof is $210 per square metre. Using mathematical calculations, decide which roof design is the most cost effective.



## Sample solution

### Appendix C – faded worked examples

#### Faded worked examples – Rectangular prism 1

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| Rectangle prism with length 45cm, height 15cm and width 25cm | Square based prism, with square of side length 6.3cm and the prism has a height of 7.1cm.  | Rectangular prism with width 40mm, length 50mm and height 20mm |
| Front face | Front face  | Front face |
| Back face | Back face | Back face |
| Right face | Right face | Right face |
| Left face | Left face | Left face  |
| Bottom face | Bottom face | Bottom face |
| Top face | Top face | top face |
| Surface area  | Surface area | Surface area  |

#### Faded worked examples – rectangular prism 2

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| A net of a rectangular prism. The rectangular cross section has dimensions 9cm by 15cm. And the prism has a height of 27cm. The faces that make up the height of the prism are labelled 1,2,3,4 respectively.  | A net of a square based prism. The square cross section has side length 6cm. And the prism has a height of 11cm. The faces that make up the height of the prism are labelled 1,2,3,4 respectively.  | A net of a rectangular prism. The rectangular cross section has dimensions 3cm by 3.5cm. And the prism has a height of 1.7cm.  The rectangle faces that make up the height of the prism are labelled 1,2,3,4. |
| Small rectangle left | Square face top  | Big rectangle top left |
| Small rectangle right (same as the other small rectangle) | Square face bottom | Big rectangle middle right |
| Rectangle 1 | Rectangle 1 | Rectangle 1 |
| Rectangle 2 | Rectangle 2 (same as rectangle 1) | Rectangle 2 |
| Rectangle 3 (same as rectangle 1) | Rectangle 3 | Rectangle 3 |
| Rectangle 4 (same as rectangle 2) | Rectangle 4 | Rectangle 4 |
| Surface area  | Surface area | Surface area  |

#### Faded worked examples – triangular prism 1

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| Triangular prism with height 5cm, Width 12cm, length 9cm and sloping side of 13cm | Triangular prims with base of 16cm by 21cm and sloping sides of 10cm and height of 6cm | A triangular prism. The triangular cross section is an equilateral triangle with side length 8.6cm and a perpendicular height of 7.4cm. The prism has a height of 14.2cm.   |
| Front triangular face | Front triangular face | Front triangular face |
| Back triangular face (same as front) | Back triangular face | Back triangular face |
| Right/slope rectangle face  | Right/slope rectangle face | Right/slope rectangle face |
| Left rectangle face  | Left /slope rectangle face | Left/slope rectangle face |
| Bottom rectangle face | Bottom rectangle face | Bottom rectangle face |
| Surface area | Surface area | Surface area  |

#### Faded worked examples – triangular prism 2

|  |  |  |
| --- | --- | --- |
| Question 1 | Question 2 | Question 3 |
| A net of a right angled triangular prism. The triangular cross section has a base of 4cm and a height of 3cm. The prism has a height of 9cm. The rectangular faces are labelled 1,2,3.  | The net of a triangular prism. The triangular cross sections are isosceles triangles with equal side lengths of 15cm and the remaining side 24cm. The prism has a height of 17cm. The rectangular faces are labelled 1,2,3. | The net of a triangular prism. The triangular cross section is an equilateral triangle with side length 4.6cm. The height of the prism is 9.2cm. The rectangular faces are labelled 1,2,3. |
| Pythagoras theorem (hypotenuse) | Pythagoras theorem (height of triangle) | Pythagoras theorem (height of triangle) |
| Triangular face | Triangular face | Triangular face |
| Rectangle 1 | Rectangle 1 | Rectangle 1 |
| Rectangle 2 | Rectangle 2 | Rectangle 2 (same as rectangle 1) |
| Rectangle 3 | Rectangle 3 (same as rectangle 1) | Rectangle 3 (same as rectangle 1) |
| Surface area | Surface area  | Surface area |

### Appendix D – home office space renovation



1. Walls:

Ceiling:

Paint needed:

 5 L of paint will be required.

1. New roof



**Design 1**

**Design 2**

Therefore, roof design 1 is more cost effective.

## References

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