# Make it clear

Students use the order of operations to evaluate numerical expressions and examine the same expressions with algebraic terms to consider how these rules apply generally.

This lesson is designed based on students having successful previous experience with applying the order of operations to evaluate numerical expressions.

## Visible learning

### Learning intentions

* To understand how order of operations rules apply to algebraic expressions.

### Success criteria

* I can compare numerical and algebraic expressions.
* I can use the order of operations to simplify algebraic expressions.
* I can explain the difference between similar algebraic expressions, such as and .

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* generalises number properties to operate with algebraic expressions including expansion and factorisation **MA4-ALG-C-01**

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## Activity structure

### Warm up

This warm up activity is designed to activate prior knowledge of the rules for the order of operations, which are thoroughly explored in the lesson ‘The order of things’. Students demonstrating proficiency in this skill can move straight to the launch section of the lesson.

The Jigsaw strategy ([bit.ly/jigsawgroupstrategy](https://bit.ly/jigsawgroupstrategy)) asks a group of students to become ‘experts’ and then share that material with another group of students. This strategy offers a way to help students understand and retain information and to develop their collaboration skills.

1. Divide the class into groups of 4. These groups will be the ‘home teams’ of the jigsaw.
2. Explain the strategy and the topic of study: The order of operations.
3. Tell students that they are going to be responsible for revising one rule with the team they are sitting with now.
4. Hand each group a copy of Appendix A ‘Left to right’, Appendix B ‘Multiplication and division before addition and subtraction’, Appendix C ‘Powers before multiplication and division’ and Appendix D ‘Brackets first’.
5. Assign each appendix to a member from each group.
6. Students are to leave their home team to sit with a group of students assigned to the same appendix.

An example of this arrangement would be that if 7 students in the room are holding Appendix A, they could come together to form a group of 4 and a group of 3 for discussion purposes before returning to their home group in step 8.

1. Ask students to begin reading to themselves or have them take turns reading aloud to their group. When students are finished reading, the group should discuss the rule, complete any problems on their copy of the appendix and discuss what they should present to their home teams.
2. When you believe students are ready, send them back to their home team. Remind them that they are going back to their home team to share the rule they have revised.
3. Distribute Appendix E ‘Order of operations summary’ to all students.
4. Advise the experts that they will have approximately 3 minutes to explain their rule and communicate their understanding of the rule to the members of their home team. During this time, they will also need to be checking for understanding with their team members.
5. Give a total of 15 minutes for students to share their understanding of the 4 given rules, record the information and attempt the problem at the bottom of Appendix E.
6. Bring students back and select students at random to share their responses in the table.
7. Collect answers to the expression in Appendix E, , focusing on how the order of operations were applied.

It is important that teachers discuss the interpretation of the rules of the order of operations, but also how they are to be applied with one another. For example, an expression can usually be evaluated by completing the brackets first, followed by powers, before combining the rules from Appendix A and B to complete multiplication and division from left to right and then addition and subtraction from left to right.

### Launch

1. Write the expression on the board.
2. Explain to student that (just as ).
3. Have students engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to discuss the answer to this problem.

Students should be given a short time to determine if they agree or disagree with one another. Ideally some students will see this as while others will see this as .

1. Show students the video ‘6÷2(1+2) =??? (1:20)’ ([bit.ly/YouTubeProblem](https://bit.ly/YouTubeProblem)).

Be sure to focus on the conclusion in the video that both answers could be considered correct, but that this is ambiguous and not what we want.

1. Ask students to consider what simplifies to and have them share their responses and reasoning.

Students are likely to contribute the solution that .

1. If students have not suggested the solution already, write the solution following solution on the board.

1. Challenge students to explain why one solution is more valid than another.
2. Explain to students that it is important that when we write expressions for situations, that the expression is clear as to the order it should be evaluated.

### Explore

1. Write the expression on the board and again have students engage in a Think-Pair-Share to discuss the solution.

Students should complete multiplication first, so that . Students may need to be reminded that we work from left to right, but that multiplications and divisions need to be completed before additions and subtractions.

1. Remove the s from the expression on the board and replace both with the pronumeral , so that it reads .
2. Explain to students that because the replaced a in both places, the expression should work the same.
3. Use a Pause-Pose-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) to gather student contributions to how this should be simplified.

The aim is to compare the working for this expression to the numerical one.
.

1. Compare with and ask students to share what they notice and what they wonder.

If , then and .

1. Give students a copy of Appendix F ‘Generalising the order of operations’. Inform students that the numerical expressions in the first column are related to the algebraic expressions in the third column.
2. Instruct students to evaluate the expressions in the first column and then use the same order to simplify the expression in the third column.

Ideally, organise students into visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)).

1. Collect solutions to the evaluated numerical expressions as a class, focusing on the order the operations are to be completed in.
2. Instruct students to modify any incorrect answers and to consider if this change requires them to adjust the solution to the related algebraic expression.

### Summarise

1. Return to the problem from the launch, writing on the board.
2. Ask students to discuss in a Think-Pair-Share how we could adjust the expression if we want to force the answer to be .

To make the solution , we want the to be completed before the division, so a solution could be , adding in the extra set of brackets.

1. Ask students to again discuss how we could adjust the original expression if we want to force the answer to be .

To make the solution , we want the division to be completed before we multiply and , so a solution could be , where the brackets would be evaluated first but then would be evaluated from left to right.

1. Conclude with students that it is the responsibility of the person writing the expression to avoid ambiguity and to be clear with what they mean.
2. Write the numerical expressions from row 8, and row 9, of Appendix F on the board.
3. Ask students to consider what is different and how they were evaluated differently.

In , operations would have been completed from left to right and the division in the middle would have been completed before the final multiplication, resulting in a solution of . In , the operation in brackets would have been completed first, leading to .

1. Explain the different possible solutions to students.
2. Write the algebraic expressions for row 8, and row 9, underneath their respective numerical expressions.
3. Ask students to consider what is different.
4. Conclude with students that we have used to imply that has already been completed in the order of operations and that if it is being divided, the entire will be divided.
5. Have students select examples from Appendix E to use to write notes to their future self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) regarding how to use the order of operations when simplifying algebraic expressions.

### Apply

1. Distribute copies of Appendix G to students.
2. Discuss the first problem as a class, completing the first example.
3. Ask students to consider the question ‘what if we found out that the number of lollies in a packet was 10? What do we know?
4. In groups of 3, have students complete the remaining 2 problems in the table, writing and simplifying algebraic expressions.

Once students have finished the 2 problems but before revealing solutions, have them evaluate their simplified algebraic expressions based on the number of highlighter pens in a packet being 5, the number of chocolates in a small box being 4 and the number of chocolates in a medium box being 9. Have students check their result against the original problem with these values replacing the unknowns.

1. Select students at random to share their expressions and strategies for simplifying with the class.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Warm up**

* This activity provides revision of the order of operations rules. Students can engage with this part of the lesson in whatever depth the teacher believes benefits access to the remainder of the lesson.

**Launch**

* This activity is open ended, with students able to argue a perspective on what the result will be without a need for a correct answer.

**Explore**

* Students who are not ready to move into algebra could just complete the first column of Appendix F.

**Apply**

* For students who not ready to move into algebra, each scenario could be modified to state the value of the unknown quantity so that the answer can be evaluated.

### Suggested opportunities for assessment

**Warm up and launch**

* Teachers should focus on how students are using the order of operations rules together and use these sections of the lesson to determine whether students are ready to engage with the later sections.

**Explore**

* Appendix F gives evidence of how students apply the order of operations both numerically and algebraically, as well as their ability to relate numerical expressions to similar algebraic ones.

## **Appendix A**

#### Left to right

Complete the examples by evaluating the expressions from left to right, then from right to left and consider what is happening.

|  |  |  |
| --- | --- | --- |
| Expression | Left to right (correct) | Right to left (incorrect) |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Each expression is giving 2 different solutions.

**Conclusion:** We have agreed that so that everyone can get the same solution, we will always work from left to right.

|  |  |
| --- | --- |
| **Example 1:** |  |
| **Example 2:** |  |
| **Example 3:** |  |

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

## **Appendix B**

#### Multiplication and division before addition and subtraction

A teacher has arranged their class into 7 groups. There are 5 groups of 2 students and 2 groups of 3 students. How many students are there in the class?

Beth and Evelyn agreed that this can be written as , because it is 5 groups of 2 and 2 groups of 3.

They then each answered the question as shown below.

|  |  |
| --- | --- |
| Beth (correct) | Evelyn (incorrect) |
| An image from Polypad showing a grid of squares. There is one array of coloured squares that is 5 rows of 2, totalling 10 squares. There is a separate array of squares, with 2 rows of 3 squares, totalling 6 squares.  |  |

Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

Evelyn has completed the expression from left to right. Beth has completed the multiplication first and found the correct answer.

**Conclusion:** Multiplications and divisions must be completed before we evaluate additions and subtractions.

|  |  |  |
| --- | --- | --- |
| **Example 1:** |  | An image from Polypad showing a 4 by 5 array of orange squares with a 2 by 3 set of squares greyed out in the middle, leaving 14 squares around the outside still coloured orange.  |

Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

## **Appendix C**

#### Powers before multiplication and division

I have 2 boxes of chocolates that are in a tray 3 across and 3 down.

James and Olivia agree that this is the same as , because it is 2 groups of 3 by 3.

They then each answered the question as shown below.

|  |  |
| --- | --- |
| Olivia (correct) | James (incorrect) |
| Two orange squares made up of 9 smaller orange squares. |  |

Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

James has completed the expression from left to right. Olivia has completed the power first and found the correct answer.

**Conclusion:** powers need to be completed before we evaluate any operations.

|  |  |  |
| --- | --- | --- |
| **Example 1:** |  | An image from Polypad of three grids of 4 by 4 squares.  |

Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

## **Appendix D**

#### Brackets first

Complete the examples by evaluating the expressions by completing the operations in brackets first, then working from left to right.

|  |  |
| --- | --- |
| Expression | Left to right |
|  |  |
|  |  |
|  |  |
|  |  |

Each expression is giving 2 different solutions. We have agreed that so that everyone can get the same solution, we will always work from left to right.

|  |  |
| --- | --- |
| **Example 1:** |  |
| **Example 2:** |  |

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

## **Appendix E**

#### Order of operations summary

In the space below, listen to your team member and write a summary of the rule.

|  |
| --- |
| **Rule 1: Left to right** |
|  |
| **Rule 2: Multiplication and division before addition and subtraction** |
|  |
| **Rule 3: Powers before multiplication and division** |
|  |
| **Rule 4: Brackets first** |
|  |

1. Use the rules to evaluate .

## Appendix F

### Generalising the order of operations

For each row, evaluate the numerical expression and then use the same order of operations to simplify the related algebraic expression.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Numerical expression | Result | Algebraic expression | Result |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |
| 11 |  |  |  |  |
| 12 |  |  |  |  |
| 13 |  |  |  |  |
| 14 |  |  |  |  |

## Appendix G

### Writing simplified expression

For each situation, write a full algebraic expression and then apply the order of operations to simplify the expression.

|  |  |  |  |
| --- | --- | --- | --- |
| Question | Situation | Algebraic expression | Simplified expression |
| 1 | You have 5 packets of lollies and your friend has 4 packets of lollies. You don’t know how many lollies are in each bag, but you do know it is always the same number. Write an expression for the number of lollies you have altogether.  | Let be the number of lollies in each bag.  |  |
| 2 | You have 5 highlighter pens as well as 2 packets of highlighter pens but you can’t see how many are inside. You also want to lend 3 to your friend. Write an expression for the number of highlighter pens you will have once you lend 3 to your friend.  |  |  |
| 3 | You go to the shops and buy 4 small boxes and 1 medium box of chocolates. You do not know how many chocolates are in each box. You plan to share the medium box with yourself and 2 friends and give 2 small boxes to your brother, keeping the others for yourself. Write an expression for the number of chocolates you will have. |  |  |

## Sample solutions

#### Appendix A – left to right

|  |  |  |
| --- | --- | --- |
| Expression | Left to right (correct) | Right to left (incorrect) |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

#### Appendix B – multiplication and division before addition and subtraction

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

#### Appendix C – powers before multiplication and division

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

#### Appendix D – brackets first

|  |  |
| --- | --- |
| Expression | Left to right |
|  |  |
|  |  |
|  |  |
|  |  |

Practice questions:

|  |  |
| --- | --- |
|  |  |
|  |  |

#### Appendix E – order of operations summary

|  |
| --- |
| **Rule 1: Left to right** |
| Complete operations from left to right. |
| **Rule 2: Multiplication and division before addition and subtraction** |
| Complete all multiplication and divisions before completing additions and subtractions. |
| **Rule 3: Powers before multiplication and division** |
| Complete powers before completing multiplications and divisions. |
| **Rule 4: Brackets first** |
| Complete anything in brackets first. |

1. Use the rules to evaluate .

#### Appendix F – generalising the order of operations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Numerical expression | Result | Algebraic expression | Result |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |
| 11 |  |  |  |  |
| 12 |  |  |  |  |
| 13 |  |  |  |  |
| 14 |  |  |  |  |

#### Appendix G – writing simplified expression

|  |  |  |  |
| --- | --- | --- | --- |
|  | Situation | Algebraic expression | Simplified expression |
| 1 | You have 5 packets of lollies and your friend has 4 packets of lollies. You don’t know how many lollies are in each bag, but you do know it is always the same number. Write an expression for the number of lollies you have altogether. | Let be the number of lollies in each bag. |  |
| 2 | You have 5 highlighter pens as well as 2 packets of highlighter pens but you can’t see how many are inside. You also want to lend 3 to your friend. Write an expression for the number of highlighter pens you will have once you lend 3 to your friend. | Let be the number of highlighter pens in a packet. |  |
| 3 | You go to the shops and buy 4 small boxes and 1 medium box of chocolates. You do not know how many chocolates are in each box. You plan to share the medium box with yourself and 2 friends and give 2 small boxes to your brother, keeping the others for yourself. Write an expression for the number of chocolates you will have. | Let be the number of chocolates in a small box and be the number of chocolates in a medium box.  |  |

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