# Inverse journeys

Students use Grid Algebra to explore the multiplication and division of algebraic terms.

## Visible learning

### Learning intention

* To identify that multiplication is the inverse of division and vice versa.

### Success criteria

* I can identify the inverse of a given number.
* I can use inverses to calculate a variable’s original value.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* generalises number properties to operate with algebraic expressions including expansion and factorisation **MA4-ALG-C-01**
* solves linear equations of up to 2 steps and quadratic equations of the form $ax^{2}=c$ **MA4-EQU-C-01**

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## Activity structure

### Launch

1. Present the following scenario to students:

Would you rather 8 lollies from a vending machine for 25 cents or 62 of the same lollies from a 61 g bag for $1.16?

1. Use a Think-Pair-Share strategy ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) for students to discuss this question before randomly selecting students to share their thoughts.
2. Collate student strategies on the board, looking for similarities and differences. Consider which strategies are more efficient or would be more efficient with larger numbers.

We want students to realise that a multiplicative strategy will be more efficient than an additive strategy.

1. In a Think-Pair-Share, ask students if they could express this information using algebra. For instance, 8 x price A = 0.25 and 62 x price B = 1.16. Students have had some experience of using algebra to represent situations in the ‘Additive strategies’ unit.
2. Teacher to randomly call on pairs to share.

This task is designed to introduce the idea of inverses or opposites.

### Explore

1. Navigate to [gridalgebra.com/free](https://gridalgebra.com/free) and drag the file ‘Inverse journeys – multiplying.json’ onto the grid. For more information on how to use the software, please see video Grid Algebra overview (8:04) ().

Figure 1 – grid showing variables



Image created using the free virtual manipulatives at [Grid Algebra](https://gridalgebra.com/welcome).

1. Explain to students that this grid shows where variables ended up after being taken on a journey through the grid. Ask students if they can work out where the variables started.
2. Use a Think-Pair-Share strategy ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) for students to discuss this question before randomly selecting students to share their thoughts.
3. Test student’s theories by clicking on the inverse button shown below and slowly selecting and dragging the variable to the cell indicated by the students. If they are correct, the variable will return to its original form.

Figure 2 – image displaying the return button on Grid Algebra



Image created using the free virtual manipulatives at [Grid Algebra](https://gridalgebra.com/welcome).

If students have chosen the incorrect cell, click the delete expression button (cross) and then click on the appropriate cells to remove incorrect expressions. Click on the delete expression again to toggle off the feature, before trying again.

1. Repeat with as many variables as students need to feel comfortable with the concept.

Introduce the terminology of ‘inverse’ and explain that addition and subtraction are the inverse of each other. You could refer back to zero pairs, used in the lesson *Negative groups of negatives* and *Sharing, grouping and negating* where 1 and (-1) were considered a zero pair because they cancelled each other out. You could now use the terminology of inverse and explain that zero pairs are the inverse of each other.

1. Students should then complete the worksheet from Appendix A ‘Back to the beginning’.

### Summarise

Students should write notes to their future forgetful self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) to summarise their learning from the explore section.

### Apply

Students begin to informally look at one step equations involving multiplying and dividing. In each example a letter has been taken on a journey, landing on a particular numerical cell. Students are challenged to find the numerical value of the cell that the letter started on.

1. Present the following scenario to students:

A letter went on this journey through the grid ‘$3x$’ and landed on the cell containing the number 9.

1. Challenge students to find the value of the cell the letter started from. (For instance, 3)
2. What if the letter finished on the cell containing the number 6? What about the cell containing the number 3?
3. Navigate to Grid Algebra ([gridalgebra.com/free](https://gridalgebra.com/free)).
4. Watch the video Grid Algebra overview (8:04) (<https://bit.ly/GridAlgebraOverview>) from the 5:40 minute mark to learn how to demonstrate the above scenario using the Grid Algebra software.
5. Students to complete the ‘Where did I come from?’ worksheet in Appendix B.
6. If students have access to devices and the internet, they could complete the Grid Algebra tasks ‘21 Where are the letters?’, ‘22 Where is the letter?’ and ‘18 Solving equations’ available at <https://gridalgebra.com/tasks>.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Teachers could begin by taking a number on a journey, rather than starting with variables.

**Summarise**

* Students may require a scaffold to help them write notes to their ‘future forgetful selves’.

**Apply**

* Students could be challenged by landing on a cell with a fraction or decimal value.
* Students who are struggling with the concept of a variable could work with numbers. For instance, ‘7 was taken on a journey by moving 3 places down and ended up on 21. How could he get back to his 7 cell?’

### Suggested opportunities for assessment

* The worksheets from Appendices A and B could be collected to check for understanding.
* An exit ticket with a question from the lesson could be given to students to check for understanding.
* Teachers should monitor student language during class discussions and pair/shares to check for correct terminology usage and to identify and misconceptions.

## Appendix A

### Back to the beginning

Each letter in the following grids has been taken on a journey. Colour and name the cell where it started its journey.

|  |  |  |
| --- | --- | --- |
| A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the 4th row. The expression 4p is contained. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the 2nd row. The expression 2m is contained. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the first row. The fraction e over 3 is contained. |
| A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the first row. The fraction h over 5 is contained. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the sixth row. The expression 6y is contained. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the third row. The fraction g over 2 is contained. |
| A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the fourth row. The expression 2(2k) is contained. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the sixth row. The expression 2(3r) is contained. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the third row. The fraction q over 2 over 3 is contained. |
|  |  |  |

#### Combining moves

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1** |  |  |  |  |  |
| **2** |  |  |  | $$2(c+2)$$ |  |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **1** |  |  |  |  |  |  |
| **3** |  | $$3b-3$$ |  |  |  |  |
| **4** |  |  |  |  | $$4t+4$$ |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1** |  | $$\frac{g}{4}-1$$ |  |  |  |
| **2** |  |  |  |  | $$\frac{f}{2}+4$$ |
| **4** |  |  |  |  |  |

## Appendix B

### Where did I come from?

Can you find the number value of the cell that each letter started its journey from? The first row has been done for you.

|  |  |  |  |
| --- | --- | --- | --- |
| Journey | Value of final cell | Working | Value of starting cell |
| $$3x$$ | 18 | $$18÷3$$ | 6 |
| $$3x$$ | 15 |  |  |
| $$3x$$ | 12 |  |  |
| $$\frac{x}{3}$$ | 6 |  |  |
| $$\frac{x}{3}$$ | 5 |  |  |
| $$\frac{x}{3}$$ | 4 |  |  |
| $$3x$$ | -15 |  |  |
| $$3x$$ | -12 |  |  |
| $$\frac{x}{3}$$ | -6 |  |  |
| $$\frac{x}{3}$$ | -5 |  |  |
| $$-3x$$ | -15 |  |  |
| $$-3x$$ | -12 |  |  |
| $$2x$$ | 7 |  |  |
| $$\frac{x}{2}$$ | 1.5 |  |  |

## Sample solutions

### Appendix A – back to the beginning

|  |  |  |
| --- | --- | --- |
| A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the 4th row. The expression 4p is contained. The expression p is in the first row. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the 2nd row. The expression 2m is contained. The expression m is in the first row. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the first row. The fraction e over 3 is contained. The expression e is in the third row. |
| A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the first row. The fraction h over 5 is contained. The expression h is in the fifth row. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the sixth row. The expression 6y is contained. The expression y is in the first row. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the third row. The fraction g over 2 is contained. The expression g is in the sixth row. |
| A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the fourth row. The expression 2(2k) is contained. The expression k is in the first row. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the sixth row. The expression 2(3r) is contained. The expression r is in the first row. | A 2 by 6 table layout. The first column contains the numbers 1 to 6. The second column contains blank cells in all but the third row. The fraction q over 2 over 3 is contained. The expression q is in the sixth row. |
|  |  |  |

#### Combining moves

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1** |  | $$c$$ |  |  |  |
| **2** |  |  |  | $$2(c+2)$$ |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1** |  | $$b$$ | $$t$$ |  |  |
| **3** | $$3b-3$$ |  |  |  |  |
| **4** |  |  |  | $$4t+4$$ |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1** |  | $$\frac{g}{4}-1$$ |  |  |  |
| **2** |  |  |  |  | $$\frac{f}{2}+4$$ |
| **4** |  |  | $g$ **and** $f$ |  |  |

### Appendix B – where did I come from?

|  |  |  |  |
| --- | --- | --- | --- |
| Journey | Value of final cell | Working | Value of starting cell |
| $$3x$$ | 18 | $$18÷3$$ | 6 |
| $$3x$$ | 15 | $$15÷3$$ | 5 |
| $$3x$$ | 12 | $$12÷3$$ | 4 |
| $$\frac{x}{3}$$ | 6 | $$6×3$$ | 18 |
| $$\frac{x}{3}$$ | 5 | $$5×3$$ | 15 |
| $$\frac{x}{3}$$ | 4 | $$4×3$$ | 12 |
| $$3x$$ | -15 | $$-15÷3$$ | -5 |
| $$3x$$ | -12 | $$-12÷3$$ | -4 |
| $$\frac{x}{3}$$ | -6 | $$-6×3$$ | -18 |
| $$\frac{x}{3}$$ | -5 | $$-5×3$$ | -15 |
| $$-3x$$ | -15 | $$-15÷-3$$ | 5 |
| $$-3x$$ | -12 | $$-15÷-3$$ | 4 |
| $$2x$$ | **7** | $$7÷2$$ | $$\frac{7}{2} or 3.5$$ |
| $$\frac{x}{2}$$ | **1.5** | $$1.5×2$$ | 3 |

## References

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