# The order of things

Students consider the implications of not using the order of operations and establish the need for rules before applying them. This lesson does not introduce the use of brackets in expressions, instead focusing on the conventions to evaluate powers, multiplication, division, addition and subtraction within a single expression.

## Visible learning

### Learning intentions

* To understand the need for rules around the order of operations.
* To know the rules established to evaluate expressions.

### Success criteria

* I can apply the order of operations to evaluate expressions.
* I can explain how expressions can give multiple results if we don’t apply the order of operations correctly.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* **compares, orders and calculates with integers to solve problems MA4-INT-C-01**
* **represents and operates with fractions, decimals and percentages to solve problems MA4-FRC-C-01**
* **operates with primes and roots, positive-integers and zero indices involving numerical bases and establishes the relevant index laws MA4-IND-C-01**

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Please use the associated PowerPoint *The order of things* to display images in this lesson.

## Activity structure

### Launch

1. Display the following expression on the board for students.
2. Organise students into visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)).
3. Ask students to evaluate this expression, informing them that they will be required to explain how they achieved their solution.
4. Bring the class back together once they have had sufficient time to find a solution.
5. Use a Pose-Pause-Pounce-Bounce questioning strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) to collect solutions from each group and record it on the teacher’s whiteboard. Ask for reasoning as students contribute their solution.

Sample answers include:

* , if students worked from left to right.
* , if students completed multiplication, division, addition and subtraction in that order.
* , if students completed division, multiplication, addition and subtraction in that order.
* , if students correctly applied the order of operations.

Encourage debate without concluding any correct method. If students try to suggest correct methods or strategies, such as ‘you need to do multiplication before addition’, ask them ‘why do we need to do this?’.

1. Use an online calculator, such as the Google scientific calculator ([bit.ly/GoogleSciCalc](https://bit.ly/GoogleSciCalc)) to determine the solution to .

Teachers could also demonstrate using this online scientific calculator ([bit.ly/SciCalc2](https://bit.ly/SciCalc2)) that we cannot reliably get the correct solution on a calculator without a good understanding of how to evaluate the operation. Entering this operation into the calculator from left to right should give the solution, 17.

1. Conclude with students that there are instances where the order we complete a calculation in makes a significant difference.

It is likely at this stage that some students will share their experiences with rules for the order of operations. In this lesson, students are exploring the need for conventions and rules when performing multiple operations while evaluating a single expression. While overarching rules can serve to guide students, there are instances where most tend to fail and as such this lesson does not recommend memorising acronyms.

Examples of acronyms that define the order for operations include BIDMAS – Brackets, indices, division, multiplication, addition and subtraction; PEMDAS – Parentheses, exponents, multiplication, division, addition and subtraction; and BODMAS – Brackets, order, division, multiplication, addition and subtraction. No such acronym is identified or referred to in the Mathematics 7–10 Syllabus. Each of these acronyms suggests defined orders of evaluating multiplication and division within expressions, as well as for addition and subtraction. For example, a student using BIDMAS to evaluate would likely find an incorrect result of by adding before subtracting, while a student using PEMDAS to evaluate an expression such as would incorrectly find a result of by multiplying first.

### Explore

#### The commutative and associative properties

1. Working in their groups of 3, give each student a copy of Appendix A ‘When order matters’ and provide each group with 18 counters to represent their calculations.
2. Have students determine whether the operations in each row give the same or a different answer.
3. Use a Pose-Pause-Pounce-Bounce question strategy to discover what students notice and wonder.
4. Conclude with the entire class that the commutative property applies to addition and multiplication, for instance that and that for any integers and .
5. Conclude also that the commutative property does not apply to subtraction or division, but that repeated subtractions and divisions can be applied in any order, for instance that and that .

#### Left to right

1. Hand groups of students Appendix B ‘Forwards and backwards’.
2. Still working in their groups of 3, have students evaluate and record the result of each expression both forwards and backwards to see if it remains the same.
3. Use a Pose-Pause-Pounce-Bounce question strategy to discover what students notice and wonder.

Students should identify that while some expressions are the same evaluated both ways, some are not.

1. Encourage students to suggest a general rule for when the order we evaluate in matters and interrogate that rule. For example, if students suggest that in any expression that involves 2 different operations the order we evaluate in will matter, look for a counter-example, such as .
2. Conclude with the entire class that since we cannot find a rule for when the answer is the same both ways, we need to decide whether we should work left to right, or right to left, all of the time.
3. Have students type each expression into a calculator to observe that evaluating operations from left to right has been agreed upon by mathematicians.

#### Multiplication and division before addition and subtraction

1. Display the following problem for students.

A teacher has arranged their class into 7 groups. There are 5 groups of 2 students and 2 groups of 3 students. How many students are there in the class?

1. Have students discuss, in their groups, how they could write this total as an expression using the numbers in the problem.

Possible responses from students include or .

1. Conclude with students that is the most sophisticated expression, as it saves space compared to repeated addition and can be literally read as ‘5 groups of 2 and 2 groups of 3’.
2. Have students calculate this total from left to right as we have established in the previous section. Students should find the total to be:
3. Display Figure 1 on the teacher screen. This can be found on slide 2 of the *The order of things* PowerPoint.

Figure 1 – five groups of 2 and 2 groups of 3

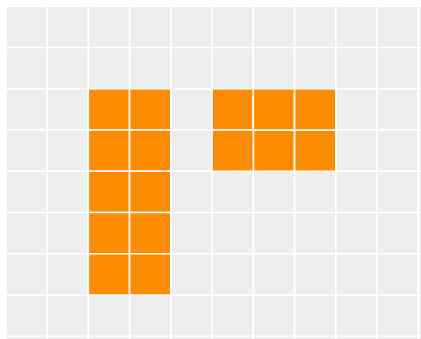


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Explain to students that the image in Figure 1 shows ‘5 groups of 2 and 2 groups of 3.’
2. Ask students to again discuss in their groups the result of from this diagram.
3. Use student responses to establish that they worked out the total in each group first and that clearly and so our previous answer of is incorrect.
4. Conclude with the entire class that we should complete multiplications (and divisions) before we attempt additions (and subtractions).

#### Powers first

1. Display the following problem for students.

I have 2 boxes of chocolates, with 4 rows of 4 chocolates in each box. How many chocolates do I have?

1. Have students discuss in their groups how they could write this total as an expression using the numbers in the problem.

Possible responses from students include or or .

1. Conclude with students that is the most sophisticated expression, as it saves space compared to repeated addition and can be literally read as ‘2 groups of 4 squared’.
2. Have students calculate this total from left to right as we have established in the previous section. Students should find the total to be:
3. Display Figure 2 on the teacher screen. This can be found on slide 3 of the *The order of things* PowerPoint.

Figure 2 – two boxes of chocolates



[This Photo](https://www.foodiggity.com/body-part-chocolates/) by Unknown Author is licensed under [CC BY-NC-ND](https://creativecommons.org/licenses/by-nc-nd/3.0/)

1. Explain to students that the image in Figure 2 shows ‘2 groups of 4 by 4 chocolates’. Have students examine whether this is 64 as we previously found and pick students at random to share their answer and methods.
2. Conclude with the entire class that we should complete powers before we attempt multiplications.
3. Have students evaluate by evaluating the power first, so .

### Summarise

1. Use slides 4–15 from the *The order of things* PowerPoint for explicit teaching of simple order of operations.

The explicit teaching technique used in the associated PowerPoint is ‘Your turn.’ The first slide is a worked example which should be displayed for the students and then use the following steps.

* + - 1. Reveal the question to students and its solution.
      2. Students read in silence.
      3. Students individually think and explain to themselves what is happening in each step.
      4. Students hold a thumbs up to the teacher when they have finished reading and have some sort of understanding.
      5. Think-Pair-Share. Students explain the solution to their partner.
      6. In pairs, students then answer the self-explanation questions.
      7. Finally, randomly select students to share their answers with the whole class.

1. Hand students a copy of Appendix C ‘Order of operations’. Students are required to evaluate a range of expressions by completing multiplications from left to right before completing additions and subtractions from left to right. This resource has been designed using Variation Theory ([variationtheory.com/introduction/](https://variationtheory.com/introduction/)).
2. Have students complete Appendix C individually. Once completed, select students at random to share their responses.

### Apply

#### The numbers game

1. Students are given 6 numbers and are instructed to use the 6 numbers and the 4 operations (powers are not allowed) to obtain a given 3-digit target number. Students may use each of their 6 numbers only once.
2. The 6 numbers are made up of ‘small’ and ‘large’ numbers.
3. ‘Small’ numbers can be anything from 1–10. ‘Large’ numbers can be 25, 50, 75 or 100. The 6 numbers are chosen randomly, but the teacher can allow a student to select a student to choose how many ‘large’ and ‘small’ numbers they want in their 6 numbers.
4. Teachers can randomly obtain their 6 numbers and specific target number at the ‘Maths starters’ website ([bit.ly/MSNumbersGame](https://bit.ly/MSNumbersGame)).

For an illustration of how to play, an episode from the SBS program ‘Letters and Numbers’ can be viewed ([bit.ly/LANEpisode](https://bit.ly/LANEpisode)).

An example would be a target number of 712 with 6 numbers being 75, 25, 10, 5, 10 and 8. A solution would be .

1. Students should be challenged to write their solution in such a way that the order of operations will give the correct target answer.

Solutions can be obtained by entering the target number and the 6 numbers into the ‘Crossword tools’ website ([bit.ly/CTNumbersgame](https://bit.ly/CTNumbersgame)). These solutions will be presented in ways that use brackets to change the order of operations and will need to be considered and possibly edited before being shown to students.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* The emphasis of this activity is not on correct or incorrect answers, but rather different students or calculators finding different solutions. This supports all students to participate.

**Apply**

* The size of the target number can be modified depending on student readiness. Two-digit target numbers or giving students 7 digits to operate with can make the calculations simpler.

### Suggested opportunities for assessment

**Summarise**

* Teachers can collect Appendix C as evidence of students’ ability to interpret numerical expressions and apply the order of operations.

**Apply**

* Teachers can interrogate and record student responses to the Numbers game to assess mathematical communication skills of students. Teachers should verify how students are writing the expression to reach their target number to ensure that the student is using the order of operations correctly when choosing how to write their expression.

## **Appendix A**

### When order matters

|  |  |  |  |
| --- | --- | --- | --- |
| Expression | Rearranged | Rearranged 2 | Is the value the same or different? |
|  |  |  |  |
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1. What do you notice?
2. What do you wonder?

## **Appendix B**

### Forwards and backwards

|  |  |  |  |
| --- | --- | --- | --- |
| Expression | Evaluated left to right | Evaluated right to left | Is the value the same or different? |
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1. What do you notice?
2. What do you wonder?

## Appendix C

### Order of operations

Complete the table below by first evaluating multiplications and divisions in each expression.

|  |  |  |
| --- | --- | --- |
| Expression | Evaluate and from left to right | Evaluate and from left to right |
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**Complete the table below by first evaluating the powers in each expression.**

|  |  |  |  |
| --- | --- | --- | --- |
| Expression | Evaluate powers | Evaluate and from left to right | Evaluate + and from left to right |
|  |  |  |  |
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|  |  |  |  |

## Sample solutions

### Appendix A – when order matters

|  |  |  |  |
| --- | --- | --- | --- |
| Expression | Rearranged | Rearranged 2 | Is the value the same or different? |
|  |  |  | Same. Both equal . |
|  |  |  | Different. The results are and . |
|  |  |  | Same. Both equal . |
|  |  |  | Different. The results are and . |
|  |  |  | Same. All 3 results are equal to . |
|  |  |  | Different. The first 2 results are equal to and the final result is equal to . |
|  |  |  | Same. All 3 results are equal to . |
|  |  |  | Different. The first 2 results are equal to and the final result is equal to . |

### Appendix B – forwards and backwards

|  |  |  |  |
| --- | --- | --- | --- |
| Expression | Evaluated left to right | Evaluated right to left | Is the value the same or different? |
|  |  |  | Same |
|  |  |  | Different |
|  |  |  | Different |
|  |  |  | Same |
|  |  |  | Same |
|  |  |  | Different |

### Appendix C – order of operations

|  |  |  |
| --- | --- | --- |
| Expression | Evaluate and from left to right | Evaluate and from left to right |
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| --- | --- | --- | --- |
| Expression | Evaluate powers | Evaluate and from left to right | Evaluate + and from left to right |
|  |  |  |  |
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