# A shopping plan

Students solve problems involving division of fractions. They compare fractions of foods they use daily with the quantity remaining. This leads into formally representing division of fractions.

## Visible learning

### Learning intentions

* To understand the division of fractions as forming sized groups.
* To be able to represent division of fractions and determine solutions from the representation.

### Success criteria

* I can represent the division of fractions by comparing area models.
* I can explain the division of fractions as forming groups of a certain size.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* **represents and operates with fractions, decimals and percentages to solve problems MA4-FRC-C-01**

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Please use the associated PowerPoint *A shopping plan* to display images in this lesson.

## Activity structure

### Launch

1. Present to students the following scenario, also shown on slide 2 of the PowerPoint *A shopping plan*.

Ivy bakes a muesli bar slice on Monday morning and takes $\frac{1}{12}$ of the slice to work with her each weekday. By next Wednesday night, she has $\frac{1}{3}$ of the slice left.

**Figure 1 – muesli bar slice with only one third left**



Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

How many more $\frac{1}{12}$ pieces can she make?

1. Organise students into visibly random groups ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) of 3 to discuss this problem.
2. Use a Pause-Pose-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) to share student responses.
3. Display Figure 2 and discuss the solution.

Figure 2 – four twelfths of the slice remain



Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Explain to students that when we find how many $\frac{1}{12}$’s we can make from $\frac{1}{3}$ of a slice, we are calculating $\frac{1}{3}÷\frac{1}{12}$ by making specifically sized groups.

The language of ‘sized groups’ is introduced in Lesson 2, ‘Sharing, grouping and negating’ as an approach to viewing division, as opposed to ‘equal sharing’.

1. Ask students to consider in their groups, if $\frac{1}{3}÷\frac{1}{12}=4$, what might $\frac{1}{5}÷\frac{1}{15}$ be equal to?
2. Use a Pause-Pose-Pounce-Bounce question strategy to collect answers from students, encouraging them to explain their thinking and build upon suggestions from other students.

The intention of this discussion is to introduce the idea that we may be able to predict the result of a division of fractions, without formalising any process.

### Explore

1. Hand students Appendix A ‘Ivy’s lunch supplies’.
2. Remaining in their groups of 3, have students use Appendix A to determine and record how many more serves of each product Ivy has left. The overall goal is to determine which product she will need to return to the shops for first.

If possible, have groups work at vertical, non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).

### Summarise

1. Use slides 5–12 from the *A shopping plan* PowerPoint for explicit teaching of dividing fractions.

The explicit teaching technique used in the associated PowerPoint is ‘Your turn.’ The first slide is a worked example which should be displayed for the students and then use the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually think and explain to themselves what is happening in each step.
4. Students hold up a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs, students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. Hand students a copy of Appendix B ‘Representing division of fractions’ and have students follow the given example to represent division of fractions.

For students with access to a device with internet, they can use the Desmos graph ([bit.ly/DesmosDivideFrac](https://bit.ly/DesmosDivideFrac)) to construct representations of the fraction divisions.

1. Have students engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to discuss patterns they notice in the question and again in the solution.

Processes such as inverting the second fraction before multiplying the fractions, or dividing the numerators and the denominators before simplifying can produce solutions. They do, however, run the risk of students losing connection with the visual and real concept of division. Teachers may choose to summarise processes for dividing fractions with students by noticing patterns in step 3 above, encouraging the use of visual representations to explain solutions.

### Apply

#### Open middle problem

* Challenge students to complete the open middle problem displayed on slides 13–14 of the *A shopping plan* PowerPoint. This problem is also displayed in Figure 3 and is inspired by the problem at the ‘Open middle’ website ([bit.ly/OMDivideFrac](https://bit.ly/OMDivideFrac)).

Figure 3 – open middle problem



Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

* Students are to use the integers from 1 to 9 at most once to obtain the largest possible result. Teachers can print this from Appendix C ‘Open middle problem’ if desired.
* Similar open middle problems can be found at the ‘Open middle’ website ([bit.ly/DivideFracOM](https://bit.ly/DivideFracOM)).

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Students can be encouraged to either draw bar models or use counters to represent the quantities Ivy has and will use for each product.

**Summarise**

* The use of the Desmos graph for fraction division supports more students to have access to these activities and the ability to reflect on trends in results.
* Appendix B encourages students to use visual representations to perform fraction division while maintaining connection to the underlying concept.

**Apply**

* The general link to the ‘Open middle’ website ([bit.ly/DivideFracOM](https://bit.ly/DivideFracOM)) includes a range of additional problems, furthering the challenges available.

### Suggested opportunities for assessment

**Explore**

* Student responses to Appendix B provide evidence of their interpretation of division of fractions.

**Summarise**

* Teachers may choose to collect Appendix C as evidence of students’ abilities to represent quantities as fractions, as well as interpreting division of fractions as forming sized groups.

## **Appendix A**

### Ivy’s lunch supplies

For each item, the table displays how much Ivy has left and how much she takes to work each day. Find how many more servings Ivy can have of all her items and record it in the final column of the table.

|  |  |  |  |
| --- | --- | --- | --- |
| Item | Fraction left | Fraction required | How many more servings can Ivy have? |
| Muesli bar slice | $$\frac{1}{3}$$ | $$\frac{1}{12}$$ | $$4$$ |
| Loaf of bread | $$\frac{1}{2}$$ | $$\frac{1}{10}$$ |  |
| Coffee | $$\frac{1}{2}$$ | $$\frac{1}{16}$$ |  |
| Watermelon | $$\frac{5}{6}$$ | $$\frac{1}{6}$$ |  |
| Salad | $$\frac{2}{3}$$ | $$\frac{1}{6}$$ |  |
| Nuts | $$\frac{2}{5}$$ | $$\frac{1}{10}$$ |  |

How many days will it be before Ivy runs out of something and needs to go to the shops?

## **Appendix B**

### Representing division of fractions

|  |  |  |
| --- | --- | --- |
| Calculation | Representation | Solution |
| $$\frac{1}{2}÷\frac{1}{8}$$ | An image of a rectangle that represents one half, divided into two equal pieces and one piece shaded in blue. There is then another identical rectangle underneath, divided into 8 equal parts with 4 shaded red, with each piece representing 1 eighth. The four shaded areas are labelled 1, 2, 3, 4 and underneath it shows that 4 times 1 eighth is equal to 1 half.  |  |
| $$\frac{1}{2}÷\frac{1}{4}$$ | An image of two identical rectangles. The top rectangle is divided in half. The bottom rectangle is divided into 4 equal parts.  |  |
| $$\frac{3}{2}÷\frac{1}{4}$$ | An image of four identical rectangles. The top 2 rectangles are divided in half. The bottom 2 rectangles are divided into 4 equal parts.  |  |
| $$\frac{1}{2}÷\frac{2}{8}$$ | An image of two identical rectangles. The top rectangle is divided in half. The bottom rectangle is divided into 4 equal parts, and then these parts are divided in half by a dotted line.  |  |
| $$\frac{3}{2}÷\frac{1}{6}$$ | An image of 4 identical rectangles, with 2 in line on top and 2 in line on the bottom.  |  |
| $$\frac{3}{2}÷\frac{1}{12}$$ | An image of 4 identical rectangles, with 2 in line on top and 2 in line on the bottom.  |  |
| $$\frac{3}{2}÷\frac{2}{12}$$ | An image of 4 identical rectangles, with 2 in line on top and 2 in line on the bottom.  |  |
| $$\frac{3}{2}÷\frac{1}{10}$$ | An image of 4 identical rectangles, with 2 in line on top and 2 in line on the bottom.  |  |
| $$\frac{16}{10}÷\frac{1}{10}$$ | An image of 4 identical rectangles, with 2 in line on top and 2 in line on the bottom.  |  |
| $$\frac{16}{20}÷\frac{1}{10}$$ | An image of 2 identical rectangles, one directly above the other.  |  |
| $$\frac{16}{20}÷\frac{1}{5}$$ | An image of 2 identical rectangles, one directly above the other.  |  |
| $$\frac{16}{20}÷\frac{2}{5}$$ | An image of 2 identical rectangles, one directly above the other.  |  |
| $$\frac{16}{20}÷\frac{4}{5}$$ | An image of 2 identical rectangles, one directly above the other.  |  |

## **Appendix C**

### Open middle problem

Use the integers from 1 to 9 at most once to make a product of fractions that makes this equation correct.



## Sample solutions

### Appendix A – Ivy’s lunch supplies

|  |  |  |  |
| --- | --- | --- | --- |
| Item | Fraction left | Fraction required | How many more can Ivy have? |
| Muesli bar slice | $$\frac{1}{3}$$ | $$\frac{1}{12}$$ | $$4$$ |
| Loaf of bread | $$\frac{1}{2}$$ | $$\frac{1}{10}$$ | $$5$$ |
| Coffee | $$\frac{1}{2}$$ | $$\frac{1}{16}$$ | $$8$$ |
| Watermelon | $$\frac{5}{6}$$ | $$\frac{1}{6}$$ | $$5$$ |
| Salad | $$\frac{2}{3}$$ | $$\frac{1}{6}$$ | $$4$$ |
| Nuts | $$\frac{2}{5}$$ | $$\frac{1}{10}$$ | $$4$$ |

### Appendix B – representing division of fractions

|  |  |  |
| --- | --- | --- |
| Calculation | Representation | Solution |
| $$\frac{1}{2}÷\frac{1}{8}$$ | An image of a rectangle that represents one half, divided into two equal pieces and one piece shaded in blue. There is then another identical rectangle underneath, divided into 8 equal parts with 4 shaded red, with each piece representing 1 eighth. The four shaded areas are counted 1, 2, 3, 4 and underneath it shows that 4 times 1 eighth is equal to 1 half.  | $$4$$ |
| $$\frac{1}{2}÷\frac{1}{4}$$ | An image of a rectangle that represents one half, divided into 2 equal pieces and one piece shaded in blue. There is then another identical rectangle underneath, divided into 4 equal parts with 2 shaded red, with each piece representing 1 quarter. The 2 shaded areas are counted 1, 2 and underneath it shows that 2 times 1 quarter is equal to 1 half.  | $$2$$ |
| $$\frac{3}{2}÷\frac{1}{4}$$ | An image of 2 rectangles that represent one and one half, divided into 2 equal pieces each and 3 pieces shaded in blue. There is then another identical pair of rectangles underneath, divided into 4 equal parts each, with 6 shaded red, with each piece representing 1 quarter. The 6 shaded areas are counted 1, 2, 3, 4, 5 and 6 and underneath it shows that 6 times 1 quarter is equal to 1 and 1 half.  | $$6$$ |
| $$\frac{1}{2}÷\frac{2}{8}$$ | An image of a rectangle that represents one half, divided into 2 equal pieces and one piece shaded in blue. There is then another identical rectangle underneath, divided into 8 equal parts in four groups of 2, with 2 groups shaded red, with each piece representing 1 eighth and each group representing 2 eighths. The 2 shaded areas are counted 1, 2 and underneath it shows that 2 times 2 eighths is equal to 1 half.  | $$2$$ |
| $$\frac{3}{2}÷\frac{1}{6}$$ | An image of 2 rectangles that represent one and one half, divided into 2 equal pieces each and 3 pieces shaded in blue. There is then another identical pair of rectangles underneath, divided into 6 equal parts each, with 9 shaded red, with each piece representing 1 sixth. The 9 shaded areas are counted 1, 2, 3, 4, 5, 6, 7, 8 and 9 and underneath it shows that 9 times 1 sixth is equal to 1 and 1 half.  | $$9$$ |
| $$\frac{3}{2}÷\frac{1}{12}$$ | An image of 2 rectangles that represent one and one half, divided into 2 equal pieces each and 3 pieces shaded in blue. There is then another identical pair of rectangles underneath, divided into 12 equal parts each, with 18 shaded red, with each piece representing 1 twelfth. The 18 shaded areas are counted and labelled 1-18 and underneath it shows that 18 times 1 twelfth is equal to one and one half.  | $$18$$ |
| $$\frac{3}{2}÷\frac{2}{12}$$ | An image of a pair of rectangles that represent one and one half, divided into 2 equal pieces each with 3 pieces shaded in blue. There is then another identical pair of rectangles underneath, divided into 12 equal parts each in 6 groups of 2, with 9 groups shaded red, with each piece representing 1 twelfth and each group representing 2 twelfths. The 9 shaded areas are counted 1-9 and underneath it shows that 9 times 2 twelfths is equal to one and one half.  | $$9$$ |
| $$\frac{3}{2}÷\frac{1}{10}$$ | An image of 2 rectangles that represent one and one half, divided into 2 equal pieces each and 3 pieces shaded in blue. There is then another identical pair of rectangles underneath, divided into 10 equal parts each, with 15 shaded red, with each piece representing 1 tenth. The 15 shaded areas are counted and labelled 1-15 and underneath it shows that 15 times 1 tenth is equal to one and one half.  | $$15$$ |
| $$\frac{16}{10}÷\frac{1}{10}$$ | An image of 2 rectangles that represent sixteen tenths, divided into 10 equal pieces each and 16 pieces shaded in blue. There is then another identical pair of rectangles underneath, divided into 10 equal parts each, with 16 shaded red, with each piece representing 1 tenth. The 16 shaded areas are counted and labelled 1-16 and underneath it shows that 16 times 1 tenth is equal to 16 tenths.  | $$16$$ |
| $$\frac{16}{20}÷\frac{1}{10}$$ | An image of a rectangle that represents 16 twentieths, divided into 20 equal pieces and 16 pieces shaded in blue. There is then another identical rectangle underneath, divided into 10 equal parts with 8 shaded red, with each piece representing 1 tenth. The 8 shaded areas are counted 1-8 and underneath it shows that 8 times 1 tenth is equal to 16 twentieths.  | $$8$$ |
| $$\frac{16}{20}÷\frac{1}{5}$$ | An image of a rectangle that represents 16 twentieths, divided into 20 equal pieces and 16 pieces shaded in blue. There is then another identical rectangle underneath, divided into 5 equal parts with 4 shaded red, with each piece representing 1 fifth. The 4 shaded areas are counted 1-4 and underneath it shows that 4 times 1 fifth is equal to 16 twentieths.  | $$4$$ |
| $$\frac{16}{20}÷\frac{2}{5}$$ | An image of a rectangle that represents 16 twentieths, divided into 20 equal pieces and 16 pieces shaded in blue. There is then another identical rectangle underneath, divided into 5 equal parts, with the first 2 grouped, the third and fourth grouped and the final area by itself. The 2 groups are shaded red and labelled 1 and 2, with each piece representing 1 fifth and the 2 shaded groups each representing 2 fifths each. Underneath it shows that 2 times 2 fifths is equal to 16 twentieths.  | $$2$$ |
| $$\frac{16}{20}÷\frac{4}{5}$$ | An image of a rectangle that represents 16 twentieths, divided into 20 equal pieces and 16 pieces shaded in blue. There is then another identical rectangle underneath, divided into 5 equal parts, with the first 4 grouped, and the final area by itself. The group is shaded red and labelled 1, with each piece representing 1 fifth and the shaded group each representing 4 fifths. Underneath it shows that 1 times 4 fifths is equal to 16 twentieths.  | $$1$$ |

### Appendix C – open middle problem



## References

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