# Stepping through unknowns

Students use pronumerals to represent unknown distances and add like terms to simplify expressions.

## Visible learning

### Learning intentions

* To be able to simplify algebraic expressions.

### Success criteria

* I can identify like terms.
* I can simplify algebraic expressions by adding and subtracting like terms.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* generalises number properties to operate with algebraic expressions including expansion and factorisation **MA4-ALG-C-01**

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## Activity structure

### Launch

1. Use the *Stepping through unknowns* PowerPoint (Slide 2) to present the following scenario to students:

Johnny and his mother were measuring his height, but they only had a 30 cm ruler. 

Johnny’s mother recorded his height as 30 cm + 30 cm + 30 cm + 30 cm + 30 cm. Johnny told his mum ‘It would have been easier to record it as 5 × 30 cm*’.*

1. Ask students to discuss the following questions in pairs
2. Who is right?
3. Could Johnny’s height also have been written in other ways?
4. Can you draw a diagram to show who is right?

This is a chance to revisit the commutative property of multiplication that says

5 × 30 = 30 × 5.

1. Randomly choose pairs to share their thoughts and diagrams to justify their reasoning.
2. Move to Slide 3 in the *Stepping through unknowns* PowerPoint to present the following scenario.

Johnny realises that the ruler they were using is broken, so they are not sure exactly how long it is.

1. Ask students to discuss ‘*How could we record Johnny’s height now?*’ in their pairs.
2. Call on random pairs to explain their strategy.

Highlight strategies that record the height using pronumerals to represent the length of the ruler. For instance, 5 × ruler or 5 × r.

1. Move to slide 4 in the *Stepping through unknowns* PowerPoint to present the following scenario.

Johnny found out that his Mum rounded off his height to the nearest ruler. He was a bit taller than 5 rulers. They find a smaller object and use it to measure the rest of his height.



1. Ask students to discuss ‘How could we record Johnny’s height now?’ in pairs. They could also consider whether it would have mattered if Johnny’s Mum had used the smaller object first and then finished measuring with the ruler?

We want students to understand that 5 × r + 2 × p = 2 × p + 5 × r.

1. Johnny and his mum finally find an intact ruler and measure the original ruler as being 28 cm long, and the smaller object that they used as 8 cm long.
2. Ask students to discuss ‘How tall is Johnny?’ in their pairs.
3. Ask random pairs to share their strategy for calculating Johnny’s height.

We are looking to see whether students used an additive strategy such as:

* 28 + 28 + 28 + 28 + 28 + 8 + 8

or if they used a multiplicative strategy such as:

* 5 × 28 + 2 × 8.

If students use a mixture of strategies, you could hold a discussion around which strategy is more efficient.

### Explore

In visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)), students will use their individual walking paces to calculate and compare the perimeters of designated areas around the school.

#### Equipment

* Tape measures or meter rulers (one per group of 3)
* Pens
* Appendix A ‘Perimeter recording sheet’, printed (one copy per group)

#### Method

You might like to model the steps below using the example in the *Stepping through unknowns* PowerPoint (slides 5–8), prior to taking students outside.

1. Begin with one student from each group pacing out and recording the length of each side of a predetermined perimeter. The group collectively decides on the best method to document the total perimeter. For example, they might record ‘Perimeter = 125 paces’.
2. With a different perimeter, assign 2 different students to pace out different lengths of the perimeter. Given that each student's pace length will vary, they need to devise a way to record the number of paces for each portion of the perimeter and the total perimeter. For example, they might record

8 × J + 30 × M + 37 × J + 40 × M + 20 × J

This will be the first-time students consider adding like terms. Students should identify that they cannot add Jack and Mary’s paces together as they are different lengths, but they can find the total number of paces that Mary walked and the total number of paces that Jack walked.

For example, 8 × J + 30 × M + 37 × J + 40 × M + 20 × J = 55 × J + 70 × M

1. With a different perimeter, assign different lengths to all 3 members of the group. They need to figure out how to record each individual length and the total perimeter. For example, they might record ‘3 × J + 20 × M + 22 × M + 25 × B + 17 × J’.

Challenge students to consider the order in which they added their side lengths. For instance, would it matter if we re-arranged

3 × J + 20 × M + 22 × M + 25 × B + 17 × J

to

25 × B + 22 × M + 20 × M + 3 × J + 17 × J?

Ask students to draw a diagram to justify their answer.

1. Each student walks 10 paces, records the overall distance (using a tape measure or meter ruler), and then divides this total distance by 10 to determine their average pace length.
2. With their individual pace lengths determined, students use this data to calculate the actual perimeter (in standard units like meters or centimetres) of the regions they previously measured using paces.

This should be done by students at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) in the classroom and is not on the perimeter recording sheet.

### Summarise

1. Use the *Stepping through unknowns* PowerPoint (slides 9–12) to establish the definitions for term, expression, coefficient, pronumeral and simplify.
2. Students will write notes to their future forgetful selves ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) for each definition.
3. Use the worked examples and ‘your turn’ problems to summarise how to represent scenarios and add with like terms (slides 13–18).
4. Students should complete the questions from Appendix B ‘Independent practice’.

### Apply

#### Sorting terms activity

The activities in this lesson are based on ‘Collecting Like Terms – Sorting Terms’ by Chris McGrane (<https://startingpointsmaths.com/2018/09/20/collecting-like-terms-sorting-terms/>).

#### Equipment

* Appendix C ‘Sorting terms’, printed (one copy per pair of students)
* Scissors (one per pair of students).

#### Method

1. Assign students pairs.
2. Print and distribute Appendix C.
3. Students cut out each card.
4. Ask pairs to group the cards together in any way they like. Using a questioning strategy such as Pose-Pause-Pounce-Bounce question strategy ([PDF 200KB] [bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)), ask students to share how they grouped the terms and why.
5. Pose the following challenge to students (this is also on slide 19 of the *Stepping through unknowns* PowerPoint):
6. What do cards 1 and 2 make?
7. What do cards 3 and 4 make?
8. What do cards 1, 2, 3 and 4 make?
9. What do cards 1 and 8 make?
10. What do cards 1, 2 and 8 make?
11. What do cards 1, 3 and 8 make?
12. What do cards 1, 4 and 9 make?
13. What do cards 2, 4, 7 and 9 make?
14. What do cards 1 and 7 make?
15. What do cards 1, 2, 7 and 6 make?
16. What do cards 3, 4, 5 and 6 make?
17. What do cards 5, 6, 7, 8 and 9 make?
18. The following challenge can then be posed (this is also on slide 20 of the *Stepping through unknowns* PowerPoint):
19. Which cards do I require to make ?
20. Which cards do I require to make -2?
21. Which cards do I require to make 0?
22. Which cards do I require to make 7 + 3 +4?
23. Which cards do I require to make an expression with 3 negative terms?

If students argue that they cannot make any of the expressions in question 6, challenge them to convince you why, and then allow them to use the blank card to complete the challenge.

#### Equivalent expressions

1. Display this problem from Open Middle ([www.openmiddle.com/equivalent-expressions-4](http://www.openmiddle.com/equivalent-expressions-4)).
2. Students should work on mini whiteboards ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)).
3. There are multiple answers, so students should compare their answer with a partner and attempt to find all variations.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Assign 3 predetermined perimeters in a common location. This way, students are clear on the activity directions and teacher can supervise, assess, and redirect.
* By combining groups at the end of the activity, students can compare the perimeter they calculated for the same spots and discuss why answers might not be the same, as a precursor to error in measurement.

**Apply**

* The blank card can be used to pose this challenge:

1. Fill in the blank card so that you can make the expression 3 + 3 + 3.
2. How many new expressions can be made with the introduction of the blank card?

### Suggested opportunities for assessment

**Explore**

* If students require additional scaffolding and/or supervision, they could complete a similar activity using smaller objects. For example, how many paperclips is the perimeter of an object.

**Summarise**

* Hinge point questions could be asked using a finger voting strategy to ensure students have understood the concepts from the PowerPoint before commencing the independent practice.
* Students’ notes to their future forgetful selves can be collected or uploaded to an online classroom. The teacher can then assess students’ notes and address misconceptions in the following lesson.

## **Appendix A**

### Perimeter recording sheet

**Perimeter 1**

|  |  |
| --- | --- |
| Perimeter location: |  |
| Total perimeter (paces) by student 1: |  |

**Perimeter 2**

|  |  |  |
| --- | --- | --- |
| Perimeter location: |  |  |
| Student 1: |  | paces |
| Student 2: |  | paces |
| Total perimeter (paces): |  |  |

**Perimeter 3**

|  |  |  |
| --- | --- | --- |
| Perimeter location: |  |  |
| Student 1: |  | paces |
| Student 2: |  | paces |
| Student 3: |  | paces |
| Total perimeter (paces): |  |  |

**Determining pace length**

|  |  |
| --- | --- |
| Student 1 pace length: |  |
| Student 2 pace length: |  |
| Student 3 pace length: |  |

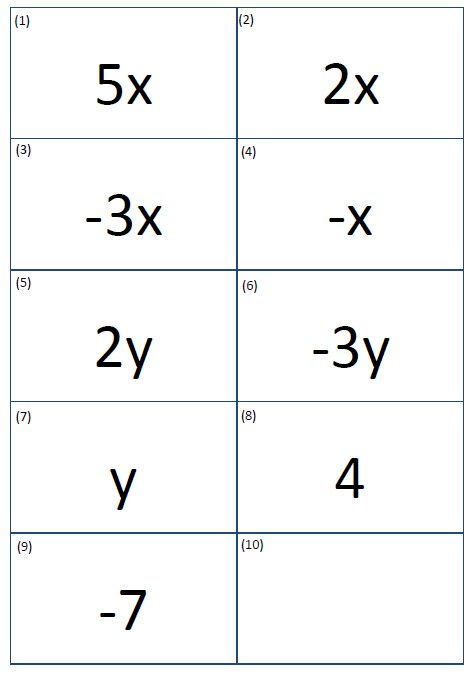
## Appendix B

### Independent practice

|  |  |
| --- | --- |
| Question | Answer |
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## Appendix C

### Sorting terms



## Sample solutions

### Appendix B – independent practice

|  |  |
| --- | --- |
| Question | Answer |
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