# Why fractions with different denominators don’t align

Students recognise the problem that arises when comparing fractions with different denominators.

## Visible learning

### Learning intention

* To understand why fractions with different denominators can’t be added in their current form.

### Success criteria

* I can explain why fractions with the same denominator can be added in their current form.
* I can determine if an expression involving fractions can be fully evaluated in its current form.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* represents and operates with fractions, decimals and percentages to solve problems **MA4-FRC-C-01**

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## Activity structure

Students will need at least one digital device per pair to interact with Desmos during this lesson.

### Warm up

1. Display the following 4 expressions on the board:
2. $\frac{2}{5}+\frac{1}{5}$
3. $\frac{2}{7}+\frac{3}{7}$
4. $1+\frac{2}{3}$
5. $\frac{2}{5}+\frac{1}{7}$
6. Have students determine in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), which expression doesn’t belong ([bit.ly/wodbstrategy](https://bit.ly/wodbstrategy)).
7. After students have had time to discuss their reasoning collect the class’s thoughts on which one doesn’t belong using a finger vote where they could hold up one finger if they thought expression ‘a’ didn’t belong, hold up 2 fingers if they thought expression ‘b’ didn’t belong, 3 fingers for expression ‘c’ and 4 fingers for expression ‘d’.

The purpose of this activity is for students to recall that fractions with the same denominator can be added without change. ‘c’ is the only sum that is greater than one, however students can add the expression by giving the sum as a mixed number of $1\frac{2}{3}$. ‘d’ cannot be added without changing the denominators.

### Launch

1. Write the following list of fractions on the board for students to see:
2. $\frac{1}{3}$
3. $\frac{1}{4}$
4. $\frac{1}{5}$
5. $\frac{2}{7}$
6. Ask students which 2 fractions have a sum closest to $\frac{1}{2}$.
7. Students should consider the problem independently before discussing with a partner.
8. Using a question technique such as Pause-Pose-Pounce-Bounce question strategy ([PDF 200KB] [bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) ask students to share some of their results.

Students have previously compared fractions with different denominators and should be able to represent each of these fractions on a number line. The purpose of this launch is to highlight the headache of trying to describe the sum of 2 fractions with different denominators.

### Explore

1. Direct students to the Desmos activity ‘Adding fractions with the same denominator’ ([bit.ly/desmossamedenominators](https://bit.ly/desmossamedenominators)).
2. Students work through the activity, discussing each slide with their partner.

The purpose of the Desmos activity is to connect students’ use of the number line to add fractions with the same denominator to the bar model used in the following activity.

1. Display the Desmos graph ‘Addition of fractions’ ([bit.ly/desmosaddfractions](https://bit.ly/desmosaddfractions)) which shows a bar model of $\frac{1}{3}$ and $\frac{1}{5}$, followed by a bar model split into thirds that adds these two fractions.
2. Move the switch **Check fraction,** at the bottom left, to demonstrate how $\frac{1}{3}$ and $\frac{1}{5}$ have been added to the bar model at the bottom.



1. Ask students to speak aloud the fraction that is shaded in the bottom bar model. Explain to students that it is challenging to see the result of adding $\frac{1}{3}$ and $\frac{1}{5}$, because the sum cannot be expressed as thirds.
2. Move the switch **Switch partition to 5**,at the bottom and centre, to change the resulting bar model into fifths. Ask students again if they can speak aloud the fraction that is coloured when $\frac{1}{3}$ and $\frac{1}{5}$ are added.



1. Allow students time to independently explore the Desmos graph ‘Addition of fractions’([bit.ly/desmosaddfractions](https://bit.ly/desmosaddfractions)). They should spend time moving the sliders under **Equivalent fractions**. This switch introduces a slider that allows students to convert both $\frac{1}{3}$ and $\frac{1}{5}$ to other equivalent fractions which also changes the bar model.



Depending on the device used, students may need to zoom out and drag the screen to position the graphics so that they do not overlap.

1. Once students have explored this set of fraction additions, either give students 2 more fractions to explore or allow them to select their own to be added. To change the starting fraction, move the switch **Modify fractions** which is on the top right of the page.



1. After students have explored a few pairs of fraction additions, ask students to discuss the following questions in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)):
2. What numbers in the denominator align in the final addition model? Can you explain why?
3. What numbers in the denominators do not align nicely in the final addition model? Can you explain why not?
4. How does the numerator change the diagram?
5. Is it the numerator or the denominator, that determines if the final addition bar model will align?
6. Conduct a class discussion to share the answers to some of the prompting questions using a Pause-Pose-Pounce-Bounce questioning strategy [PDF 200KB] ([bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)).

### Summarise

1. Conclude with students that fractions with different denominators can’t be added in their current form because they don’t divide a whole in the same way.

Develop a definition with your class based on their vocabulary and responses to the previous activities of how and when fractions can be added.

Students should gain an understanding from their previous exploration that only fractions with common denominators can be added.

1. Write the following expressions on the board. Which one doesn’t belong?
2. $\frac{1}{2}+1$
3. $\frac{2}{7}+\frac{3}{7}$
4. $\frac{2}{5}+\frac{1}{9}$
5. $\frac{3}{7}+1+\frac{4}{7}$
6. Students work in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to decide which expression doesn’t belong.
7. Collect results by using either an online poll or using a finger vote, where holding up one finger refers to expression ‘a’ not belonging, 2 fingers refer to expression ’b’, and so on.
8. From this, students should recognise that there is one expression that does not belong, expression ‘c’: $\frac{2}{5}+\frac{1}{9}$, as all the other expressions can be evaluated in one step as they share a common denominator or add to make a whole number.
9. Students write notes to their future forgetful self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) summarising why fractions with different denominators don’t add. Encourage students to use visual representations to enhance their notes.

### Apply

1. Print and distribute Appendix A ‘Can we evaluate?’, one sheet per pair.
2. Students work in pairs, using the Desmos graph as a tool to assist in answering the questions and checking their solutions.

## Assessment and Differentiation

### Suggested opportunities for differentiation

* Students may benefit from revision of Stage 3 multiplicative thinking prior to this lesson.
* Retrieval questions could be posed at the beginning of the lesson to recall adding fractions with the same denominator.
* Students could benefit from experimenting with real measuring cups. Students can pour water from the $\frac{1}{8}$ cup multiple times into the $\frac{2}{3}$ cup to verify that it will not fill exactly.
* Some students will benefit from using manipulatives to verify that fractions with different denominators don’t align. For example, comparing pieces from a fraction kit.
* Encourage students to convert their improper fractions into mixed numbers to better understand the size of their total fraction.

### Suggested opportunities for assessment

* Through the 2 ‘Which one doesn’t belong’ activities, teachers can informally assess students’ understandings, vocabulary and reasoning.
* The questions asked in Appendix A can be used to assess students’ fluency and reasoning when operating with fractions.

## Appendix A

### Can we evaluate?

1. State whether each of these expressions can be fully evaluated in its current form. If it can be, then evaluate it fully.
2. $\frac{1}{3}+\frac{2}{3}$
3. $\frac{3}{4}+\frac{1}{7}$
4. $\frac{1}{4}+\frac{2}{4}+\frac{3}{4}$
5. $1-\frac{1}{5}$
6. $\frac{3}{8}+\frac{4}{7}+\frac{5}{8}$
7. $\frac{4}{7}-\frac{3}{8}+\frac{5}{8}$
8. $-\frac{4}{7}-\frac{7}{8}-\frac{1}{8}$
9. Fill in the missing numerators and denominators in each fraction so that the expressions can be simplified to one term.
10. $\frac{}{5}+\frac{}{7}+\frac{}{}$
11. $\frac{}{7}+\frac{}{5}-\frac{}{}$
12. $\frac{}{8}-\frac{}{3}+\frac{}{}$
13. $\frac{}{3}-\frac{}{8}-\frac{}{}$
14. Find a different way of completing each fraction in question 2.

## **Suggested solutions**

### Appendix A

1. 1
2. Can’t be evaluated in current form.
3. $\frac{6}{4}=1\frac{1}{2}$
4. $\frac{4}{5}$
5. $1\frac{4}{7}$
6. Can’t be evaluated in current form.
7. $-1\frac{4}{7}$
8. $\frac{5}{5}+\frac{1}{7}+\frac{2}{7}$
9. $\frac{2}{7}+\frac{3}{5}-\frac{3}{5}$
10. $\frac{3}{8}-\frac{1}{3}+\frac{4}{3}$
11. $\frac{3}{3}-\frac{5}{8}-\frac{3}{8}$
12. Various answers where pairs of fractions equate to an integer.

## References

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