# Bonuses and penalties

Students learn to subtract integers in the context of receiving bonuses and penalties. Students explore the concept that receiving a bonus can be considered equivalent to removing a penalty. Students model contexts using black and red counters to model positive and negative numbers.

## Visible learning

### Learning intentions

* To subtract integers with or without the aid of a visual representation.
* To construct a directed number sentence to model a situation.

### Success criteria

* I can use counters to represent positive and negative numbers.
* I can use counters to subtract directed numbers.
* I can write a directed number sentence for a given situation.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly MAO-WM-01
* compares, orders and calculates with integers to solve problems **MA4-INT-C-01**

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## Activity structure

### Launch

1. Display slide 2 from the *Bonuses and Penalties* PowerPoint or write the following on the board.

Figure 1 – equations

 

1. In pairs, students should discuss what they notice and what they wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)).
2. Select random pairs to share their thoughts.
3. Challenge students to consider how they might represent each question using counters.
4. Challenge students to think of a scenario for each one. For example, ‘I have $4 in my bank account, and I deposit another $2’.
5. Explain to students that in this learning episode we will be exploring scenarios similar to the last question 4 − (-1) = 5.

### Explore

1. Verbally explain the following scenario to students. ‘Avery earns a flat rate of $5 pocket money a week. She can then earn ‘bonus’ money by doing extra jobs around the house but can also receive ‘penalties’ for forgetting to complete jobs. If she does a job particularly well or does a job that is not her responsibility, her mother will sometimes remove penalties.’
2. Students will play the game *‘*Bonuses and penalties’as outlined below*.*

#### Bonuses and penalties

##### Equipment:

* 1 set of cards from Appendix A ‘Bonus and penalty cards’ per group.
* A collection of red and black counters (these can be physical counters, virtual counters using Polypad ([mathigon.org/polypad#number-cards](https://mathigon.org/polypad#number-cards)), or hand drawn on paper).

##### Game rules:

1. Students play in randomly selected groups of 3.
2. Each student begins with 5 black and 5 red counters.
3. Students shuffle the cards and place them face down in the centre of the group.
4. The first player selects a card and follows the card instructions, adjusting their counters accordingly. For example, ‘you vacuumed the house, 2 bonuses’ means that the player takes 2 black counters from the bank, while ‘you forgot to feed the dog, 1 penalty’ means you take a red counter.
5. At the end of the game, each player receives $1 for each black counter and loses $1 for each red counter they have. The player with the most money wins.

This game has been adapted from Bonuses and penalties [PDF 119KB] ([bit.ly/nzmathsbonuses](https://bit.ly/nzmathsbonuses)) by [NZ Maths](https://nzmaths.co.nz/).

1. After students have finished playing, lead a class discussion so students can share anything they noticed during the game. Prompts could include:
* What did you do if you had no penalties to remove?

The game is designed to demonstrate that receiving a bonus is equivalent to removing a penalty. For instance, adding on 2 is equivalent to subtracting (-2).

### Summarise

1. Ask students to model the following questions using red and black counters:
2. (-4) − (-2)
3. (-4) + 2
4. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to discuss why each question gives the same answer.
5. Pick students at random to share their thoughts.
6. Ask students to model the following question:
7. 4 − (-2)
8. Use slides 3–10 from the PowerPoint *Bonuses and penalties* to demonstrate how we deal with this situation.
9. Students are to complete the questions from Appendix B *‘*Using zero pairs’.

### Apply

#### Connect 3

1. Students are to play the NRICH game ‘Connect 3’ ([bit.ly/nrichconnectthree](https://bit.ly/nrichconnectthree)).

**Equipment**

* Device with access to ‘Connect 3’ interactive game ([bit.ly/nrichconnectthree](https://bit.ly/nrichconnectthree)).

or

* Printable game board, available at the website.
* Two spinners, one with the numbers 1, 2, 3, -4, -5, -6 and the second with the numbers -1. -2. -3, 4, 5, 6.
* Counters

**Method**

1. Students play in pairs. Player 1 spins both spinners.
2. They choose what order to place the numbers in and whether to add or subtract them to produce one of the totals shown on the board.
3. They cover their selected total with a counter.
4. Player 2 then takes their turn according to the rules above.
5. The winner is the first person to place 3 of their counters in a straight line.
6. Challenge students to consider the following questions:
7. Some numbers can only be made in one way, but some can be made in many different ways.
8. Can you work out the number of different ways of achieving each of the different totals?
9. Does this influence the way in which you might choose to play the game?

## Assessment and differentiation

### Suggested opportunities for differentiation

Counters are used throughout this learning episode to assist in developing conceptual understanding. Students should be encouraged to continue using counters for as long as necessary.

**Apply – integer Venns**

* Two versions of this task have been created. The second, uses the same initial criteria from the two-circle version, but adds in an additional criteria to increase the complexity of the task.
* Students will find it easiest to fill sections A, B and C where they only need to satisfy one condition. They can then move on to D, E and F before considering sections G and H.
* Higher achieving students should be challenged to try and make as few changes to their sums between regions as possible.
* Students could be challenged to create their own criteria for an Integer Venn, complete with sample solutions.

### Suggested opportunities for assessment

* Teachers should monitor student discussions and answers during class to assess their understanding and use of correct terminology.
* Teachers could choose to ask students to complete (using visual representations if necessary) an exit ticket containing a subtraction problem involving negative numbers.
* Teachers could choose to collect the Appendix B or C to check for understanding.

Appendix A

### Bonus and penalty cards

|  |  |  |
| --- | --- | --- |
| You cooked dinner for the family**2 bonuses** | You tidied your bedroom**1 bonus** | You slept in**1 penalty** |
| You were kicking a ball and broke a window**5 penalties** | You babysat your little brother**Remove 2 penalties** | You put the bin out**1 bonus** |
| You cleaned up the kitchen**Remove 1 penalty** | You missed the bus**1 penalty** | You got 100% in your maths test**Remove 3 penalties** |
| You emptied the dishwasher**2 bonuses** | You mowed your neighbour’s lawn**Remove 2 penalties** | You missed your music lesson**3 penalties** |
| You walked the dog**1 bonus** | You completed your homework early**Remove 1 penalty** | You spent your lunch money on lollies**2 penalties** |
| You did the ironing**2 bonuses** | You lost your phone**4 penalties** | You were put in detention at school**2 penalties** |

## Appendix B

### Using zero pairs

|  |  |  |
| --- | --- | --- |
| Question | Representation using counters | Answer |
| 5 + 1 |  |  |
| 5 − (-1) |  |  |
| 1 − (-5) |  |  |
| 3 + 5 |  |  |
| 3 − (-5) |  |  |
| 5 − (-3) |  |  |
| (-5) + 6 |  |  |
| (-5) − (-6) |  |  |
| (-6) − (-5) |  |  |
| (-6) + 5 |  |  |
| (-2) − (-3) |  |  |
| (-2) + 3 |  |  |
| (-3) − (-2) |  |  |
| (-3) + 2 |  |  |

## Appendix C

### Integer Venns

Think of a 2 number subtraction sum that could belong in each region. If you think a region is impossible to fill, convince me why!



Think of a 2 number subtraction sum that could belong in each region. If you think a region is impossible to fill, convince me why!



## Sample solutions

### Appendix B – using zero pairs

|  |  |  |
| --- | --- | --- |
| Question | Representation using counters | Answer |
| 5 + 1 | A column of five black tiles with "1" on them, followed by a + symbol and then one black tile with a "1" on it.  | 6 |
| 5 − (-1) | A column of five black tiles with "1" on them followed by a column of two tiles, the top one red with "-1" on it and the bottom one black with "1" on it.   Arrow pointing from left to right.  A column of five black tiles with "1" on them and one black tile next to this with "1" on it.  | 6 |
| 1 − (-5) | One black tile with "1" on it, followed by 10 tiles in 2 columns of 5. The first column has five red tiles with "-1" on them and then second column has five black tiles with "1" on them.   Arrow pointing from left to right.  One black tile with "1" on it followed by five black tiles in one column with "1" on them.  | 6 |
| 3 + 5 | One column of three black tiles each with "1" on them, followed by a + symbol. Next to this is one column of five black tiles each with "1" on them.  | 8 |
| 3 − (-5) | One column of three black tiles each with "1" on them, followed by a minus symbol. Next to this is two columns of five. The first column is five red tiles each with "-1" on them and the second column is five black tiles each with "1" on them.   Arrow pointing from left to right  A column of three black tiles each with "1" on them, followed by a column of five black tiles each with "1" on them.  | 8 |
| 5 − (-3) | A column of five black tiles each with "1" on them, followed by a minus symbol. Next to this is two columns of three. The first column has three red tiles with "-1" on them and the second column has three black tiles each with "1" on them.   Arrow pointing from left to right.  One column with five black tiles each with "1" on them, followed by one column of three black tiles each with "1" on them.  | 8 |
| (-5) + 6 | One column of five red tiles each with "-1" on the, followed by a + symbol. Next to this is one column of six black tiles each with "1" on them.   Arrow pointing from left to right.  One column of six tiles, the first five are grey with "0" on them and the last one is black with "1" on it.  | 1 |
| (-5) − (-6) | One column of five red tiles each with "-1" on them, followed by two tiles side by side. The tile on the left is red with "-1" on it and the tile on the right is black with "1" on it.   Arrow pointing from left to right.  A black tile with "1" on it.  | 1 |
| (-6) − (-5) | One column of six red tiles each with "-1" on them.   Arrow pointing from left to right.  One red tile with "-1" on it.  | (-1) |
| (-6) + 5 | One column of six red tiles each with "-1" on them. followed by one column of five black tiles each with "1" on them.   Arrow pointing from left to right.  A column of six tiles, then first is red with "-1" on it and the remaining five are grey with "0" on them.  | (-1) |
| (-2) − (-3) | A column of red minus ones. On the right is a column with a red minus 1 at the top and a black minus 1 at the bottom.  Arrow pointing from left to right.  Black counter labelled as 1. | 1 |
| (-2) + 3 | One column of two red tiles with "-1" on them, followed by one column of three black tiles each with "1" on them.   Arrow pointing from left to right.  One column of three tiles, the first tile is black with "1" on it and the last two tiles are grey with "0" on them.  | 1 |
| (-3) − (-2) | One column of three red tiles each with "-1" on them.   Arrow pointing from left to right.  One red tile with "-1" on it.  | (-1) |
| (-3) + 2 | One column of three red tiles each with "-1" on them, followed by one column of two black tiles with "1" on them.   Arrow pointing from left to right.  One column of three tiles, the first tile is red with "-1" on it and the remaining two tiles are grey with "0" on them.  | (-1) |

Images created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

### Appendix C – integer Venns

#### Two circle Venn

Region A: 7 − (-10) = 17

Region B: (-5) + 2 = (-3)

Region C: (-2) − (-5) = 3

Region D: (-6) + 4 = (-2)

#### Three circle Venn

Region A: 7 − (-10) = 17

Region B: (-5) + 2 = (-3)

Region C: (-7) + (-4) = (-11)

Region D: (-2) − (-5) = 3

Region E: (-5) − 6 = -11

Region F: 7 − (-4) = 11

Region G: (-5) − (-6) = 1

Region H: (-6) + 4 = (-2)

## References

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