# Banking troubles

Students learn to add integers in the context of depositing and withdrawing or transferring money. Students model addition contexts using black and red counters to model positive and negative numbers.

## Visible learning

### Learning intentions

* To add integers with or without the aid of a visual representation.
* To construct a directed number sentence to model a situation.

### Success criteria

* I can use counters to represent positive and negative numbers.
* I can use counters to add directed numbers.
* I can write a directed number sentence for a given situation.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* compares, orders and calculates with integers to solve problems **MA4-INT-C-01**

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## Activity structure

### Launch

1. Verbally present the following scenario to students.

Mandy’s banking app on her phone is playing up and will only let her deposit $2 at a time or transfer or withdraw $3.

1. Working in visibly random groups of 3, at a vertical non-permanent surface ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)), ask students to find what transactions she can make to end up with $5 in her account.
2. Extending prompts could include:
3. Can she ever get back to having no money in her account?
4. Are there totals that she couldn’t have in her account?
5. Lead a class discussion around solutions. Ask students to share how they represented a deposit and a transfer or withdrawal. Discuss any similarities between student representations as well as the advantages and disadvantages of each.

### Explore

1. Explain to students that historically bookkeepers wrote deposit amounts in black ink and wrote withdrawal amounts or losses in red ink.
2. Explain to students that we will be using black counters to represent deposits and red counters to represent transfers or withdrawals.

Figure 1 – black counter representing deposits and red counter representing withdrawals or transfers



Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Ask students to represent their transactions from the launch using black and red counters. Students can be challenged to additionally represent these transactions using positive and negative numbers. For example, to make $5 students could show:

Figure 2 – four deposits of $2 and one withdrawal of $3



Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

A focus on correct terminology is required throughout this unit of learning. Students should be encouraged to read negative numbers as ‘negative 3’ and not ‘minus 3’. The terms ‘plus’ and ‘minus’ should be reserved for operations. For example, 2 - (-3) should be read as ‘2 minus negative 3’.

1. Using a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to discuss how their counters can be simplified to show the $5 final total.

During the Think-Pair-Share listen out for language such as ‘cancelling out’ or ’zero pairs’.

1. Introduce the concept of zero pairs (if it doesn’t come up in student strategies), by asking students to consider what happens if you have $1 in the bank and then you withdraw $1. Or, if you borrow $1 from a friend and then pay back the $1.
2. Demonstrate how zero pairs can be used to simplify the launch problem. The diagram in Figure 3 could be drawn on the board or modelled using Polypad Algebra Tiles ([[mathigon.org/polypad#algebra-tiles](https://mathigon.org/polypad#algebra-tiles)](https://mathigon.org/polypad#algebra-tiles)).

Figure 3 – red and black counters being paired up to show zero pairs



Figure 4 – zero pairs have been replaced with zeroes



Images created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

When using Polypad algebra tiles, if you drag a black tile on top of a red tile, they are replaced with a zero tile.

### Summarise

1. Students are to complete worksheet in Appendix A ‘Adding integers’. They will need to represent each scenario by drawing counters and writing an appropriate mathematical expression, before calculating a final total.
2. On completion, discuss with students what they noticed and what they wonder.

This activity is designed to show that the commutative rule of addition still applies with negative numbers. For example, 3 + (-2) = (-2) + 3.

### Apply

#### Activity 1 – integer wars

In pairs, students will play ‘Integer wars’.

##### Equipment:

* One deck of cards per group.
* One pencil and paper per group.

##### Game rules:

1. Using the Ace (1) to 10 cards from a standard deck of cards, each player draws 2 cards from the deck. Red cards are negative and black cards are positive.
2. Each player adds their 2 cards together.
3. The player who has the largest total wins a point.
4. The game continues until someone reaches a score of 10.

#### Activity 2 – integer magic squares

Students complete the ‘Integer magic squares’ activity in Appendix B. All rows, columns and diagonals add to the same total.

* Students can be challenged to create magic squares that use negative fractions and/or decimals.
* Students could be further challenged by writing an algebraic equation for each row, column or diagonal of the magic square, using a variable to represent the blank cell. For example, (-4) + $x$ + 0 = (-3).

#### Activity 3 – integer Venns

Students complete the ‘Integer Venns’ activity in Appendix C. Students are asked to create a 2-number addition sum that satisfies each section of the Venn diagram. If they believe a section is impossible to fill, they need to justify their reasoning.

* Two versions of this task have been created. The second uses the same initial criteria from the 2-circle version but adds in criteria to increase the complexity of the task.
* Students will find it easiest to fill sections A, B and C where they only need to satisfy one condition. They can then move on to D, E and F before considering sections G and H.
* Higher achieving students should be challenged to try and make as few changes to their sums between regions as possible.
* Students could be challenged to create their own criteria for an integer Venn, complete with sample solutions.

## Assessment and differentiation

### Suggested opportunities for differentiation

Counters are used throughout this learning episode to assist in developing conceptual understanding. Students should be encouraged to continue using counters for as long as necessary.

**Apply – integer wars**

* Students should be encouraged to continue to draw diagrams to assist them with calculating their totals during the ‘Integer wars’ game.
* Students can be challenged by using the Jack, Queen and King cards to represent values of 11, 12 and 13.
* Students could be challenged by drawing and adding more than 2 cards at a time.

**Apply – magic squares**

* Challenge students to create a magic square that includes decimals or fractions as well as negative numbers.
* Students could be challenged to write an algebraic equation for each row, column or diagonal of the magic square, using a variable to represent the blank cell. For example, (-4) + $x$ + 0 = (-3)

**Apply – integer Venns**

* Two versions of this task have been created. The second uses the same initial criteria from the two-circle version, but adds in an additional criteria to increase the complexity of the task.
* Students will find it easiest to fill sections A, B and C where they only need to satisfy one condition. They can then move on to D, E and F before considering sections G and H.
* Higher achieving students should be challenged to try and make as few changes to their sums between regions as possible.
* Students could be challenged to create their own criteria for an integer Venn, complete with sample solutions.

### Suggested opportunities for assessment

* Teachers should monitor student discussions and answers during class to assess their understanding and use of correct terminology.
* Teachers could choose to ask students to complete (using visual representations if necessary) an exit ticket containing an addition problem involving negative numbers.
* Teachers could choose to collect the Adding integers, Integer magic squares or Integer Venns activities to check for understanding.

## **Appendix A**

### Adding integers

1. Represent each scenario using counters and as a mathematical expression.

|  |  |  |  |
| --- | --- | --- | --- |
| Scenario | Representation using counters | Mathematical expression | Answer |
| A deposit of $7 followed by a deposit of $2 | A mathematical algorithm using tiles, showing seven black tiles grouped all with "1" on them then a plus symbol and two black tiles grouped with a "1" on them.  | $7 + $2 | $9 |
| A deposit of $5 followed by a withdrawal of $3 | 5 black tiles with "1" on them in one column, followed by a + symbol then three red tiles with a "-1" on them in one column.  |  |  |
| A withdrawal of $3 followed by a deposit of $5 |  |  |  |
| A deposit of $3 followed by a withdrawal of $5 |  |  |  |
| A withdrawal of $5 followed by a deposit of $3 |  |  |  |
| A withdrawal of $1 followed by a withdrawal of $3 |  |  |  |
| A withdrawal of $3 followed by a withdrawal of $1 |  |  |  |
| A withdrawal of $4 followed by a withdrawal of $2 followed by a withdrawal of $5 |  |  |  |
| A withdrawal of $2 followed by a withdrawal of $4 followed by a withdrawal of $5 |  |  |  |
| A withdrawal of $5 followed by a withdrawal of $2 followed by a withdrawal of $4 |  |  |  |
|  |  | 2 + (-2) |  |
|  |  | (-2) + 2 |  |
|  |  | 6 + (-2) |  |
|  |  | (-2) + 6 |  |

1. What do you notice? What do you wonder?
2. How many different ways can you find to end up with a final total of $2?

## Appendix B

### Integer magic squares

1. Complete each of the magic squares by filling in numbers so that each row, column and diagonal adds up to the same total.



1. Create your own magic squares and then share with a friend to complete.

## Appendix C

### Integer Venns

Think of a 2 number addition sum that could belong in each region. If you think a region is impossible to fill, convince me why!



Think of a 2 number addition sum that could belong in each region. If you think a region is impossible to fill, convince me why!



## Sample solutions

### Appendix A – adding integers

|  |  |  |  |
| --- | --- | --- | --- |
| Scenario | Representation using counters | Mathematical expression | Answer |
| A deposit of $7 followed by a deposit of $2 | A mathematical algorithm using tiles, showing seven black tiles grouped all with "1" on them then a plus symbol and two black tiles grouped with a "1" on them.  | $7 + $2 | $9 |
| A deposit of $5 followed by a withdrawal of $3 | 5 black tiles with "1" on them in one column, followed by a + symbol then three red tiles with a "-1" on them in one column.  | 5 + (-3) | $2 |
| A withdrawal of $3 followed by a deposit of $5 | Three brown tiles in a column that all have "-1" on them, followed by a + symbol and then five black titles that all have "1" on them.  | (-3) + 5 | $2 |
| A deposit of $3 followed by a withdrawal of $5 | Three black tiles in a column all with "1" on them, followed by a + symbol and then five brown tiles all with "-1" on them in a column.  | 3 + (-5) | $(-2) |
| A withdrawal of $5 followed by a deposit of $3 | Five brown tiles all with "-1" on them in a column, followed by a + symbol and then three black tiles in a column all with "1" on them.  | (-5) + 3 | $(-2) |
| A withdrawal of $1 followed by a withdrawal of $3 | One brown tile with "-1" on it, followed by a + symbol, and then three brown tiles in a column all with "-1" on them.  | (-1) + (-3) | $(-4) |
| A withdrawal of $3 followed by a withdrawal of $1 | Three brown tiles in a column all with "-1" on them, followed by a + symbol and then one brown tile with "-1" on it.  | (-3) + (-1) | $(-4) |
| A withdrawal of $4 followed by a withdrawal of $2 followed by a withdrawal of $5 | Four brown tiles in a column all with "-1" on them, followed by a plus symbol. Then two brown tiles in a column all with "-1" on them, followed by a plus symbol. And finally five brown tiles all with "-1" on them in a column.  | (-4) + (-2) + (-5) | $(-11) |
| A withdrawal of $2 followed by a withdrawal of $4 followed by a withdrawal of $5 | Two brown tiles in a column all with "-1" on them, followed by a + symbol. Then five brown tiles in a column all with "-1" on them, followed by a + symbol. Then four brown tiles all with "-1" on them in a column.  | (-2) + (-5) + (-4) | $(-11) |
| A withdrawal of $5 followed by a withdrawal of $2 followed by a withdrawal of $4 | Five red tiles in a column all with "-1" on them, followed a + symbol. Then two red tiles in a column all with "-1" on them, followed by a + symbol and then four red tiles all with "-1" on them in a column.  | (-5) + (-2) + (-4) | $(-11) |
| A deposit of $2 followed by a withdrawal of $2 | Two black tiles with "1" on them in a column, followed by a + symbol and then two brown tiles with "-1" on them in a column.  | 2 + (-2) | $0 |
| A withdrawal of $2 followed by a deposit of $2 | Two brown tiles with "-1" on them in a column, followed by a + symbol and then two black tiles with "1" on them in a column.  | (-2) + 2 | $0 |
| A deposit of $6 followed by a withdrawal of $2 | Six black tiles in a column with "1" on them, followed by a + symbol and then two brown tiles in a column with "-1" on them.  | 6 + (-2) | $4 |
| A withdrawal of $2 followed by a deposit of $6 | Two brown tiles with "-1" on them, followed by a + symbol and then six black tiles in a column with "1" on them.  | (-2) + 6 | $4 |

### Appendix B – integer magic squares

**-6**

### Appendix C – integer Venns

#### Two circle Venn

Region A: (-10) + 7 = (-3)

Region B: (-5) + 7 = 2

Region C: (-5) + (-2) = (-7)

Region D: (-3) + 5 = 2

#### Three circle Venn

Region A: (-10) + 7 = (-3)

Region B: (-5) + 7 = 2

Region C: 7 + 4 = 11

Region D: (-5) + (-2) = (-7)

Region E: (-5) + 6 = 1

Region F: (-6) + 5 = (-1)

Region G: (-6) + (-5) = (-11)

Region H: 0 + 4 = 4

## References

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