# Comparing using place value

Students use concrete and online manipulatives, as well as visual representations, to compare common measurements in decimal form, placing them on a number line.

## Visible learning

### Learning intention

* To understand the relationship between place values of a decimal number.
* To be able to place decimals on a number line.

### Success criteria

* I can compare decimals using place value.
* I can represent a decimal to thousandths.
* I can place decimals on a number line to compare their size.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* represents and operates with fractions, decimals and percentages to solve problems **MA4-FRC-C-01**

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## Activity structure

Please use the associated PowerPoint Comparing using place value to display images in this lesson.

### Warm up

#### Equipment

* Appendix A ‘Weights of object cards’ cut into cards (one per student)
* Appendix B ‘Reasoning when comparing integers’ (one per pair of students)

##### Optional

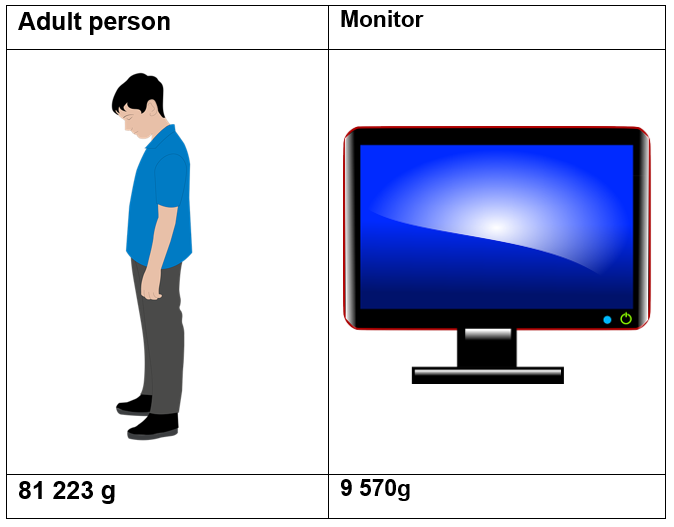
* Sets of base 10 blocks (one per pair of students)
* Device with internet access (one per pair of students)

#### Method

##### Base 10 blocks or Polypad

1. Hand students copies of Appendix A ‘Weights of object cards’ cut into cards. Have students individually organise the weights of these objects in order from lightest to heaviest, without sharing their results.
2. Display Figure 1 and have students engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to give a reason why one weight is heavier than the other.

Figure 1 – adult weight compared to weight of a monitor



Students are likely to discuss comparing the place value digits that are furthest to the left.

1. Hand students copies of Appendix B ‘Reasoning when comparing integers’.
2. Demonstrate to students how to use base 10 blocks to represent and compare the weights of the Smart phone and the textbook from Appendix A ‘Weights of object cards’.

Figure 2 – representation of the weight of a smart phone

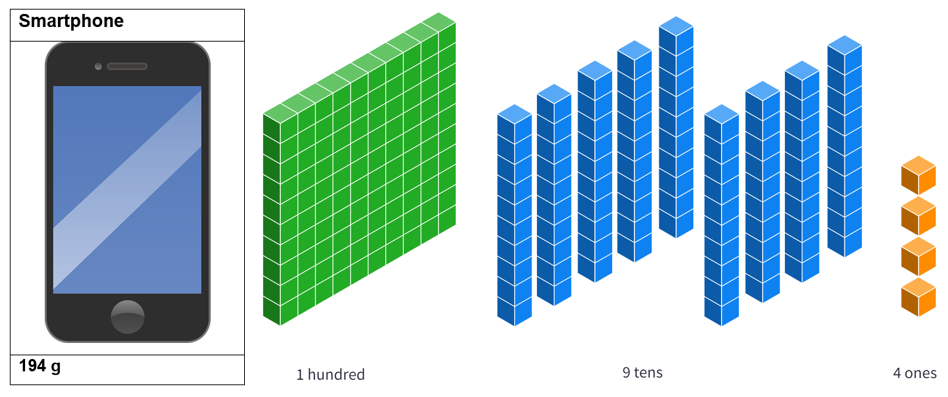


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

Figure 3 – representation of the weight of a textbook

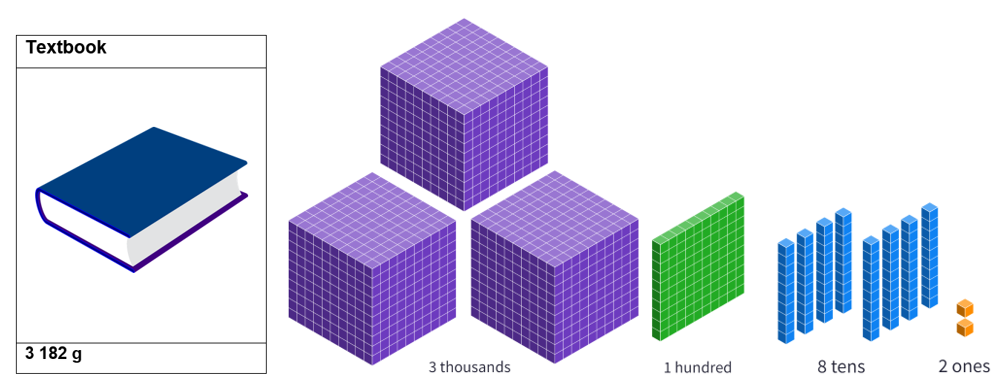


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Use this Polypad illustration ‘Renaming thousands’ ([bit.ly/PolypadRenameThousands](https://bit.ly/PolypadRenameThousands)) to demonstrate how to split thousands and rename the weight of the textbook to be 31 hundreds.
2. To demonstrate this, click on the cubes and press the **Split** button.

Figure 4 – demonstration of how to use the split feature in Polypad

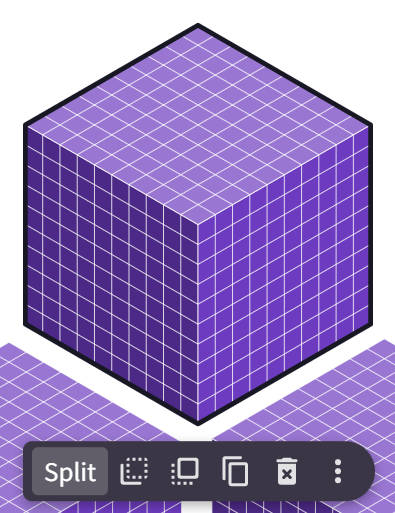


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

Teachers should emphasise the relationship between one place value and the next, describing how 10 of one makes the whole of another.

1. Students use either physical base 10 blocks or Polypad manipulatives to represent and rename the weights in the first 6 rows of the table in Appendix B ‘Reasoning when comparing integers’. Base 10 blocks can be found in Polypad under the heading **Number** and the subheading **Number Tiles and Cubes**.

Figure 5 – base 10 blocks in Polypad

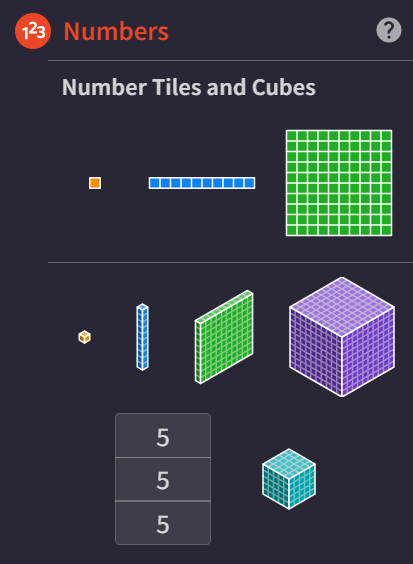


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

For students who are familiar with base 10 blocks and their application in representing the place value of integers, drawn diagrams can replace the use of physical blocks or devices in comparisons in step 6.

1. Lead a discussion with students around the limitations of this representation by focusing on the question ‘Why wouldn’t we use base 10 blocks to represent the weight of the adult person or the dog?’

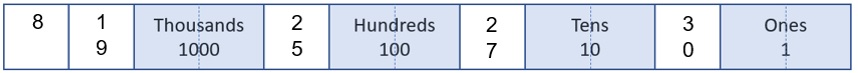
##### Number expanders

1. Using a copy of Appendix C ‘Number expander template’, demonstrate how, using the number expander, we can rename the measurements to compare the measurements as ‘81 thousands’ against ‘9 thousands’, as shown in Figure 6.

Appendix D ‘Number expander’ contains detailed instructions about how to use a number expander to rename and compare integers.

Figure 6 – number expanders from Appendix C ‘Number expander template’

A strip of paper divided into 10 sections. Going from left to right, the odd numbered sections contain numbers, 8 in the first one, 1 and a 9 directly beneath it in the third one, 2 and 5 underneath in the fifth one, 2 and 7 in the seventh and 3 and 0 in the ninth. 
The even numbered sections contain the place value descriptions. This includes ten thousands in the second section, thousands in the fourth section, hundreds in the sixth section, tens in the eighth section and ones in the tenth and final section. 



1. Have students write pairs of numbers on number expanders to complete the remaining rows of the table in Appendix B ‘Reasoning when comparing integers’.

### Launch

1. Show students Figure 7. Explain that packets of a particular brand of chips are labelled as containing 24 grams of food. This weight is never exact, and several bags have been weighed, with the results shown in the table.

Figure 7 – weights from 8 identical packets of chips

A bag of chips, with the weight of 24 grams shown on the label. 
There is then a table beneath it, showing the actual measured weight of eight bags of this brand, reading 24.1 grams, 24.12 grams, 24.2 grams, 24.036 grams, 24.3 grams, 24.24 grams, 24.31 grams and 24.127 grams. 

1. Have students engage in a ‘Think-Pair-Share’ ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to discuss which bag they would most like to have and why.

### Explore

1. Display Figure 8 to students. Inform students that we have now changed the ‘thousands’ cube to represent the number 1. Use a Pause-Pose-Pounce-Bounce question strategy ([PDF 200 KB] [bit.ly/pausepouncebouncestrategy](https://bit.ly/pausepouncebouncestrategy)) to gather student opinions about what each of the other solids now represent.

Figure 8 – base 10 blocks where the cube represents one whole

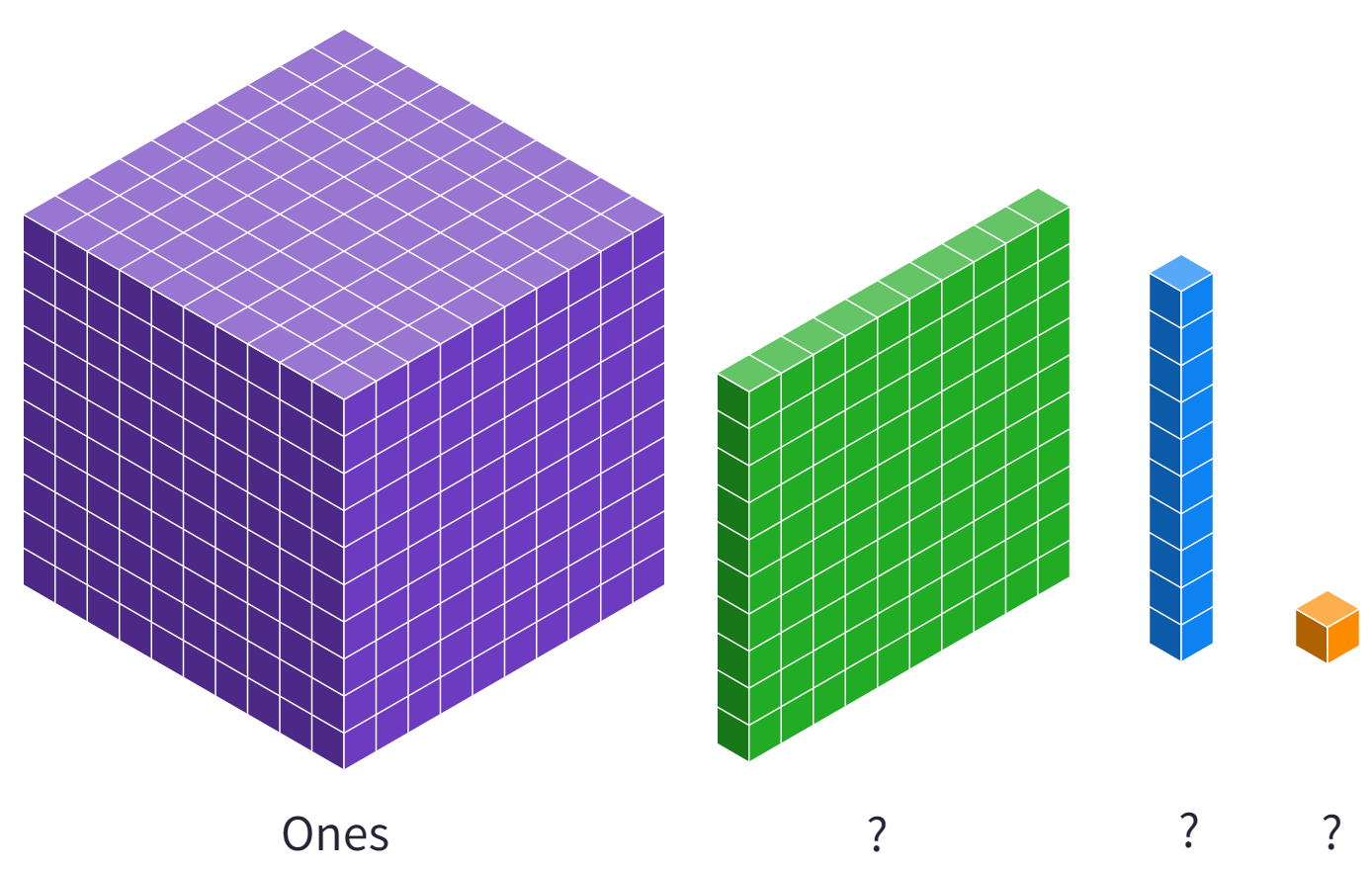


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Show students Figure 9 to clarify these definitions.

Figure 9 – base 10 blocks indicating the value on the smaller blocks

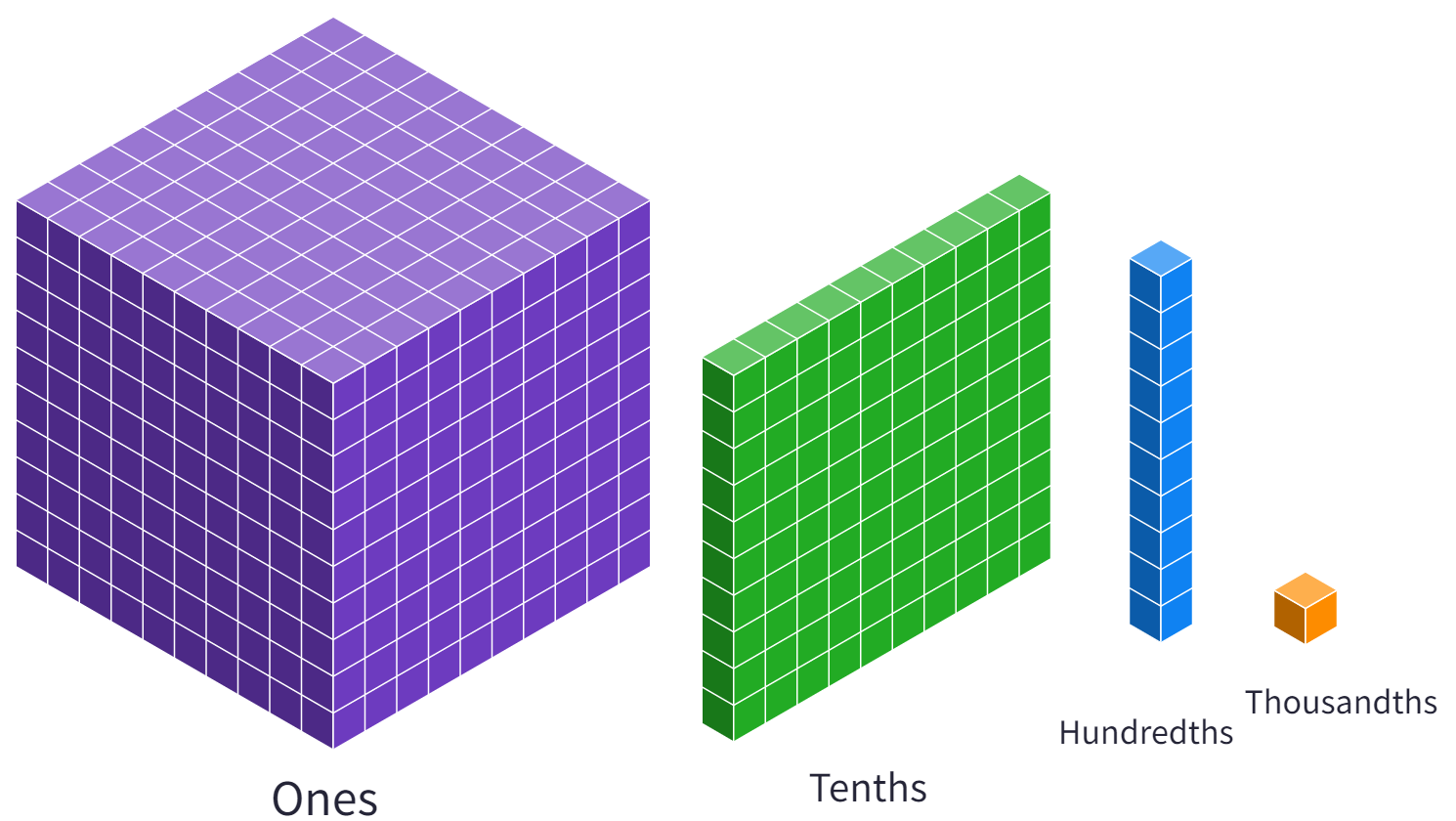


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Demonstrate to students, using Polypad ([mathigon.org/polypad](https://mathigon.org/polypad)), how to represent the number 1.1. Establish by ‘splitting’ the one, that this is the same as 11 tenths.
2. Using either physical base 10 blocks, a Polypad graph or the decimal number expander in Appendix E ‘Number expander template – decimals’, students are to use these definitions to represent and compare the numbers 2.3, 2.7, 2.1, 2.15, 2.72, 2.186 and 2.715. Appendix G ‘Saving your Polypad file’ demonstrates how students can save and share their work in Polypad.
3. Students should be challenged to give reasons for their comparisons.

### Summarise

The instructions below use an interactive Desmos graph to engage students with worked examples. Teachers can also use the *Comparing using place value* PowerPoint slides 7 to 10 for explicit teaching of the skills required to place hundredths on the number line.

1. Teachers are to use the explicit teaching technique ‘Your turn’.
2. Reveal the question to students and its solution via the Desmos graph animation *’*Placing hundredths on the number line’([bit.ly/DesmosHundredthsNL](https://bit.ly/DesmosHundredthsNL)), either on the teacher screen or sent to students to see on their own screen.
3. Students read in silence.
4. Students individually explain to themselves what happens in each step.
5. Teachers may wish to pause the animation after it has played at least once through. To pause, go to the left of screen and press the **pause** symbol near the **Play** slider, as shown below.

Figure 10 – ‘Pause’ symbol in Desmos

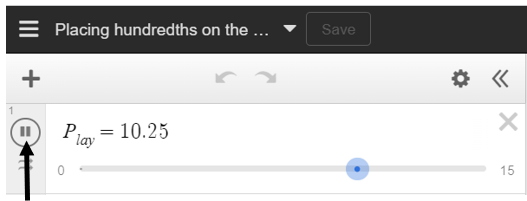


Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Students hold up a thumbs up to the teacher when they have finished reading and have some sort of understanding.
2. Students engage in a Think-Pair-Share. Students then explain the solution to their partner.
3. In pairs, students then answer the self-explanation questions. These can be turned on using the slider that appears at the bottom of the screen.

Figure 11 – ‘Turn on prompts’ slider from Desmos

A slider from Desmos labelled "Turn on prompts". 

Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Finally, randomly select students to share their answers with the whole class.
2. Students complete a ‘Your turn’ example. They will either need to draw their own number line or be handed a copy of Appendix F ‘Your turn – number lines. The example can be turned on in Desmos using the **Your turn** switch.

Figure 12 – ‘Move to ‘Your turn’’ slider from Desmos

A slider from Desmos labelled "move to your turn". 

Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Students are to order the weights of the chip packets in the launch. The decimal number expander in Appendix F ‘Your turn – number lines’, Polypad or base 10 blocks can be used to determine the order.
2. Students should also attempt to place each of the weights on the second number line in Appendix F ‘Your turn – number lines’.

### Apply

1. Have students attempt this ‘Open Middle’ problem ([bit.ly/OpenMiddleDec](https://bit.ly/OpenMiddleDec)), selecting where to place digits 0 to 9 in a scenario on the number line. Encourage students to consider how many solutions are possible.
2. Teachers can print off Appendix H ‘Open Middle problem' for students to complete this.
3. Alternatively, students can be given the Desmos Graph ‘Number line decimals: Open Middle’ ([bit.ly/DesmosOMDec](https://bit.ly/DesmosOMDec)).
4. To submit their work, students need to first ensure they have logged in, as shown in the top right corner of the screen.

Figure 13 – ‘sign-in menu’ from Desmos

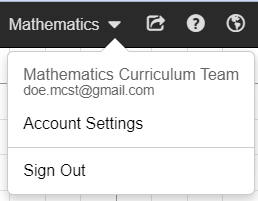


Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Students then click **save** in the top left corner of their screen.

Figure 14 – ‘save button’ in menu from Desmos

The menu from a Desmos graph at the top left of the screen, where the blue save button is clear. 

Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Students can then share the link found in the top right corner of the screen, as shown below.

Figure 15 – ‘share graph’ button in menu from Desmos

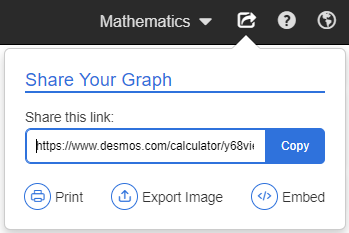


Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Have students also attempt the problem in Appendix I, ‘Decimals Venn’. Students are to determine decimals that would fit each of the categories describing sections A, B, C and D.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Warm up and Launch**

* Images and the use of common objects should give all students an opportunity to contribute to discussions about the order of the weights in the warm up activity.
* For students who demonstrate fluency with interpreting and operating with decimals, challenge them by modifying the table to include weights below the advertised 24 grams. Ask students to represent each weight with a positive or negative number, based on its distance from the desired weight.

**Explore**

* Challenge students who are excelling at using representations with base 10 blocks to consider the implications of using any of the other blocks as the ‘ones’.
* Challenge students to produce individual comparisons between different weights, giving reasons for their final solution.

**Apply**

* The Open Middle problem in Appendix H ‘Open Middle problem’ supports all students to participate and submit their work.
* For students who find the Venn diagram activity difficult to interpret, the teacher may wish to introduce the concept using a simpler example from the Maths Venns website ([bit.ly/MathsVennEasier](https://bit.ly/MathsVennEasier)).
* Students can be challenged by engaging with a more complex Venn diagram from the Maths Venns website ([bit.ly/DecimalVenn2](https://bit.ly/DecimalVenn2)).

### Suggested opportunities for assessment

**Warm up and Launch**

* This activity is an opportunity for teachers to assess students’ ability to represent integers using familiar objects. This should be monitored to determine whether students are ready to extend beyond integers.

**Summarise and Apply**

* Have students submit their solution to the problem in the launch, their representations of the numbers, including Appendix F ‘Your turn – number lines’ and an explanation involving the renaming of decimals by comparing their size.
* Student solutions to the Open Middle problem in Appendix H ‘Open middle problem’ gives evidence as to their ability to interpret the impact of different place values on the intervals between decimals.

## **Appendix A**

### **Weights of object cards**

|  |  |  |
| --- | --- | --- |
| **Smart phone** | **Pencil** | **Dog** |
| **An image of a smart phone.** | An image of a pencil. | An image of a dog. |
| **194 g** | **16 g** | **31 613 g** |
| **Adult person** | **Monitor** | **Textbook** |
| An image of an average sized man. | An image of a monitor. | **An image of a textbook.** |
| **81 223 g** | **9 570 g** | **3 182 g** |

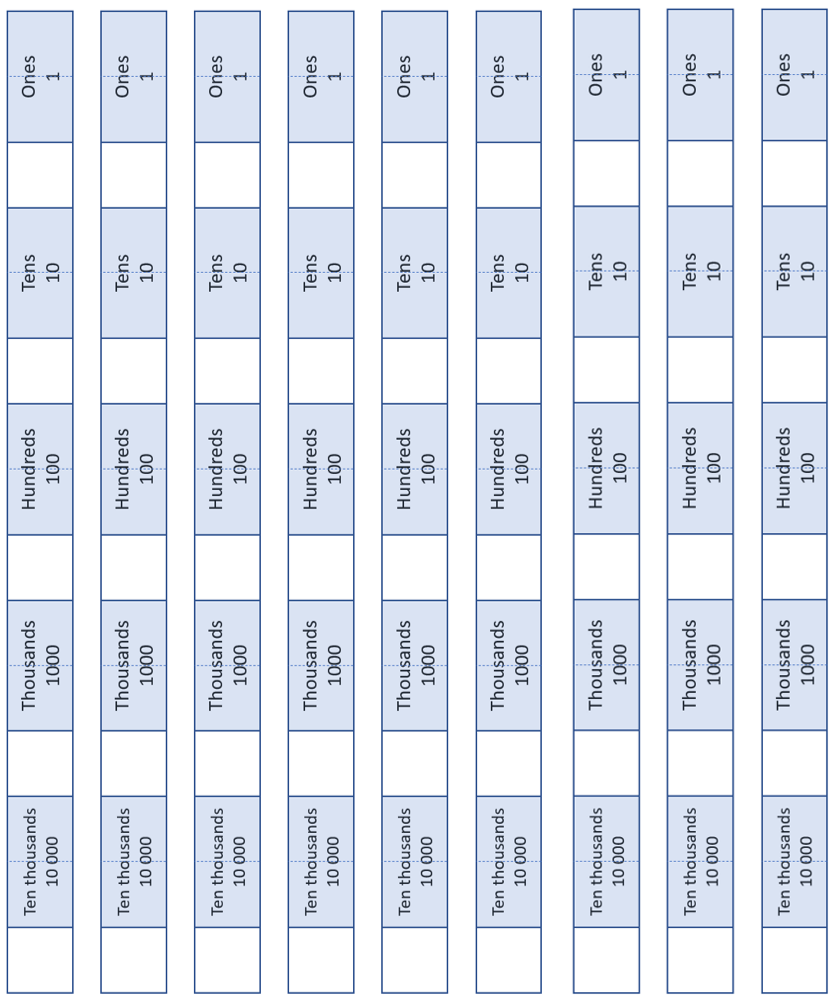
## ****Appendix B****

### Reasoning when comparing integers

|  |  |  |
| --- | --- | --- |
| First object | Second object | Compare with reasoning |
| Smart phone 194 g | Textbook 3 182 g | The textbook is heavier than the smart phone because 31 hundred grams is heavier than 1 hundred grams. |
| Smart phone 194 g | Monitor 9 570 g |  |
| Smart phone 194 g | Pencil 16 g |  |
| Pencil 16 g | Monitor 9 570 g |  |
| Pencil 16 g | Textbook 3 182 g |  |
| Monitor 9 570 g | Textbook 3 182 g |  |
| Adult person 81 223 g | Monitor 9 570 g | The adult person is heavier than the monitor because 81 thousand grams is heavier than 9 thousand grams. |
| Adult person 81 223 g | Textbook 3182 g |  |
| Dog 31 613 g | Adult person 81 223 g |  |
| Dog 31 613 g | Monitor 9 570 g |  |
| Dog 31 613 g | Textbook 3 182 g |  |
| Smart phone 194 g | Dog 31 613 g |  |
| Smart phone 194 g | Adult person 81 223 g |  |
| Pencil 16 g | Dog 31 613 g |  |
| Pencil 16 g | Adult person 81 223 g |  |

Appendix C

### Number expander template



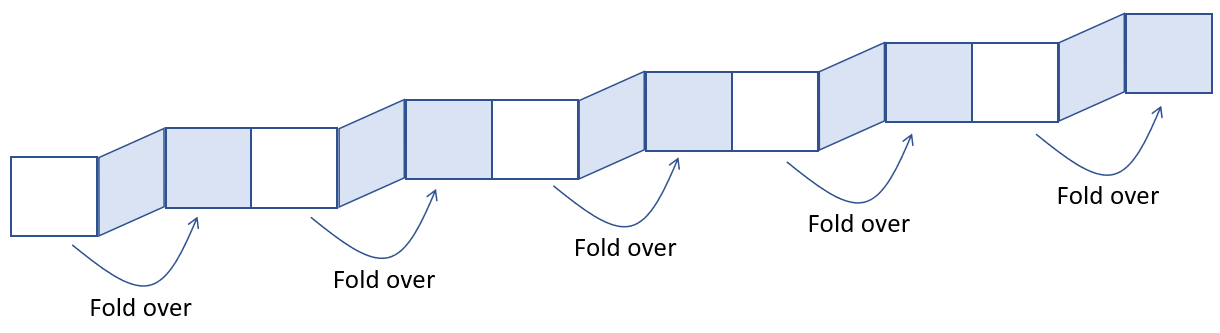
## Appendix D

### Number expander

The first 57 seconds of the [Number expander video](https://www.youtube.com/watch?v=ycGqL5_VLe0) (1:13) on Youtube.com shows how to use the number expander for integer values up to tens of thousands.

Students cut out and fold the number expander so that the white sections are folded over the blue section to hide them from view (for now).

A strip of paper divided into 10 sections. Going from left to right along the paper, every second section contains a place value description. This includes ten thousands in the second section, thousands in the fourth section, hundreds in the sixth section, tens in the eighth section and ones in the tenth and final section. 

There are arrows showing where to fold to make a number expander. 

### Comparing integers using a number expander

To compare 4 725 and 4 782, students write the digits of 4 725 so that they align with the correct place value columns. That is, write 5 in the ones column, 2 in the tens column, 7 in the hundreds column and 4 in the thousands column. Write these high in the spaces, shown below, leaving enough room for other digits below.

A strip of paper divided into 10 sections. Going from left to right, the odd numbered sections contain numbers, nothing in the first one, 4 in the third one, 7 in the fifth one, 2 in the seventh and 5 in the ninth.
 
The even numbered sections contain the place value descriptions. This includes ten thousands in the second section, thousands in the fourth section, hundreds in the sixth section, tens in the eighth section and ones in the tenth and final section. 

In the same way, write the digits of 4 782 underneath.

A strip of paper divided into 10 sections. Going from left to right, the odd numbered sections contain numbers, nothing in the first one, 4 and a 4 directly beneath it in the third one, 7 and 7 underneath in the fifth one, 2 and 8 in the seventh and 5 and 2 in the ninth. 

The even numbered sections contain the place value descriptions. This includes ten thousands in the second section, thousands in the fourth section, hundreds in the sixth section, tens in the eighth section and ones in the tenth and final section. 

Compare the integers by comparing the digits starting with the most significant (largest) place value column. In this case, the thousands column.

In this scenario, each integer has an equal number of thousands, 4 in each.

This means the digits in the next place value column need to be compared. To help this, collapse the number expander to hide the thousands label, as shown below.



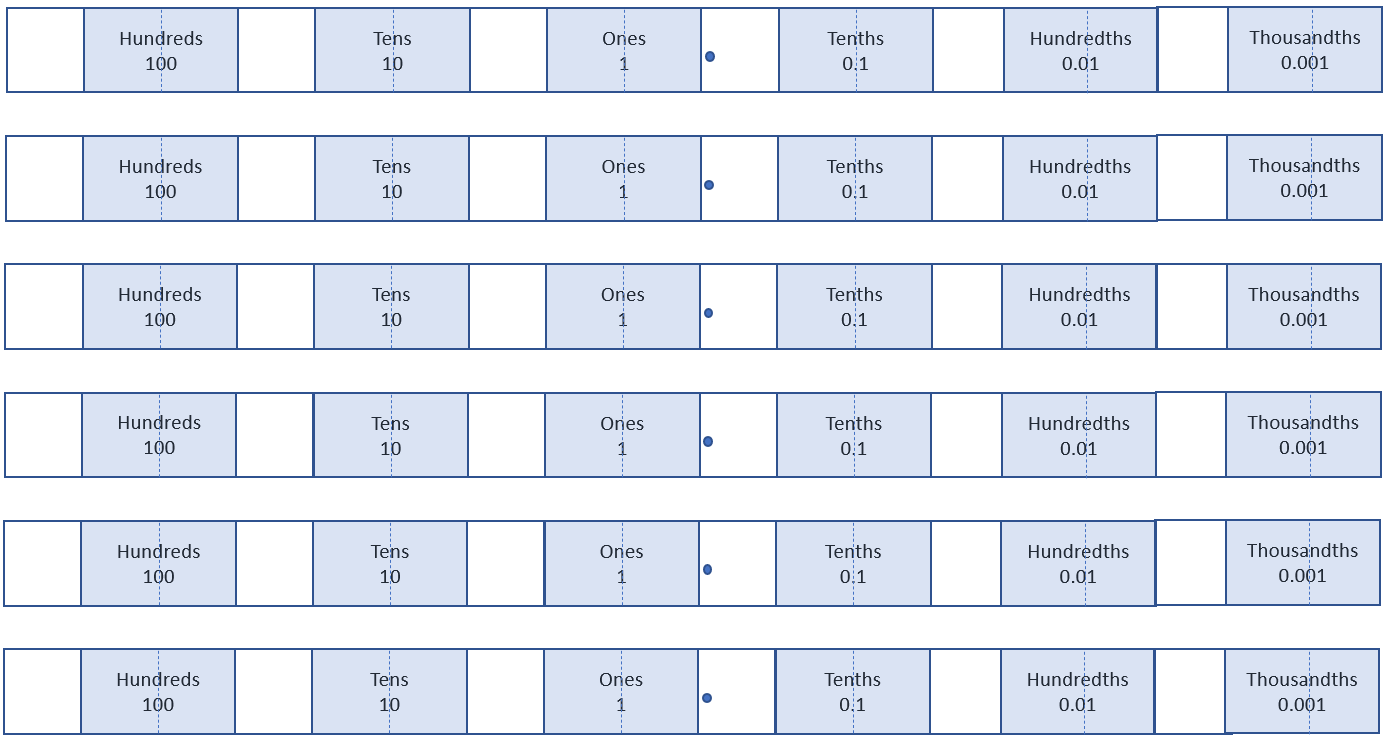
This regroups the integers to show the number of hundreds. In this case, 47 hundreds in each. Once again, the integers cannot be identified as different. Therefore, collapse the number expander to hide the hundreds label, as shown below.



This regroups the integers, once again, to show the number of tens. In this case, 472 and 478 tens. It is at this point, the integers can be identified as different. 4 782 is bigger than 4 725 as it contains 478 tens compared to 472 tens.

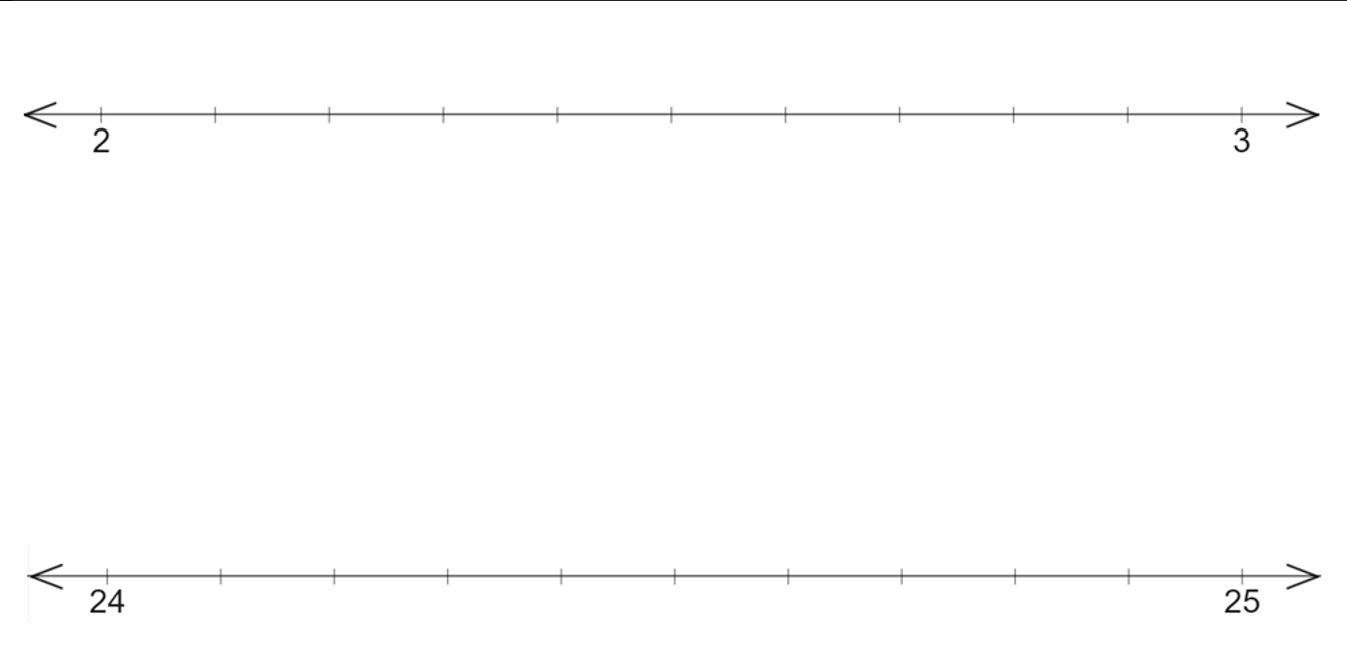
## Appendix E

### Number expander template – decimals



## **Appendix F**

### Your turn – number lines



## Appendix G

### Saving your Polypad file

1. Students should ensure they are signed in using their NSW Department of Education Gmail account, ending in ‘@education.nsw.gov.au’.

The task bar in the Polypad website, with the profile in the top right corner highlighted to show that if the letter of your first name appears, you are logged in. 

Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Click on the **file icon** in the top left of the screen.



Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Click on the words **Untitled Polypad** to give the graph a title.

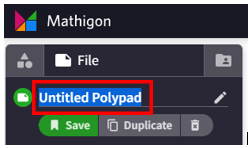


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

1. Click **Save**. A link will be generated that can be shared with the teacher.

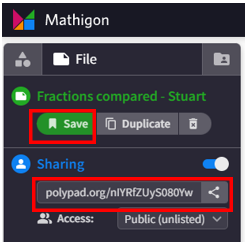
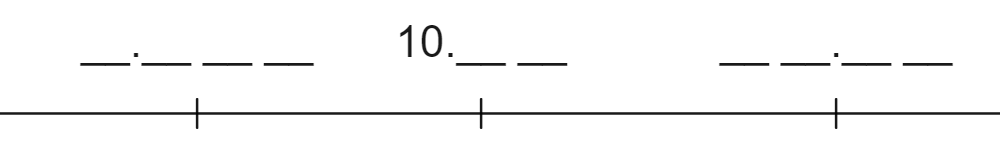


Image created using the free virtual manipulatives at [Polypad.org](https://mathigon.org/polypad/).

## Appendix H

### Open Middle problem

The number line below has 3 locations represented.



Most of the digits within each number are missing. There are 10 spots on the diagram waiting for a digit.

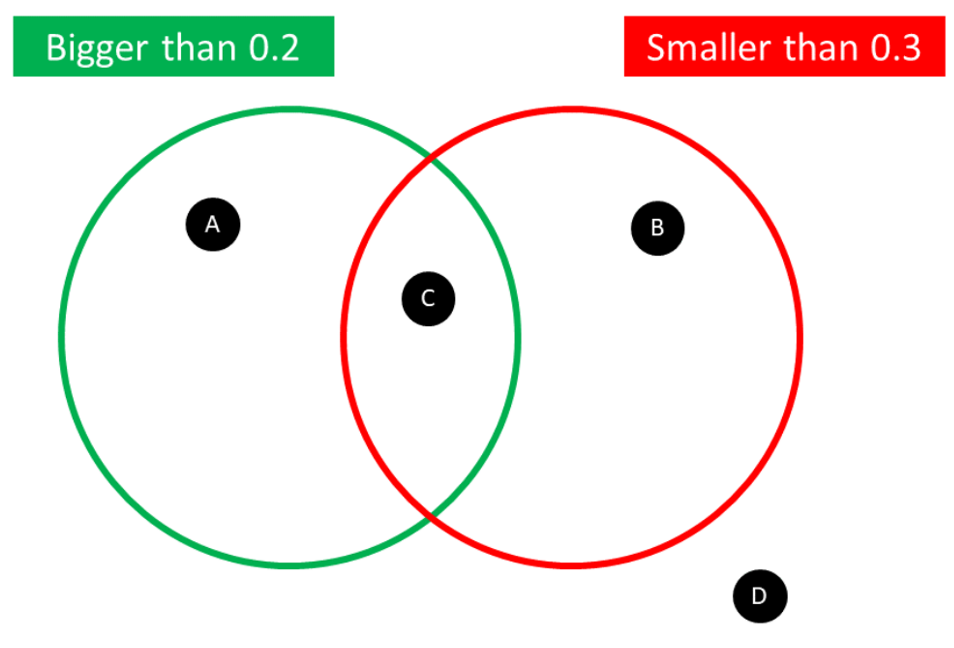
Use each of the digits 0 to 9 exactly once to fill the blank spaces.

Use the space below to explain why you’ve chosen to place the numbers where you have.

## Appendix I

### Decimals Venn

Think of a decimal that could belong in each region, A, B, C and D. If you think a region is impossible to fill, convince me with reasons why!



## ****Sample solutions****

### Appendix A

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Adult person** | **Dog** | **Monitor** | **Textbook** | **Smart phone** | **Pencil** |
| An average sized man. | An average sized dog. | A omputer monitor. | **A textbook.** | **A smart phone.** | A pencil. |
| **81 223 g** | **31 613 g** | **9 570 g** | **3 182 g** | **194 g** | **16 g** |

### Appendix B

|  |  |  |
| --- | --- | --- |
| First object | Second object | Compare with reasoning |
| Smart phone 194 g | Textbook 3 182 g | The textbook is heavier than the smart phone, because 31 hundred grams is heavier than 1 hundred grams. |
| Smart phone 194 g | Monitor 9 570 g | The monitor is heavier than the smart phone, because 95 hundred grams is heavier than 1 hundred grams. |
| Smart phone 194 g | Pencil 16 g | The smart phone is heavier than the pencil, because 19 tens of grams is heavier than 1 ten of grams. |
| Pencil 16 g | Monitor 9 570 g | The monitor is heavier than the pencil, because 957 tens of grams is heavier than 1 ten of grams. |
| Pencil 16 g | Textbook 3 182 g | The textbook is heavier than the pencil, because 318 tens of grams is heavier than 1 ten of grams. |
| Monitor 9 570 g | Textbook 3 182 g | The monitor is heavier than the textbook, because 9 thousand grams is heavier than 3 thousand grams. |
| Adult person 81 223 g | Monitor 9 570 g | The adult is heavier than the monitor, because 81 thousand grams is heavier than 9 thousand grams. |
| Adult person 81 223 g | Textbook 3182 g | The adult is heavier than the textbook, because 81 thousand grams is heavier than 3 thousand grams. |
| Dog 31 613 g | Adult person 81 223 g | The adult is heavier than the dog, because 81 thousand grams is heavier than 30 thousand grams. |
| Dog 31 613 g | Monitor 9 570 g | The dog is heavier than the monitor, because 31 thousand grams is heavier than 9 thousand grams. |
| Dog 31 613 g | Textbook 3 182 g | The dog is heavier than the textbook, because 31 thousand grams is heavier than 3 thousand grams. |
| Smart phone 194 g | Dog 31 613 g | The dog is heavier than the smart phone, because 316 hundred grams is heavier than 1 hundred grams. |
| Smart phone 194 g | Adult person 81 223 g | The adult is heavier than the smart phone, because 812 hundred grams is heavier than 1 hundred grams. |
| Pencil 16 g | Dog 31 613 g | The dog is heavier than the pencil, because 3 161 tens of grams is heavier than 1 ten of grams. |
| Pencil 16 g | Adult person 81 223 g | The adult is heavier than the pencil, because 8 122 tens of grams is heavier than 1 ten of grams. |

## References

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