# Factorising non-monics

Students practise finding the product, sum and factors of non-monic quadratics before exploring different methods to factorise them.

## Visible learning

This lesson incorporates Path content.

Reveal the learning intention and success criteria at the end of the Launch activity.

### Learning intention

* To be able to factorise non-monic quadratic trinomials.

### Success criteria

* I can find the product of the constant term and leading coefficient and determine the factors that equate to the coefficient of the term.
* I can represent a non-monic quadratic trinomial expression in factorised form.
* I can explain how to factorise a non-monic quadratic trinomial expression using my preferred method.

Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| ****Section**** | ****Summary of activity**** | ****Teaching strategy**** | ****Teaching points**** |
| ****Launch**** | Students are asked to factorise and then These are on slides 3–6 of the PowerPoint Factorising non-monics. | Visibly random groups of 3  Vertical non-permanent surfaces  Pose-Pause-Pounce-Bounce | Students realise that they need a new strategy to factorise non-monic quadratic trinomials. |
| ****Explore**** | Students expand brackets ([Appendix A](#_Appendix_A)). They are introduced to, and then practise finding, the product, sum and factors (PSF) for various non-monic quadratics (slides 7–10 of the PowerPoint Factorising non-monics). Students then work in groups, discovering different methods of factorising in [Appendix B](#_Appendix_B) and then, for some students, in [Appendix C](#_Appendix_C). | Worked examples (Your turn)  Mini whiteboards  Think-Pair-Share  Notice and wonder  Visibly random groups of 3  Vertical non-permanent surfaces | Students are exposed to a variety of different methods to factorise non-monic trinomials. |
| ****Summarise**** | Students can vote and speak for and against different strategies before settling on which one works best for them. | Notes to future forgetful selves | Students choose the method that they can use with speed and accuracy. |
| ****Apply**** | Students practice, using [Appendix D](#_Appendix_D), their chosen method of factorisation. |  | Students practise and become efficient using their method. |

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* selects and applies appropriate algebraic techniques to operate with algebraic fractions, and expands, factorises and simplifies algebraic expressions **MA5-ALG-P-02** (Path)

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-k-10-2022/overview) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

Six different strategies have been provided for teachers to consider. Students moving into the calculus-based courses in Mathematics Stage 6 will need to be efficient at factorising non-monic quadratic trinomials. This lesson is an opportunity for students to experience different methods and choose one that they feel comfortable using as they move into Stage 6.

The cross method does not rely on students finding PSF but it is slow if the leading coefficient has multiple factors.

The Lyszkowski/Howell method is very efficient with limited room for small mathematical errors.

## Activity structure

Please use the associated PowerPoint Factorising non-monics (FN PPT) to display images in this lesson.

### Launch

1. Assign students to visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).
2. Ask groups to factorise . This can be displayed using slide 3 of the PowerPoint (FN PPT).

Click on the slide to reveal the answer. A sample solution using an area model has been provided on slide 4 of the PowerPoint (FN PPT).

1. Display slide 5 of the PowerPoint (FN PPT) and remind students that a non-monic quadratic trinomial has a leading coefficient
2. Ask students to factorise in their groups.

Click on the slide to reveal the answer.

Students may notice that this expression is double the last expression. All the terms share a common factor of 2 which we could take out to create a monic quadratic expression.

1. Display slide 6 of the PowerPoint (FN PPT) which displays the quadratic trinomial and again, ask groups to factorise the expression.

Students will struggle to factorise this as they don’t yet have the skills to do so.

1. Use the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to ask students what strategies they tried to use and when they realised that wouldn’t work.
2. Reveal the lesson learning intention and success criteria.

### Explore

1. Display slide 8 of the PowerPoint (FN PPT), which displays an attempt to factorise the quadratic trinomial using an area model.
2. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students why this is difficult to factorise.

Students should recognise that since the quadratic trinomial is non-monic, that there are more than 2 factors for and this affects the terms that create .

1. State to students that there are other methods we can use when factorising non-monic quadratic expressions.
2. Distribute Appendix A ‘Product, sum, factors (PSF)’ to each student and have them complete the expanded form and quadratic expression columns only. Students can compare their solutions at any time with a peer.
3. Use slide 9–12 of the PowerPoint (FN PPT) to explicitly teach PSF for quadratic expressions using the Worked examples (Your turn) method ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
4. Have students complete the PSF columns in Appendix A. Students may like to use mini whiteboards ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)) to find different sets of factors.
5. After completing the table, in a Think-Pair-Share, students are to share what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) about the table.

Students may notice that the factors are the middle 2 terms in the expanded expression and wonder how this can help them to factorise the quadratic equation.

1. Tell students that there are many methods of factorising non-monic quadratic trinomials. Today we are going to explore 3 of these methods.

Teachers are to choose as many or as few methods as they feel necessary. There are 6 in total. Three have been chosen based on ease of understanding and speed of execution.

1. With students back in their groups of 3 at vertical non-permanent surfaces, distribute a different method of factorising to each group from Appendix B ‘Strategies for factorising’.

Appendix C ‘Alternate methods’ contains the 3 remaining methods which teachers may choose to use.

1. Have students unpack the method of factorising and attempt the 2 questions at the bottom using the method described.
2. While students are unpacking the methods, ask them questions to drive their thinking. Some suggestions include:

* Does it matter what order I put the and , when separating the term , when grouping in pairs?
* Does it matter what order I put the and , when placing them in the binomial product in the numerator, when using the Lyszkowski/Howell method?
* How efficient do you think the cross method is?

1. Rotate groups to the next method. Students may benefit from the previous group explaining the method to the next group or from observing the solutions before attempting them for themselves.
2. Repeat this process until all groups have been exposed to each method.

### Summarise

1. Bring the class back together, leaving one example of each method displayed in various places around the room.
2. Allocate a sticky note to each student.
3. Ask students to consider the different methods they have learned and to place a sticky note at the method they like the best.
4. After voting, visit each station and have students speak for and against each method.

Students should choose a method they can use with speed and accuracy.

1. Have students write notes to their future forgetful selves ([bit.ly/notestofutureself](https://bit.ly/notestofutureself)) outlining their preferred method of factorising.

### Apply

Distribute Appendix D ‘Practice questions’ to each student. Students are to complete the questions using their preferred method.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students may need to be reminded to take out a common factor before factorising.

**Explore**

* Support students by providing a list of factors to assist in finding the PSF or use the area model.
* Teachers should be mindful of which strategy groups are given initially and may restrict the number of strategies for some students.
* Extend students by encouraging them to explain why a strategy works or reinforce that it also works with monic quadratic trinomials.
* Students may practice more questions using the quadratic equations from Appendix A.

**Summarise**

* Students can choose their preferred method of factorising, allowing for differentiation between students.
* The syllabus recommends grouping in pairs as a method for non-monic quadratic equations. This may be a suggested solution for students who are having difficulty choosing a method.

**Apply**

* Students could be provided with the PSF for Appendix C to support their progress.

### Suggested opportunities for assessment

**Explore**

* Monitor responses in class discussions to check for student understanding of different methods of factorising.
* When placed in groups of 3, students provide and receive peer feedback on their understanding.

**Summarise**

* Teachers could review students’ notes to their future forgetful selves to check for understanding.

**Apply**

* Teachers could create an exit ticket, collecting an example from Appendix C for review.

## Appendix A

### Product, sum, factors (PSF)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Factorised form | Expanded form | Quadratic expression | Product | Sum | Factors |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

## Appendix B

### Strategies for factorising

#### Grouping in pairs

Factorise

|  |  |
| --- | --- |
| **P** | −24 |
| **S** | −5 |
| **F** | 3, −8 |

Split the middle term into 2 factors

Group in pairs by collecting like terms

Factorise each pair

Factorise using as a common factor

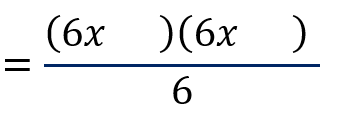
##### Practice questions

#### Lyszkowski/Howell method

Factorise

|  |  |
| --- | --- |
| **P** | −24 |
| **S** | −5 |
| **F** | 3, −8 |

Create 2 brackets with the first term and divide both brackets by 6



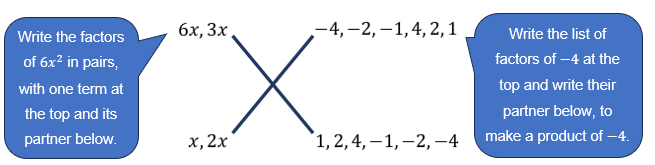
Add factors into the brackets

Factorise and simplify

##### Practice questions

#### Cross method

Factorise



Test the combinations by multiplying diagonally and adding the 2 terms to see if the sum equals the middle term of the quadratic:

A table with 12 cells, 3 rows of 4. Each cell has a cross in it. 

On the right of each cross are the pairs of factors of 6x squared. On the right of the cross are the factors of negative 4.

Below each cross is an equation generated from multiplying the terms diagonally and adding them.

It’s not necessary to draw a cross for each of the combinations.

You can stop testing the combinations when you’ve found the right one.

A cross with 3x at top left, -4, at top right, 2x at bottom left, 1 at bottom right. 
A red, horizontal oval is encircling the top two terms. another red, horizontal oval is encircling the bottom 2 terms.
An equation below the cross says 6x squared minus 5xminus 4 equals, in brackets, 3x minus 4, times, in brackets, 2x+1.

##### Practice questions

## Appendix C

### Alternate methods

#### Magic X

Factorise

|  |  |
| --- | --- |
| **P** | −24 |
| **S** | −5 |
| **F** | 3, −8 |

Three crosses. From the top, clockwise, they read:
Cross 1: −24, 3, −5, −8.
Cross 2: −24, 3/6x, −5, −8/6x.
Cross 3: −24, 1/2x, −5 and −4/3x.

Three speech bubbles with the following instructions:
Place the product, sum and factors into a cross.
Divide the factors by 6x.
Simplify the fractions and create the brackets using the denominator as the first term and the numerator as the second.


##### Practice questions

#### Area model

Factorise .

|  |  |
| --- | --- |
| **P** | −24 |
| **S** | −5 |
| **F** | 3, −8 |

Three boxes 2 x 2.
Box 1 top row, 6x^2, empty square. Bottom row empty square, 
Box 2 top row 6x^2, 3x and bottom row, -8x, -4
Box three is the same as box 2. Above the box is 2x + 1 and down the left side of the box 3x-4. The equation =(3x-4)(2x+1)

Three speech bubbles with the following instructions:
Place the leading term and constant term in a grid.
Place the factors to complete the grid.
Factorise pairs across and down.

##### Practice questions

#### Non-monic to monic

Factorise

|  |  |
| --- | --- |
| Process | Example |
| Make the expression an equation | Let |
| Multiply by 6 |  |
| Expand the brackets |  |
| Factor from all possible terms |  |
| Replace with | Let |
| Factorise the monic quadratic |  |
| Replace with |  |
| Factorise out the common factor in each binomial |  |
| Divide by 6 |  |

##### Practice questions

## Appendix D

### Practice questions

|  |  |
| --- | --- |
| Question | Expression |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

## Sample solutions

### Appendix A – product, sum, factors (PSF)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Factorised form | Expanded form | Quadratic expression | Product | Sum | Factors |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

### Appendix B – strategies for factorising

|  |  |  |
| --- | --- | --- |
| Method |  |  |
| Grouping in pairs |  |  |
| Lyszkowski/Howell |  |  |

|  |  |  |
| --- | --- | --- |
| Method |  |  |
| **Cross method** | xbig cross. Top left 3x, top right -4,-2-1,4,2,1 Bottom left and bottom right 1,2,4,-1,-2,-4  A set of 5 cross products, 3 in a row, then 2 in the next row. On the left of each cross are the pairs of factors of 3x squared. On the right of the cross are the factors of negative 4. Below each cross is an equation generated from multiplying the terms diagonally and adding them.  Cross with 3x + 2 circled in red at top and x -2 circled in red at bottom (3x+2)(x-2). | A blue x with 3x at top left, 2,1,-2,-1 at top right. x at bottom left and 1,2,-1,-2 at bottom right.  A of 4 cross product solutions, 2 rows of 2. On the left of each cross are the pairs of factors of 3x squared. On the right of the cross are the factors of 2.  Cross with 3x -1 circled in red at top and x -2 circled in red at bottom (3x-1)(x-2) |

### Appendix C – alternate methods

|  |  |  |
| --- | --- | --- |
| Method |  |  |
| Magic X | 3 crosses. Cross 1 from top, clockwise -12,2,4, -6 Cross 2 from top clockwise, -12,2/3x,-4,-6.3x. Cross 3 from top, clockwise, -12,2/3x, -4 and -2/x. | Cross 1 from top, clockwise 6,-6,-7,-1 Cross 2 from top clockwise, 6,-6/3x,-7,-1/3x Cross 3 from top, clockwise, -6,-2/x,-7,-1/3x. |
| Area model | Three boxes 2 x 2. Box 1 top row, 3x^2, empty square. Bottom row empty square, -4 Box 2 top row 3x^2, 3x and bottom row, -6x, -4. Box 3, same as box 2 but above the top row is 3x + 2 and down the left is x - 2 | Three boxes 2 x 2. Box 1 top row, 3x^2, empty square. Bottom row empty square, 2 Box 2 top row 3x^2, -x and bottom row, -6x, +2. Box 3, same as box 2 but above the top row is 3x -1and down the left is x - 2 |

|  |  |  |
| --- | --- | --- |
| Method |  |  |
| Non monic to monic |  |  |

### Appendix D – practice questions

|  |  |  |  |
| --- | --- | --- | --- |
| Question | Expression | Factorised expression | PSF |
| 1 |  |  | P = , S =, F = |
| 2 |  |  | P =, S = , F = |
| 3 |  |  | P =, S = , F = |
| 4 |  |  | P =, S = , F = |
| 5 |  |  | P =, S = , F = |
| 6 |  |  | P =, S = , F = |
| 7 |  |  | P =, S = , F = |
| 8 |  |  | P =, S = , F = |
| 9 |  |  | P =, S = , F = |
| 10 |  |  | P =, S = , F = |

## References

This resource contains NSW Curriculum and syllabus content. The NSW Curriculum is developed by the NSW Education Standards Authority. This content is prepared by NESA for and on behalf of the Crown in right of the State of New South Wales. The material is protected by Crown copyright.

Please refer to the NESA Copyright Disclaimer for more information <https://educationstandards.nsw.edu.au/wps/portal/nesa/mini-footer/copyright>.

NESA holds the only official and up-to-date versions of the NSW Curriculum and syllabus documents. Please visit the NSW Education Standards Authority (NESA) website <https://educationstandards.nsw.edu.au> and the NSW Curriculum website <https://curriculum.nsw.edu.au>.

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-k-10-2022/overview) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

**© State of New South Wales (Department of Education), 2024**

The copyright material published in this resource is subject to the Copyright Act 1968 (Cth) and is owned by the NSW Department of Education or, where indicated, by a party other than the NSW Department of Education (third-party material).

Copyright material available in this resource and owned by the NSW Department of Education is licensed under a [Creative Commons Attribution 4.0 International (CC BY 4.0) license](https://creativecommons.org/licenses/by/4.0/).

[](https://creativecommons.org/licenses/by/4.0/)

This license allows you to share and adapt the material for any purpose, even commercially.

Attribution should be given to © State of New South Wales (Department of Education), 2024.

Material in this resource not available under a Creative Commons license:

* the NSW Department of Education logo, other logos and trademark-protected material
* material owned by a third party that has been reproduced with permission. You will need to obtain permission from the third party to reuse its material.

**Links to third-party material and websites**

Please note that the provided (reading/viewing material/list/links/texts) are a suggestion only and implies no endorsement, by the New South Wales Department of Education, of any author, publisher, or book title. School principals and teachers are best placed to assess the suitability of resources that would complement the curriculum and reflect the needs and interests of their students.

If you use the links provided in this document to access a third-party's website, you acknowledge that the terms of use, including licence terms set out on the third-party's website apply to the use which may be made of the materials on that third-party website or where permitted by the Copyright Act 1968 (Cth). The department accepts no responsibility for content on third-party websites.